



**Public Domestic Debt Optimization and the Term Structure of
Interest Rates: Dynamic Nelson Siegel Approach**

Anne Keru Munene: 062424

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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Anne Keru Munene

Signature AKMunene

Date 4th DECEMBER 2015

This Research Proposal has been submitted for examination with my approval as the Supervisor

Ferdinand Othieno

Signature F Othieno

Date 4. DEC. 2015

School of Finance and Applied Economics

Strathmore University

Abstract

In this study, Markowitz's mean-variance optimization approach is applied to Kenya's public debt portfolio to identify the optimal domestic debt portfolio. The study first adopts the Dynamic Nelson Siegel model to parameterize the yields only model of the Term Structure of Interest rates in Kenya between March 2010 and October 2014. This is done so as to account for the importance of elaborate interest rate models in incorporating interest rate uncertainty. The expected interest rate cost is estimated based on the yield curve dynamics as established under the Dynamic Nelson Siegel framework. The optimization scheme adopted minimizes interest rate cost and variance of the different bonds across the different maturities subject to the constraints of no short sales, sufficient liquidity and a sufficient portion is allocated to long term maturities to cater for long term development projects. The results indicate that the optimized bond portfolio is in favour of medium to long dated bonds in order to reduce the overall risk level.

Key words: Markowitz's mean-variance optimization, Dynamic Nelson Siegel

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Abbreviations List

ATM	-	Average Term to Maturity
CaR	-	Cost at Risk
CBK	-	Central Bank of Kenya
CFaR	-	Cash Flow at Risk
CIR	-	Cox, Ingersoll and Ross
CPI	-	Consumer Price Index
DNS	-	Dynamic Nelson-Siegel
GDP	-	Gross Domestic Product
IMF	-	International Monetary Fund
MTDS	-	Medium-Term Debt Strategy
OU	-	Ornstein Uhlenbeck
RPI	-	Retail Price Index
VaR	-	Value at Risk

1 Introduction

1.1 Background of the study

The surge in public debt and ‘twin deficits’ of many developing nations from early 1990s has received increased attention from practitioners, policymakers and scholars. Much of this attention has been devoted to understanding the optimal size and structuring of public debt. Hahn & Kim (2003) and Pick & Anthony (2006) assert that optimal design of public debt involves assessing the cost-risk trade off of borrowing strategies to ensure that over the long-term, debt service costs are minimized taking into account risk while ensuring that debt management policy is consistent with the aims of monetary policy.

The optimal design of public debt could be determined using Markowitz (1952) portfolio optimization approach which is based on the mean-variance relationship among assets such as bonds of different maturities. By subjecting the portfolio to various key constraints such as liquidity constraints of the government, various strategies are determined. To assess alternative debt strategies, a clear definition of cost and risk measures is required. The most common cost measure is interest expenditures stated in the government budget. Pick & Anthony (2006) reasoned that the use of ratio of annual interest payments to GDP is superior to interest expenditures because debt cost ratio provides a better indication of the debt cost burden to the Government. Conventional risk measures used include: Average Term to Maturity, Duration and Maturity Profile to determine refinancing risk of debt. Risbjerg & Holmund (2005) accuses these risk measures as being deterministic and thus fail to incorporate uncertainty. To overcome this limitation, stochastic risk measures adopted from Value at Risk approach have been used such as Cost at Risk (CaR) and Cash Flow at Risk (CFaR). Garcia (2002) points out the advantage of CFaR of being able to measure impact of fluctuations in market prices on public debt service costs and other components of fiscal accounts while Choi, Kim, & Lee (2010) recognize superiority of CaR by being able to provide an estimate of variability of debt service costs under various simulation for interest rates and other macroeconomic factors.

Alternative financing strategies are then compared to various risk measures such as the absolute CaR and relative CaR which are computed from simulated cost distributions obtained from a stochastic simulation model, (Pick & Anthony, 2006). Three types of risks will be assessed: (1) Interest rate risk which deals with the desired proportion of floating as opposed to fixed interest rate debt (2) Refinancing Risk (Roll-Over) risk which addresses the desired maturity structure and redemption profile of the debt (3) Foreign Currency Risk which addresses the

desired currency composition of the debt portfolio in terms of domestic currency versus foreign currency, (Melecky, 2007).

To use a Cost-Risk analytical framework, evolution of term structure of interest rates is a key consideration for determining the optimal debt strategy. Despite interest rates being a major source of uncertainty, there is limited literature on how to incorporate simulated future path of interest rates using a rigorous term structure of interest rates model into a cost-risk analytic framework. See for instance (Bergström & Holmlund, 2000), (Bolder, 2003) and (Hahm & Kim, 2003). A majority of these studies incorporate single factor and two factor yield curve models which do not reflect the dependency between other macroeconomic factors such as the growth rates, short term interest rates, inflation and interest rates. Under this vein, we have studies by Pick & Anthony (2006) and Choi, Kim, & Lee (2010) that have modelled and incorporated various macroeconomic variables into the Dynamic Nelson Siegel Yield curve Model in Diebold, Rudebusch, & Aruoba (2006) which is founded on the Nelson Siegel yield curve model.

This study contributes to public debt management literature by adopting a Dynamic Nelson Siegel model into a cost-risk analytic framework in order to determine the composition of Kenya's domestic debt portfolio which minimizes the debt service cost given the cost-risk trade-offs associated with different financing strategies.

1.2 Evolution of Kenya's public debt

As at May 2015, Kenya's public debt stood at KES 2.79 trillion and 50 per cent of it being external, (Figure 1). This has led to various warnings from International Monetary Fund noting that foreign debt repayment is expected to become expensive with a stronger U.S. economy and possible increase in Fed rates exposing the country to exchange rate risk. In addition, there is a risk that Kenya might deteriorate its credit rating from 'B+' rating stable outlook due to projection of increased external borrowing to finance mega infrastructure projects under Kenya Vision 2030 such as standard gauge railway, plan to add 5,000 megawatts by 2017, rural electrification and annuity roads programme that will increase the tarmac road network by 10,000 km.

Other concerns for increased levels of Kenya's public debt is because: (1) Kenya has the highest public debt ratio of at least 44 percent in the region compared to its peers, Tanzania with about 34.7 per cent and Uganda with 35 per cent. (2) Leakages through corruption which

hinders proper utilization of borrowed funds (3) Continue rise in twin deficits is a major concern based on a report by (ControlRisks, 2015).

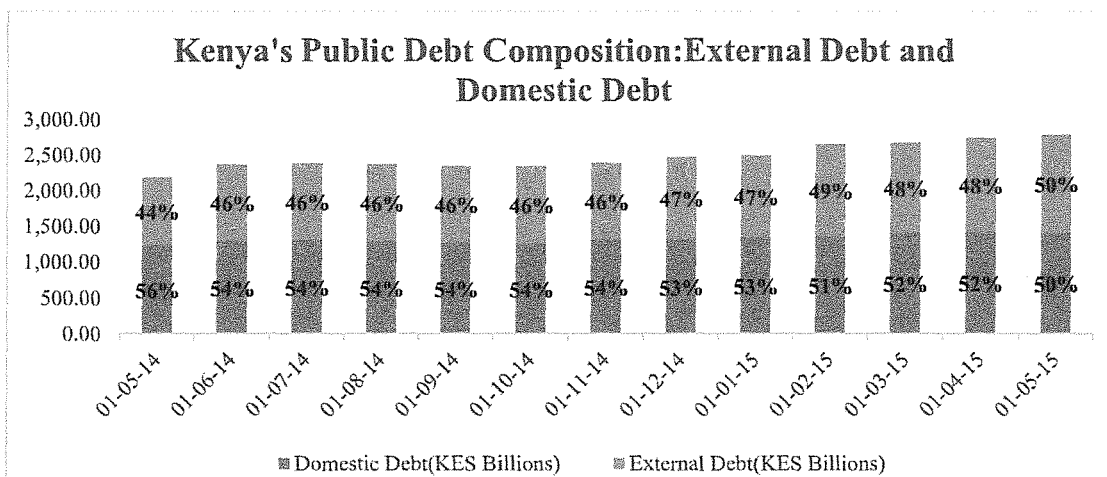
Despite these concerns, history indicates that Kenya's public debt management strategy and dynamics has improved over time. In the 1980s, Kenya was among the major aid recipients in Africa, largely to finance various infrastructure projects that could enhance economic growth. During the 1990s, Kenya witnessed a steady decline in aid as a result of poor governance and corruption. Corruption scandals such as Goldenberg and Anglo Leasing coupled with 1980s financial crisis led to a debt crisis in the country and reduction of development assistance funds. As a result, the government resorted to occasional debt rescheduling and domestic borrowing to finance its expenditure.

