



STRATHMORE UNIVERSITY

JAN-APRIL 2020: END OF SEMESTER EXAMINATION

STA 8101: STOCHASTIC PROCESSES/ STOCHASTIC MODELLING

Date: Wednesday, 2nd September, 2020. Time: 3 hours Open-Book Exam

Instruction: Answer Question one and any other two

Question 1

1. Suppose that $\{X_n : n \geq 1\}$ is a random walk with $X_0 = 0$ and probability p of a step to the right, find:
 - a. $P\{X_5 = -1, X_8 = 2, \}$
 - b. $P\{X_5 = 1, X_{10} = 4, X_{16} = 2\}$

(2 marks)

2. A telephone attendant receives 110 calls during the busy hour. Each call takes, on average, 30 seconds to process.
 - a) What percentage of the attendant's time is devoted to answering calls?
 - b) How long must people wait, on average, before their call is processed?
 - c) How many people are in the queue, on average?

(3 marks)

3. Find the 3-transition matrix of the following i.e. M^3 [If you use R, ensure you attach R scripts]

$$M = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.5 \\ 0.5 & 0.2 & 0.3 \end{pmatrix}$$

(5 marks)

4. For a semi-Markov process, describe the two possible scenarios of $U_{ij}(t)$. Where, $U_{ij}(t) = P\{\text{system in state } j \text{ at time } t | X_0 = i\}$ and that $U_{ij}(t)$ is a function of external time

(4 marks)

5. Two gamblers, Tom and Jerry make a series of \$5 wagers where Jerry has .65 chance of winning and Tom has a .35 chance of winning on each wager. What is the probability that Jerry wins \$50 before Tom wins \$25?

(4 marks)

6. Consider, a branching process $N(t)$ with independent discrete random variables X_1, X_2, \dots, X_N , each with range $\{0, 1, 2, \dots\}$. Suppose $N \sim G_0(p)$ and each $X_i \sim \text{Binomial}(1, \theta)$ (independent). Find the distribution of $Z = \sum_{i=1}^n x_i$

(6 marks)

7. Suppose that $\{X_n : n \geq 1\}$ is a random walk with $X_0 = 0$ and probability p of a step to the right. Show that:

- a. If $m \leq n$, then $\text{Cov}(X_m, X_n) = m4p(1 - p)$.
- b. If $m \leq n$, then $\text{Var}(X_n - X_m) = (n - m)4p(1 - p)$.
- c. $\text{Var}(aX_n + bX_m) = (a + b) 2n4p(1 - p) + b2(m - n)4p(1 - p)$

(6 marks)

Question 2

Distinguish the following terms as used in Markov processes and provide practical examples

- a. Recurrent and transient states
- b. Periodicity and aperiodic
- c. Communicating class and absorption state
- d. Semi-Markov and Hidden Markov Chain

(15 marks)

Question 3

- a. Derive the Poisson process using Chapman-Kolmogorov Postulates
- b. Show that Galton Watson Process is a Martingale

(15 marks)

Question 4

Given the HIV2 dataset with id: Patient identification number, state.h: Starting state (1 for optimal, 2 for suboptimal and 3 for unacceptable control state), state.j: Arrival state (1 for optimal, 2 for suboptimal and 3 for unacceptable control state), time: Waiting (sojourn) time in state state.h, Opportunistic_Infection: Presence of OI (1=present, 0=none), Regimen_Naive: Patients with protein-related drug naive (1=Yes, 0=No), DCM: Patient on differentiated care (1=Yes, 0=No). The data implies that, for a given patient, the visited states are the sequence of state.h and the follow-up time is the cumulated sum of time.

Using R, compute and plot

- i. Hazard rates of waiting time
- ii. Hazard rates of the semi-Markov process
- iii. Fit semi-Markov model with a covariate " Opportunistic_Infection "
- iv. Fit semi-Markov model with two covariates Rgimen_Naive and DCM

(15 marks)

Question 5

- i. Consider the Markov chain with state space $E = \{1, 2, 3\}$ and transition matrix

$$P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.4 & 0.4 & 0.2 \\ 0 & 0 & 1 \end{pmatrix}$$

- a) Using R program, draw a well labeled transitional plot of the Markov chain P

(3 marks)

- b) Identify the recurrent states, transient states and the absorbing states

(3 marks)

- ii. Using R scripts determine the limiting transition probability of P

(9 marks)