From 2000 to 2008, public debt to Nominal GDP reduced substantially from 62 per cent to 39 per cent,(Figure 2).This was a result of implementation of sound public debt management practices, increased confidence into the country that led to more investments hence increased economic growth from 3.8 per cent in 2000 to 6.9 per cent in 2007 and also change in public debt structure of which a large proportion of external debt was on concessional terms, (Ryan & Maana, 2014).From 2008, public debt increased to stabilize the country after 2008/2009 Post Election Violence, to promote economic growth and to manage various macroeconomic shocks such as the 2011 escalating oil prices and a weaker shilling against major international currencies.

Despite considerable improvements in implementation of a sound debt management strategy over time, the concerns on ballooning Kenya's public debt are based on Kenya's integration with the international financial markets from the issue of a Eurobond. This integration exposes the country to more currency risks thus macroeconomic instability, (Burnside & Dollar, 2000).

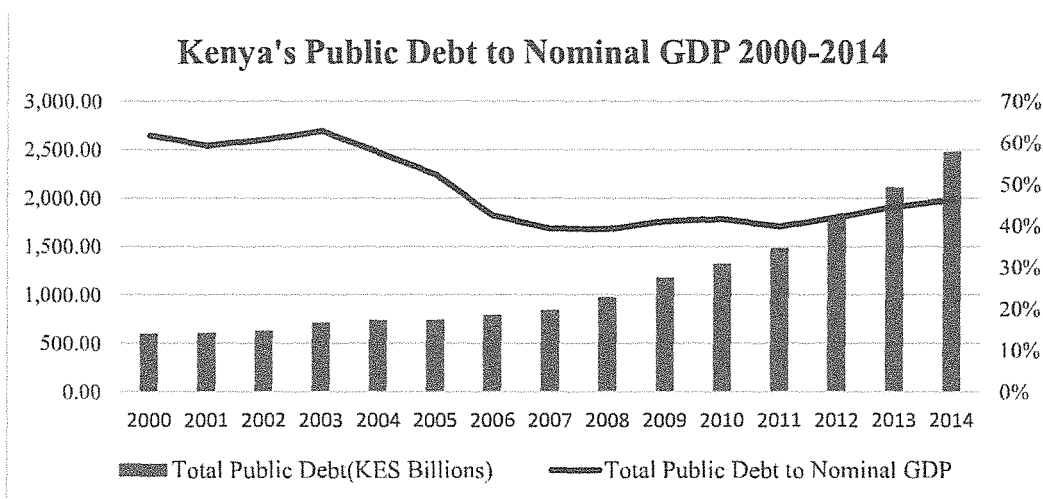
The public debt dynamics in Kenya are discussed below:

Figure 1: Kenya's Public Debt Composition: External Debt and Domestic Debt



Source: Central Bank of Kenya Website

Figure 2: Kenya's Total Public Debt to Nominal GDP



Source: Central Bank of Kenya Website

1.3 Problem statement

Literature suggests that the ultimate objective of sovereign debt management is to minimize long term debt cost given the trade-off between expected debt service costs and risk associated with various borrowing strategies to finance borrowing requirements. Though there has been notable development to achieve this objective by considering interest rate risk, major focus has been laid on using single and multifactor equilibrium interest rate models to incorporate this risk, (Bolder, 2003). Bolder (2003) highlights that lack of using an elaborate term structure

model underestimates both risk and costs associated with a given strategy. This study contributes to public debt management literature by adopting a Dynamic Nelson Siegel model into a cost-risk framework that adopts the Markowitz's mean-variance optimization approach in order to determine the composition of Kenya's debt portfolio which minimizes the debt service cost given the cost-risk trade-offs associated with different financing strategies.

1.4 Research objectives

To identify whether the current public domestic debt portfolio in Kenya is optimal.

1.5 Research questions

What is the optimal public domestic debt composition for Kenya?

2 Literature review

2.1 Introduction

This chapter discusses theoretical and conceptual underpinnings upon which this study builds upon. Section 2.2 discusses theoretical literature on optimal public debt. Section 2.3 discusses the optimization framework built by different debt management offices and scholars based upon Markowitz's Modern Portfolio Theory. Section 2.4 presents considerations for determining optimal public debt. The considerations discussed include: types of risks, a term structure of interest rates and cost and risk measures.

2.2 Theoretical literature on optimal public debt

Government needs resources to finance its budget and development efforts either through taxes or borrowing. Taxes which form the bulk of government revenue might not be sufficient to cover the government expenditure. So the government might increase taxes to cover the deficit but this might distort the structure of relative prices, hence borrowing, (Brainard, Okun, & Perry, 2010). With borrowing, the government is concerned about an unexpected increase in debt servicing costs that could lead to adverse financial and economic consequences. Faced with unexpected increase in debt servicing costs, governments have two policy alternatives:

First, government can reduce its expenditure which may lead to high social costs because less services are being provided to its citizens. Secondly, government can raise taxes. Tax smoothing literature proposes that if taxes are lump sum and other conditions for Ricardian Equivalence are present, there are no real effects from shifts between taxes and issuances of public debt as modes of financing fiscal imbalances. However, if taxes are distorting, they create deadweight welfare losses, wasted resources from tax avoidance and volatility of tax rates increases these losses, (Barro, 1979). To ensure sustained growth, tax variability must be minimized because volatility creates inefficiencies, complicates long-term investment decisions, depresses consumption and leads to capital flight, (Alesina & Tabellini, 1989).

To minimize variability of taxes, Barro (1979) proposes a tax smoothing model by spreading the burden of taxes over time by ensuring fiscal imbalances exist. Bohn (1990) extends Barro's tax smoothing model by demonstrating that government can use its debt portfolio to act as a hedge against any macroeconomic shocks that might affect tax revenues. Therefore, government's objective under the optimal taxation approach is to minimize welfare losses resulting from distortionary taxation. The risk being considered is budgetary risk which is the

risk of having to change taxes in response to shocks hitting government budget, (Missale, 1997).

If volatility in debt service is so great and the government is either unable or unwilling to reduce spending or raise taxes, they must default, which cause severe economic costs. These include: collapse of financial institutions and higher costs of future borrowing both for the public and private sector as a result of the government's reputational loss.

To ensure that above costs are avoided and to maintain the reputation of government in international financial markets, governments are expected to have sound debt management practices and a risk management culture, (Storkey, 2001). According to IMF and World Bank (2002), the government should choose its debt strategy that will enhance liquidity in the secondary markets through issuance of benchmark securities at key maturities and to broaden the investor base by diversifying public debt among different maturities through a range of different market instruments with an objective of minimizing the costs of borrowing over the long term.

2.3 Debt management strategy in Kenya

Public debt management is the process of establishing and executing a strategy for managing the government's debt in order to raise the required amount of funding at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk. (World Bank & IMF, 2014)

Literature suggests that whatever borrowing strategy the Government chooses should ultimately depend on cost and risk considerations such as debt costs, the cost to the tax payer and also the cost associated with a given borrowing strategy should not be too volatile nor expose the Government to unexpected and large increases in debt costs nor should it pose a threat to the attainment of the Government's overall fiscal goals, (Pick & Anthony, 2006).

The consideration of cost and risk of borrowing strategies is very important to the Kenyan government and it has put efforts to ensure that the level of public debt has been consistent with the overall fiscal framework supporting macro-economic stability for sustainable growth over the medium term. It is anchored on the Medium Term Debt Strategy (MTDS) revised annually and tabled in Parliament as part of the documents supporting Budget Estimates. The Government debt management objective remains two-fold: to raise resources through borrowing to meet central government budgetary requirements at minimum cost and prudent

level of risk; and to promote the development of domestic debt markets, (National Treasury, 2015)

According to National Treasury (2015), the key drivers for the 2014 MTDS were a desire to minimize overall cost by issuing medium term debt to reduce cost associated with longer dated securities and to further develop and deepen the domestic debt market. The Government also highlighted the need to minimize the degree of foreign exchange rate risk exposure associated with the external debt portfolio by borrowing more concessional debt, while maintaining a limited window for borrowing on commercial terms to minimize costs and refinancing risks. The 2014 MTDS emphasizes that financing on non-concessional terms will be highly restricted to projects with high expected risk-adjusted rates of return including critical infrastructure that would otherwise not be undertaken due to lack of concessional financing.

The 2014 MTDS presents as the optimal strategy after taking into account both cost and risk considerations, the need to develop the domestic debt markets and the feasibility of implementing the strategy over the medium term. The strategy comprises of the following actions: 1)55% gross domestic borrowing and 45% net external borrowing to finance the central government budget; 2)Issuance of medium term domestic debt through benchmark bonds is recommended. Some of the benchmark bonds are indicated in table 1 .3) External borrowing will comprise of 28% on concessional terms, and 17% on non-concessional terms, (National Treasury, 2015).

Table 1: Composition of bonds issued in Kenya

Classification of maturity length of bonds	Maturity Length of Bonds	Composition over the recent months
Short term	<5 years	<15%
Medium term	>5 years but less than 15 years	Between 50%and 60%
Long term	>15 years	>25%

Source: Central Bank of Kenya website and researcher's estimates.

2.4 Mean-Variance optimization framework

Most literature agree that debt matters, however there is no agreed way of finding the optimal level of debt. There are two main approaches used: One approach is the theoretical approach optimal taxation approach which I discussed earlier. The second approach is through adoption of the Markowitz's Mean-Variance optimization approach which evaluates various alternative financing strategies with respect to risk measures such as variance; and Cost at Risk or Cash Flow at Risk computed from simulated cost distributions. An efficient portfolio is then constructed and the portfolio with the minimal cost is chosen as the optimal portfolio.

Markowitz (1952) highlights that an optimal portfolio is the one that provides the best combination of risk and return for a specific investor depending on said investor's degree of risk tolerance based on the variance and standard deviation. This approach is mainly used by market practitioners due to its cost effectiveness, computational simplicity and familiarity. There is limited academic literature on the second approach for public debt portfolios see Hahn & Kim (2003) and Choi, Kim, & Lee (2010) however various debt management offices have come up with their own optimization models for their respective countries.

Bergström & Holmlund (2000) describes the Swedish Model as a Cash Flow Monte Carlo Simulation Model, with auto-regressive process for inflation, GDP, long-term interest rates and exchange rates, as well as a Taylor rule for short term interest rates. Different portfolio strategies in terms of currency compositions and target durations are examined over a ten year horizon to get an efficient portfolio that will minimize costs under given level of risk assuming an upward-sloping yield curve. The author concludes that foreign currency debt leads to greater variation in costs and only limited diversification gains. However, Bolder (2003) comments that assuming an upward-sloping yield curve might favour a portfolio of short term securities which might increase roll over risk.

Bolder (2003) describes a framework developed for the Bank of Canada. The framework combines a macro-yield curve model using Markov-switching approach for the real GDP growth rate and a two-factor CIR model to permit the steepness of the term structure of interest to evolve over time in a manner that is consistent with business cycle. Further, the author calibrates the parameters of the financing requirement. To consider the costs, the author uses a cost at risk measure where the author finds out that debt costs are not normally distributed. The author comments that more flexible models are required to describe the stochastic interest rate

environment, impact of inflation when it comes to index linked bonds and relative supply of bonds should be factored in the model.

Hahm & Kim (2003) employs Cost at Risk and measure on actual Korean government data as of March 2000 and considers debt service cost as the only risk. The various cost-risk tradeoffs are summarized as efficient frontiers constructed over expected value and variability of future debt service costs and thus an optimal benchmark portfolio is identified. Hahm & Kim (2003) try to achieve the objective of minimizing the costs of long-term borrowing at a given risk by first computing cost and risk of various financing strategies. An efficient portfolio set is obtained by selecting the portfolio that yields lowest expected cost at each level of standard deviation. Given an efficient portfolio set and exogenous risk targets such as duration and CaR, an optimal portfolio is selected by minimizing CaR and deviations from other exogenous risk targets. The author is able to achieve the optimal debt portfolio for the Korean Government debt, however the author recognizes the limitation to this approach is that important risks such as liquidity, market and refinance risk are not incorporated.

The impact of inflation which was not included by Bolder (2003) is incorporated by (Pick & Anthony, 2006). Pick & Anthony (2006) model is made up of three main components: (1) A macroeconomic model comprising five equations for the output gap, the government's net primary financing requirement, CPI and RPI inflation and the central bank's policy rate. (2) A nominal and real yield curve specifications using the Nelson Siegel model. (3) Cost Distributions of various issuance debt strategies such as varying the terms of the bonds or issuing inflation-linked bonds or fixed interest rate bonds are generated. The main advantages of this model is that interest rates and inflation rates risks are incorporated. However, the author points out some limitations of this model: First, relative supply of bonds which might affect the term structure of interest rates are not incorporated. This limitation is also encountered by (Bolder, 2003). Secondly, the first component of the model is backward-looking model and should accommodate extreme adverse economic conditions in the true spirit of the new-Keynesian models that have been developed.

The Brazilian approach, as described by Silva, Cabral, & Baghdassarian (2010) is basically an efficient frontier analysis, where costs and risks are measured in terms of debt/GDP ratios. To achieve this, it involves: Financial stochastic models namely Cox, Ingersoll and Ross (CIR) model for domestic and external interest rates, a Brownian process for inflation and a Chan, Karolyi, Longstaff and Sanders (CKLS) model for real exchange rates are constructed. Residuals are correlated using a Cholesky decomposition; A macro-structural model to

describe the evolution of the main economic variables (an IS and Phillips curve, a Taylor rule and equations for exchange rates and risk Premium behaviour); Various debt compositions are simulated through a number of different periods based on stochastic scenarios and assumptions about the pricing of assets. The debt compositions are evaluated in terms of cost and risk, as well as the correlation matrix and an efficient frontier is drawn. Having observed efficient frontier, debt managers would choose, based on their attitude towards risk, the point (portfolio) to represent the benchmark.

The above models discussed though they recognize that interest rate is a major source of uncertainty, there is limited use of a rigorous term structure simulation model. Choi, Kim, & Lee (2010) attempt to incorporate a more rigorous term structure models of interest rates into a cost-risk model by extending the work of Hahm & Kim (2003) to assess alternative sovereign debt financing strategies. The author considers two models: time series-based dynamic Nelson-Siegel (DNS) model proposed by Diebold & Li (2006) and the arbitrage free Nelson-Siegel (AFNS) proposed Christensen , Diebold, & Rudebusch (2011).

2.5 Considerations for an optimization framework on public debt

2.5.1 Types of risk

Public debt management is the process of establishing and executing a strategy for managing government's debt in order to raise the required amount of funding at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk, (World Bank & IMF, 2014). The most relevant types of risks to which public debt is exposed to include: Market risk, refinancing risk, budget risk and demand-side risk. Market Risk is defined as the risk of unexpected increase in the debt service cost due to fluctuations in market conditions such as interest rates, exchange rates and inflation, (IMF & World Bank, 2001). The most important factor when selecting a given financing strategy is future interest rates. Based on expectations theory and liquidity preference theory of yield curves, the yield curve is upward sloping due to uncertainty perceived and thus short term interest rates are usually lower than the longer term. Because of this argument, one might conclude that it is better to issue short term financial instruments. However, a portfolio composed of short term instruments will lead to the need of renewing debt more frequently which sudden changes in the debt payment structure might affect the government 's ability to roll over part or all debt falling due at a given time giving rise to refinancing risk, (Balibek & Memis, 2012). For refinancing risk, time diversification is very important as lack of it could expose government to substantial refinancing risk in future

periods, (Bolder, 2003). The other risks which are also crucial include :budget risk which is the risk that the debt service within a fiscal year will exceed the amount originally approved by the 'Parliament' and Demand-side Risk which is the risk of sudden changes in the demand for government bonds, (Silva, Cabral, & Baghdassarian, 2010) .The common factor that influences all these risks is interest rates uncertainty. To cater for these risks, various classes of yield curve models are applied.

2.5.2 A review of term structure models

Term Structure Modelling has evolved from traditional equilibrium models to no-arbitrage class of models (evolutionary) and currently descriptive class models. The equilibrium models are based on identifiable external factors and start with the assumptions about external economic variables and then derive a process for the short interest rate. These models include: Vasicek (1977), Cox, Ingersoll, & Ross (1985) and Duffie & Kan (1996).

The Vasicek (1977) model as noted by Chaplin (1998) shaped term structure modelling and has remained so fundamental. The model explains that yield levels at different maturities in relation to a single exogenous factor-the level and movement of money market rates, money rates are assumed to follow an 'Ornstein-Uhlenbeck (OU) process', sum of a 'deterministic' component (a reversion from the stochastic money rate to a fixed long-term level at a rate which is proportional to the difference between current yield levels and the long-term level) and a stochastic element with a fixed variance. The main criticism to the model is that the constant volatility term in the OU process governing the evolution of money market rates allows for negative money rates. This goes against one of the desirable features of a term structure model that requires that term structure models should allow for positive rates only.

Cox, Ingersoll, & Ross (1985) address the above criticism by replacing the constant volatility in the OU formula by a volatility term proportional to the square root of interest rate levels thereby eliminating the possibility of negative rates. However, as noted by Chaplin (1998), the model suffers from an inability to produce a humped yield curve shape. In addition, just like any other one-factor model, it assumes that instantaneous interest rate changes across the maturity spectrum are 100% correlated. This led to development of two-factor and multifactor models. Two factor models such as Richard (1978) model normally known as the two-factor Vasicek Model and two-factor Cox, Ingersoll-Ross Model by Longstaff & Schwartz (1992) provide a good compromise in terms of flexibility of fitting a variety of shapes of term structures, Nowman (2010) and more parameters can be added for calibration, (Babbs &

Nowman, 1999). However, they are not sufficient to reflect the different shapes adopted by real yield curves over time, (Chaplin, 1998).

No-arbitrage models developed as a result of development of derivatives which required a model that eliminated any arbitrage between synthetic bonds and actual bonds. Their main aim is to reflect the initial term structure and are primarily concerned with the future development of the initial term structure. The models include: Ho & Lee (1986) model, a single factor model, Hull & White (1990) and Heath, Jarrow, & Morton (1992) models. The Hull & White (1990) model is a modification of the Vasicek (1977) model to ensure that it produces yields that are consistent with the initial term structure which is the major focus of no arbitrage models. The researchers replaced the constant long run equilibrium rate by a deterministic rate which depends on time. The Heath, Jarrow, & Morton (1992) model unlike the above discussed single factor no-arbitrage models is a multifactor model and is based on forward as opposed to spot rates.

Finally, Descriptive models aim at providing an accurate description of the yield curve and by implication an accurate indication of yields which can be obtained in the market, then a model which fits the market accurately is preferable, (Chaplin, 1998). The models include Nelson & Siegel (1987) and an extension of the Nelson Siegel by (Svensson, 1994).

2.5.2.1 The Nelson & Siegel model

Nelson & Siegel (1987) developed a factor model that was parsimonious and flexible enough to represent the range of shapes generally associated with yield curves. To achieve this, difference equations were developed which represented a class of functions that could readily generate the typical yield curve shapes and the solution to these difference equations were forward rates being forecast rates of spot rates in accordance to expectations theory of term structure of interest rates. The study further adopted the second order difference equation and obtained its solution which formed the initial formulation of the Nelson & Siegel 1987 forward rate.

Defining $r(m)$ as the instantaneous forward rate at maturity (m) , $r(m)$ is given by:

Equation 1

$$r(m) = \beta_0 + \beta_1 * e^{\left(\frac{-m}{\tau_1}\right)} + \beta_2 * e^{\left(\frac{-m}{\tau_2}\right)}$$

Where τ_1 and τ_2 are time constants associated with the equations and β_0 , β_1 and β_2 are parameters.

Fitting the above forward rate model to US Treasury Bill yields, Nelson and Siegel(1987) found that it was over-parameterized. They noted that a more parsimonious model could be obtained by using equal roots ($\tau_1 = \tau_2 = \tau$). This model could still generate the same range of yield curve shapes that is monotonic, humped and S shaped. The result equation seen below was adapted as the initial forward rate formulation of the Nelson & Siegel model.

Equation 2

$$r(m) = \beta_0 + \beta_1 * e^{\left(-\frac{m}{\tau}\right)} + \beta_2 * e^{\left(-\frac{m}{\tau}\right)}$$

From theoretical literature, the yield to maturity $R(m)$ is the average of the forward rates.

Equation 3

$$R(m) = \frac{1}{m} \int_0^m r(x) dx$$

From the forward rate formulation of the Nelson & Siegel model, the yield to maturity is given by:

Equation 4

$$R(m) = \beta_0 + \beta_1 \left(\frac{1 - e^{-m/\tau}}{m/\tau} \right) + \beta_2 \left(\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau} \right)$$

The three parameters also known as factors measure the strength of the short, medium and long term components of the forward curve. For instance:

β_0 is known as the contribution of the long term component because β_0 is the limit of $r(m)$ as m tends towards infinity.

$$\lim_{m \rightarrow \infty} r(m) = \beta_0$$

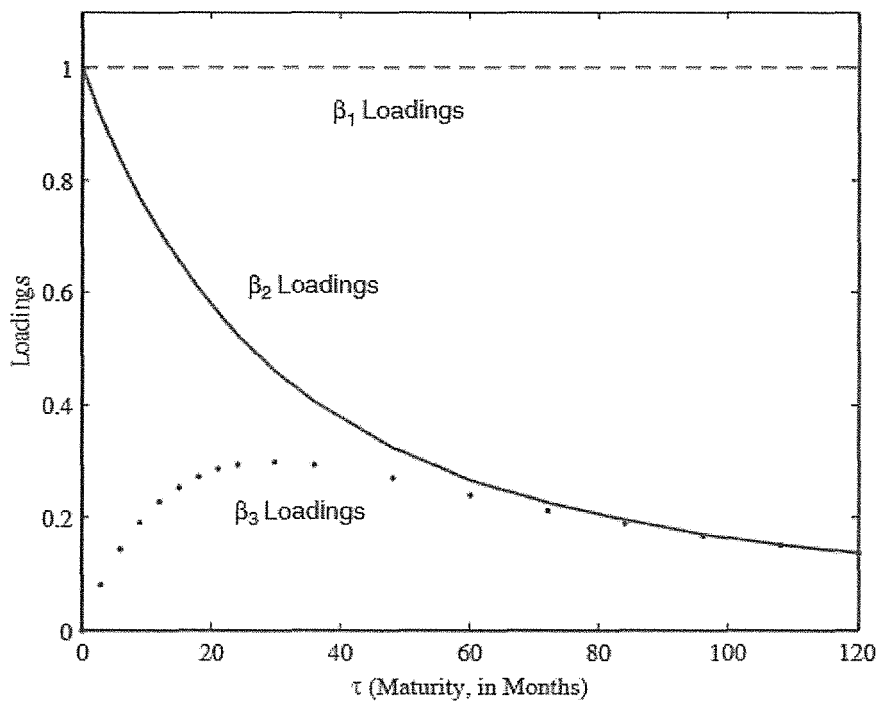
β_1 is known as the contribution of the short term component. It has loading factors which contribute to its rate of decay. The $1/\tau$ measures the rate of the exponential decay and its small values produce slow decay and provide a better fit of the yield curve at long maturities while its large values produce fast decay and a better fit of the yield curve at short maturities.

β_2 is known as the contribution of the medium term component and which starts off at zero and decays to zero eventually making it at the middle of long and short term component.

The development of the Nelson & Siegel Model has contributed greatly towards term structure modelling. According to Patakova (2011), factor models have overcome the major problem that lies with compressing all the price information for the large volumes of bonds traded as at any point

in time by summarizing information of all bond prices in a small set of variables called factors. From the Nelson & Siegel model, further developments have been in term structure modelling. For instance, Diebold & Li (2006) interprets the parameters β_0 , β_1 and β_2 as time varying latent factors represent the level, slope and curvature factors respectively based on the aspect of the curve each factor governs. For β_0 is the same loading across all maturities leading to an equal change of yields across all maturities hence governing the level factor. On the other hand, β_1 is labelled as the slope factor and any increase of the slope increases short rates more than long yields while β_2 interpreted as the parameter governing medium term rates load heavily on them increasing the yield curve curvature.

Replacing β_0 , β_1 and β_2 with time varying parameters L_t , S_t and C_t represent the level, slope and curvature factors respectively, the dynamic form of the Nelson & Siegel model was formulated. This is very useful especially during the application of term structure modelling in portfolio optimization problems.



2.5.3 Cost and risk measures

2.5.3.1 Cost measures

A clear definition of “cost” and “risk” measures is a precondition for developing a sound debt management strategy, (IMF & World Bank, 2009). The cost of debt measure is determined by

country-specific factors such as the risk profile of public debt, market conditions and methods used in measuring and reporting debt.

Interest cost in terms of interest expenditures is a common measure for cost of borrowing. Another approach used is a share of nominal interest expenditures in budget items and/ or GDP. Towe, D., & K. (2009) focus on two specific cost measures: annual interest payments-to-GDP and nominal stock of debt-to-GDP. Pick & Anthony (2006) uses ratio of annual interest payments to GDP (debt cost ratio) which has the advantage of providing a better indication of the debt cost burden to the Government than what nominal cost of debt does.

2.5.3.2 Risk measures

According to Velandia (2002), there are two categories of risk measures used: deterministic and stochastic measures. The conventional/ deterministic risk measures models used include: First, average Term to Maturity (ATM) which is the average remaining time to maturity of debt instruments that make up the debt is used. The longer the ATM indicates that debt instruments are rolled over less frequently and therefore there is a lower refinancing risk and less uncertainty regarding future debt cost. Secondly, duration which is the weighted average of the remaining maturity of the debt stock is normally used. The most commonly used duration is Macaulay duration. However, duration considers not only the principal payment but the present value of all expected cash flows through the lifespan of a debt instrument. Longer duration lowers the rollover risk, (Missale, 1997). The disadvantage of this risk measure is that two debt portfolios with the same duration may respond differently to an increase in the interest rates due to different maturity structures thus not being sufficient. To overcome limitation for duration, Riksgaldskontoret (2002) uses Maturity profile as a risk measure which is the amount of debt that matures in any given year. Well-distributed maturity profile reduces refinancing risk.

Developing deterministic models are simple and efficient method in cost-risk modelling of public debt since they are also easy to communicate to policy makers. However, they fail to incorporate uncertainty into the model and thus “not possible to achieve a direct quantification of the risk by relating potential outcomes to their likelihood,” (Risbjerg & Holmund, 2005). To overcome this challenge of uncertainty, various stochastic measures derived from ‘Value at Risk’ and standard deviation are used. However, the key challenges in these stochastic simulation measures include complexity, the distributional assumptions of financial variables and stability of estimated relationships, (Anderson, 2011).

One stochastic risk measure used is the Cost at Risk (CaR) measure which is the maximum cost that could occur with some probability in a particular time period.. The other risk measure is Cash Flow at Risk (CFaR), normally used under Asset and Liability Management, which takes into account of the impact of risk factors on the firms cash flows, (Jorion, 2001). Besides the 'at Risk' measures, standard deviation is also used to measure the deviation from the mean.

3 Methodology

3.1 Introduction

The researcher will adopt Markowitz mean-variance optimization methodology as proposed by Hahn & Kim (2003) later adopted by Choi, Kim, & Lee (2010) in an attempt to determine the optimal level of Kenya's debt. This method entails three steps: First, researcher will indicate the debt portfolio optimization problem which involves minimizing costs of domestic debt within a given risk measure. Secondly, to recognize that the cost of public domestic debt is influenced mainly by term structure of interest rates. Thirdly, the optimal domestic debt portfolio is obtained based upon Markowitz's mean-variance optimization approach.

3.2 Research design

The proposed design will involve an exploratory study to determine the level of optimal public debt. The study seeks to explore whether the current government financing strategy is optimal and what is that optimal debt portfolio. This design is justified by the need to introduce more rigorous term structure of interest rates into a cost-risk model since interest rates is the major source of uncertainty as emphasized by Bolder (2003) and Choi, Kim, & Lee (2010).

3.3 Sampling design

This study is split into two: First, it determines the composition of Kenya's domestic debt in terms of maturity structure as at October 2014 to determine whether the strategy is optimal or how far is the strategy far away from the optimal. Secondly, it will analyse term structure of interest rates from March 2010 to October 2014 to determine the mean cost and variances of interest rates. The lack of complete historical term structure data led to limited time period of study.

3.4 Nature and source of data

This study will employ secondary data that is quantitative in nature. The data set includes Kenya's public domestic debt and historical term structure of interest rates. The data on public domestic debt is available in the Central Bank of Kenya (CBK) and its composition is extracted from CBK's monthly economic reports.

In addition, the study uses monthly average yields on fixed coupon paying government bonds derived from the daily trades of fixed coupon paying government bonds at the Nairobi Securities Exchange (NSE). The use of fixed coupon paying bonds is not used in the Nelson &

Siegel model due to the complexities such as differences in capital gains and coupon income taxation hence the use of zero coupon bonds when fitting the yield curve. For the Kenyan scenario, fixed coupon paying yields are the most appropriate due to the lack of zero coupon bond yields in the Kenyan bond market. For this study, complexities inherent in fixed coupon paying bonds are assumed and thus not adjusted for.

3.5 Empirical approach

3.5.1 Introduction

The objective of this paper is to find the composition of debt portfolio issues every month which minimizes a specific cost function under a certain level of risk. This becomes a portfolio optimization problem. According to Claessens, Kreuser, Seigel, & Wets (1998) as quoted by (Hahm & Kim, 2003), for any optimization problem including the sovereign debt portfolio benchmarking can be represented as a variant of the following stochastic dynamic program:

Equation 5

Minimize $E\{f(s; d)\}$, where d is in D

Where d stands for a sequence of decisions to be made such as on maturity composition or currency composition and D is the set of all acceptable decisions. E is an expectations operator and function f is an objective function that serves as decision criteria. The value of f depends upon d as well as random variables s that provide a description of possible future environments when the consequences of the decisions d are evaluated.

In sovereign debt management, d may correspond to decision on the maturity of government debt portfolio or on the currency composition in the case of external sovereign debt. The vector of random variables s may correspond to the term structure of interest rates, exchange rates, GDP growth rates and so forth. The possible set D represents the range where decisions are restricted and reflects policy constraints such as minimum exposure to a specific currency, borrowing limits or maximum percentage of refinancing. This depends on the realizations of random variables as well as previous decision choices. While the control function f can be specified in various forms such as debt service costs of the government debt or nominal values and usually involves the target risk measure and is often represented in the form of a penalty function, (Hahm & Kim, 2003).

To implement this, first is to model the term structure of interest rates which will be useful in finding the optimal composition of domestic debt portfolio in terms of the maturity structure

and thereafter find the optimal domestic debt portfolio under the mean-variance optimization framework.

3.5.2 Yield curve model

3.5.2.1 Dynamic Nelson-Siegel model

Diebold & Li (2006) draws upon the Nelson Siegel Model, which provides a parsimonious representation of the term structure of interest rates at a particular period. They adopt the Nelson Siegel model framework to fit the term structure by approximating the forward rate curve by a constant plus a polynomial times an exponential decay term with the only adjustment made was to make the latent factors dynamic. This is given as follows:

Equation 6

$$f_t(\tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}e^{-\lambda_t\tau}$$

where $f_t(\tau)$ the instantaneous forward rate. This yields the following term structure

Equation 7

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau} \right)$$

Where $\beta_{1t}, \beta_{2t}, \beta_{3t}$ and λ_t are parameters and $1, \left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} \right)$ and $\left(\frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau} \right)$ are their loadings. Diebold and Li (2006) interpret the parameters β_{1t}, β_{2t} and β_{3t} as time varying or dynamic latent factors which represent the level, slope and curvature factors respectively. The parameter λ_t controls both the exponential decay rate. Diebold & Li (2006) noted that the state-space representation provides a framework for analysis and estimation of dynamic models.

State space representation of the dynamic Nelson & Siegel model is as follows:

Equation 8

$$F_t - \mu = A(F_{t-1} - \mu) + \eta_t$$

This can be expressed in matrix form as:

Equation 9

$$\begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{bmatrix}$$

Equation 9 is the transition equation which specifies the dynamic of the state vector which for the three factor Nelson & Siegel model is given by the unobservable vector $F_t =$

$(\beta_{1t} \beta_{2t} \beta_{3t})'$. According to Diebold and Li(2006), it is assumed that these time-varying factors follow a vector autoregressive process of first order, where the mean state vector μ is a 3×1 vector of coefficients, the transition matrix A is a 3×3 matrix of coefficients and η_t is a white noise transition disturbance with a 3×3 non-diagonal covariance matrix Q .

Equation 10

$$\begin{bmatrix} y_t(\tau_1) \\ \dots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} - e^{-\lambda_t \tau_1} \\ \dots & \dots & \dots \\ 1 & \frac{1 - e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} & \frac{1 - e^{-\lambda_t \tau_N}}{\lambda_t \tau_N} - e^{-\lambda_t \tau_N} \end{bmatrix} \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \dots \\ \epsilon_{Nt} \end{bmatrix}$$

The Equation 10 is a measurement equation which specifies the term structure and relates N observable yields to the three latent factors. The vector of yields Y_t contains N different maturities $Y_t = (y_t(\tau_1) \dots y_t(\tau_N))'$. The measurement matrix Λ is a $N \times 3$ matrix whose columns are the loadings associated with the respective factors, and ϵ_t is a white noise measurement disturbance with a $N \times N$ non-diagonal covariance matrix H .

The factors are estimated in a state space model with equation 9 as the transition equation and equation 10 as the measurement equation using the Expectation-Maximization (EM)-algorithm in conjunction with Harvey (1989) Kalman filter. According to Harvey (1989), the Kalman Filter is a mathematical tool that operates by means of a prediction and correction mechanism and its two basic building blocks are the measurement and transition equation. Jalles (2009) further highlights that the aim of the Kalman Filter is to compute a linear, unbiased and optimal estimator of a systems state as at time t based on information available as at time $t-1$. The Kalman Filter entails first formulation of the model in state space form and for a predefined set of initial parameters, the model prediction errors are generated from the filter which are then recursively used to evaluate the likelihood function until it is maximized. Jalles (2009) further comments that because of the state space representation of the DNS model and the time varying parameters within the model it makes the Kalman Filter appropriate to be used.

3.5.3 Mean-Variance optimization

Markowitz H. (1959) concluded that the optimal portfolio is one that provides the best combination of risk and return for a specific investor, depending on said investor's degree of risk tolerance. As a result, Markowitz introduced the concepts of variance and standard deviation with regards to the calculation of portfolio risk. To find the optimal portfolio, the following equations were developed by Markowitz (1952, 1959):

For a portfolio consisting of k assets such as bonds with different maturities with expected returns for our case interest rates μ_i . Let w_i be weight of the portfolio's value invested in asset i such that $\sum_{i=1}^n w_i = 1$, and let $w = (w_1, \dots, w_k)^T$, $\mu = (\mu_1, \dots, \mu_k)^T$.

Then, expected portfolio return is given by

Equation 11

$$E(R_p) = w^T \mu$$

Where $w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{bmatrix}$ which is $k \times 1$ weights vector and $\mu = \begin{bmatrix} E(R_1) \\ E(R_2) \\ \vdots \\ E(R_k) \end{bmatrix}$ which is $k \times 1$ vector of

expected returns to each of the asset.

In addition, the portfolio standard deviation is given by

Equation 12

$$\sigma_p = (w^T \Sigma w)^{1/2}$$

where w is $k \times 1$ weights vector and Σ represents $k \times k$ variance-covariance matrix of k assets by monthly returns of k assets.

Mathematically, the covariance between asset i and asset j , σ_{xy} , is given by

Equation 13

$$\sigma_{xy} = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$$

where n represent the no of observations. Therefore the $k \times k$ var-cov matrix is calculated as follows:

Equation 14

$$\Sigma_{k \times k} = \left(\frac{1}{n}\right) X^T X \text{ where } n \text{ is a scalar and } X \text{ is the } n \times k \text{ is the monthly returns of } k \text{ assets.}$$

Based on the expected portfolio return, Markowitz characterizes an efficient portfolio by its weight vector w_{eff} that solves the optimization problem:

Equation 15

$$w_{eff} = \arg \min w^T \Sigma w \text{ subject to } w^T \mu = \mu, w^T 1 = 1, w \geq 0$$

When short selling is allowed, the constraint $w \geq 0$ for all i from equation above can be removed, yielding the following problem that has an explicit solution:

Equation 16

$$w_{eff} = \arg_{w:w^T \mu = \mu, w^T 1 = 1} \min w^T \Sigma w$$

Based on equation, the optimization problem is modified to cater for the government's strategy to minimize the risk subject to additional constraints other than sum of weights being equal to 1, policy constraints on the minimum composition of debt portfolio at certain maturities to enhance liquidity and development of the domestic debt market.

3.5.4 Cost and risk measures

Debt Service cost is a primary concern for the government in managing sovereign debts and this depends on the debt issuance requirements and the realizations of interest rates. This is because sovereign debt managers are more concerned about debt service costs which directly government budget expenses rather than day-to-day fluctuations in the market value of debt portfolios. To identify the various decision choices sets $D_1, D_2 \dots$ which are the various alternative borrowing strategies, then the expected cost-risk trade-off over various borrowing strategies is identified, (Hahm & Kim, 2003).

4 Findings and Discussion

4.1 Yields only model

4.1.1 Introduction

The study examines the monthly averages of the Kenyan Treasury Bond Yields with maturities 1-30 years during the period March 2010 to October 2014. In computing the three time varying components of the yield curve: Level, Slope and Curvature, the study adopts the parsimonious Nelson Siegel approach to modelling the yield curve. The factors are derived from the set of yields observed during the study period.

4.1.2 Results and discussion

The three-factor term structure Nelson Siegel model provides a good fit of the Kenyan curve as observed in *Figure 3*. Diebold & Li (2006) interpret the model as a model of level, slope and curvature. These factors L_t , S_t and C_t also known as latent or unobserved factors contribute immensely to the shape of the curve. According to Afonso & Manual 2010, negative mean values of S_t and C_t imply an ascending and concave yield curve. Based on the *Figure 3*, it indicates that the Kenyan yield curve is an upward sloping and concave yield curve. This is supported by the negative mean values of S_t and C_t .

Figure 3: Fitting Nelson-Siegel yield curve

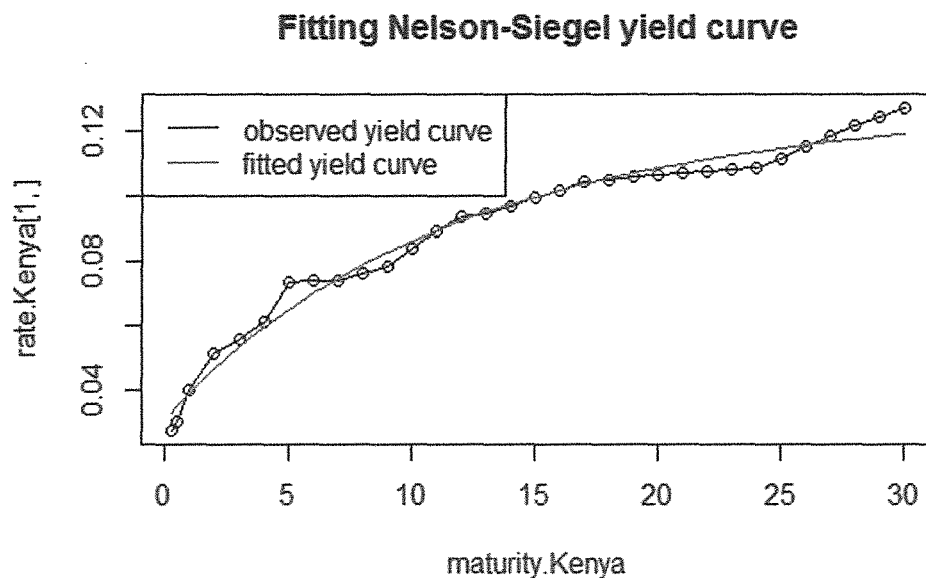


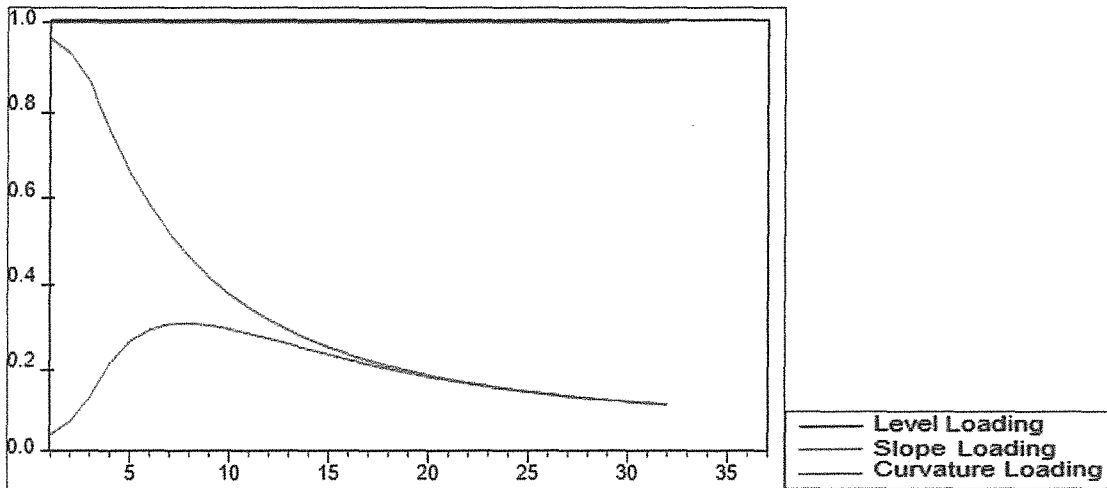
Table 2: Mean and Variance of time series parameters

	Mean	Variance
Level	0.1314	0.0002
Slope	-1.7090	5.8891
Curvature	-0.7141	5.8793

From table 2, the level factor has the least variance, followed by the slope and then curvature. Also, the variances of the slope and curvature are nearly equal. From the observation, the author can indicate that the level factor displays very high persistence followed by the slope and curvature. This observation is consistent with observation by Diebold and Li (2006).

Theoretically, the level factor is the long term component, slope is the short-term and curvature is medium term component. The long term component is constant across maturities; the medium term starts out at zero and decays to zero; and the short term curve has the fastest decay decaying to zero. This is observed in the curve below:

Figure 4: Factor Loadings as at March 2010



The time series estimates of the Nelson & Siegel model are found in Appendix 2

4.2 Markowitz mean-variance optimization approach

4.2.1 Introduction

The study proceeds to identify efficient cost-risk trade-offs and optimal debt portfolio which is based on the expected debt cost as a function of interest rates and bond portfolio. The interest rates are modelled through the dynamic Nelson Siegel model. The period of study for the

optimal domestic debt portfolio is similar to the yields only model period of study. Additional data used for the bond portfolio is the debt portfolio of nominal Kenya Treasury Bonds at maturities of 1-12, 15, 20, 25 and 30 years as of 31 October 2014, which amounts to KES 0.922 trillion. These weights for each maturity is summarized in table These weights are regarded as the reference borrowing strategy and alternative strategies are then constructed by allowing each of these weights to vary by minimizing the interest rate risk. These strategies are composed of nominal bonds issued with short, medium and long maturities therefore the optimal domestic debt portfolio is based on the maturity structure of the bonds.

4.2.2 Results and Discussion

4.2.2.1 Expected portfolio cost

To determine the portfolio expected cost, two basic components are required: the weights of bonds at different maturities (Appendix 4) and the expected costs for each maturity (Appendix 5) are determined.

4.2.2.2 Portfolio standard deviation

The variance of a portfolio is a function of the different bond maturities and the covariance between each of the different bond maturities. Standard deviation is determined by taking out the square root of the variance. This indicates the key components of the standard deviation of a portfolio include the variance-covariance matrix indicated in Appendix 3 and the weights of the different bond maturities indicated in Appendix 4.

This results to expected portfolio mean and standard deviation as at October 2014 as:

Table 3: Mean and Standard Deviation of portfolio comprised of different bond maturities

Portfolio Mean Cost	Portfolio Standard Deviation
0.121106	0.028436

4.2.2.3 Alternative Debt Strategies

Using approach used by Choi, Kim, & Lee (2010), alternative strategies are derived from the reference borrowing strategy in our case as at October 2014 by minimizing the portfolio mean costs subject to various constraints. The constraints include: no short selling allowed, the sum of weights of different bond maturities should equal to 1 and liquidity constraints where a mix

of short, medium and long term bond maturities should be issued for the development of the domestic market. Strategies are formulated as follows with varying constraints:

Table 4: Constraints for the various strategies

	Constraints
Strategy 1	No short selling; sum of weights equal to 1
Strategy 2	Strategy 1 constraints plus constraint on weights of key maturities for government. The weights should be maintained at: 10-13%, 10-15%, above 17%, above 15%, above 15% for 2-year, 5-year, 10-year, and 12-year and 15 year maturities respectively.
Strategy 3	Strategy 2 constraints plus constraint on total weights of short term, medium term and long term. The weights are maintained as indicated in Table 1

From the above constraints, the corresponding weights are shown in *Table 7*:

Table 5: Composition of various strategies at different bond maturities

	Strategy 1	Strategy 2	Strategy 3
2_YR		13.00%	10.00%
4_YR		0.00%	5.00%
5_YR		15.00%	10.00%
10_YR		17.00%	17.00%
12_YR		10.00%	23.00%
15_YR		15.00%	15.00%
30_YR	100.00%	30.00%	20.00%
Total Weights	100.00%	100.00%	100.00%

As a result, the portfolio mean cost and standard deviation of the three strategies are indicated as follows:

Table 6: Portfolio mean cost and standard deviation of the three strategies

	Portfolio Mean	Portfolio Standard Deviation
Strategy 1	0.1247	0.0239
Strategy 2	0.1220	0.0272
Strategy 3	0.1221	0.0270

Regarding the standard deviation as a risk property, the long issuance strategy [Strategy 1] turns out to have both marginally lower standard deviation than the other strategies that contain mixtures of short and medium dated bonds. This is in accordance with theoretical literature which indicate that long dated bonds lower the volatility of interest rates .However, the strategy is expensive relative to the other strategies indicating that bond investors are compensated for investing in long term bonds which is in line with the liquidity risk premium theory.

The optimal strategy which minimizes both interest rate cost and interest rate risk is [Strategy 3].It indicates that the government should issue medium and long dated bonds especially a higher proportion of 12-year bonds.

5 Conclusions

This paper adopts the mean-variance approach introduced by Markowitz (1952) to domestic debt portfolio based on models for the term structure of interest rates to determine the optimal domestic debt portfolio as at October 2014 is optimal. The study also adopts the dynamic version of the Nelson Siegel model in order to extract the yield curve latent factors. The use of the Nelson Siegel model was majorly motivated by its ability to provide an accurate description of the term structure movements observed from March 2010 to October 2014 with only three factors that could easily be extracted from observed yields. Once the parameters are extracted, yield rates for the various maturities are determined in order to determine the expected interest rate.

Using the composition of the domestic debt portfolio across maturities as weights, the expected interest rate and standard deviation of the portfolio are determined. Various strategies are determined from the portfolio as at October 2014 by minimizing the risk subject to constraints such as sum of weights being a total of 1, no short selling allowed and constraining the weights of key benchmark maturities such as the 2-year, 5-year, 10-year, 12-year and the 15 year bond to a certain weight. From these constraints, three strategies are constructed. The results of the strategies indicate that the government should issue more medium to long dated bonds to lead a desirable outcome of cheaper and less risky funding for the government.

Application of yield curve modelling using the dynamic latent factor approach to determine the optimal domestic debt levels offers a number of avenues for further research. This study focuses to determine whether the domestic debt portfolio is optimal; however, foreign debt portfolio should be assessed. To assess whether the debt portfolio is optimal, more up-to date term structure models incorporating the effect of various macroeconomic factors such as growth rates, short term rates, inflation rates and exchange rates should be used. This recognizes that term structure models are affected not only by unobserved factors but also macroeconomic factors. In addition, further research should be conducted to inform the government how much to borrow in the future based on future macroeconomic factors, government's financing requirement and budget deficit. This requires more complex models than the one used to be adopted which will also consider the government's risk preference. Such models could lead to better decisions to be made on what borrowing strategy that the government should undertake.

6 References

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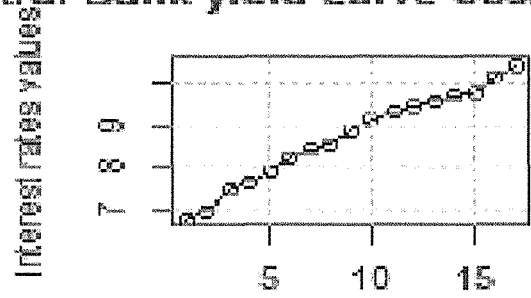
7 Appendices

Appendix 1: R Code for Fitting the Nelson Siegel Yield Curve Model

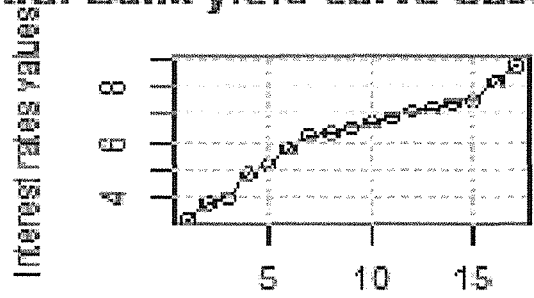
```
NS<-NSKenya[-1,]
Date<-as.Date(NS[,1],"%m/%d/%y")
NS<-NS[,-1]
NSK<-NS[,1:30]
library(YieldCurve)
library(xts)
NSA<-as.xts(NSK,Date)
rate.Kenya=first(NSA,'30 month')
maturity.Kenya=c(1:30)
NSParameters<-Nelson.Siegel(rate=rate.Kenya,maturity=maturity.Kenya)
y<-NSrates(NSParameters[5,],maturity.Kenya)
plot(maturity.Kenya,NSA[5,],main="Fitting Nelson-Siegel yield curve",
     xlab=c("Pillars in months"), type="o")
lines(maturity.Kenya,y, col=2)
legend("topleft",legend=c("observed yield curve","fitted yield curve"),col=c(1,2),lty=1)
grid()
par(mfrow=c(2,3))
for( i in c(1:30) ){
  plot(maturity.Kenya,NSA[i,], type="o", xlab="Maturities structure in years", ylab="Interest
rates values")
  title(main=paste("Kenya Central Bank yield curve observed at",time(NSA[i],jun=" ")))
  grid()
}
```

Appendix 2: Graphs of the Fitted Yield Curve as at October

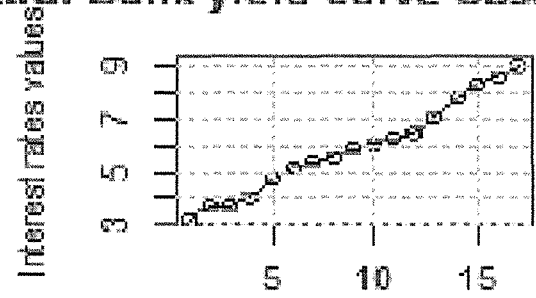
trial Bank yield curve observetrial Bank yield curve observetrial Bank yield curve observe



Maturities structure in years

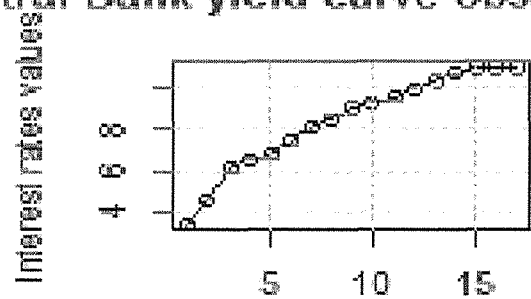


Maturities structure in years

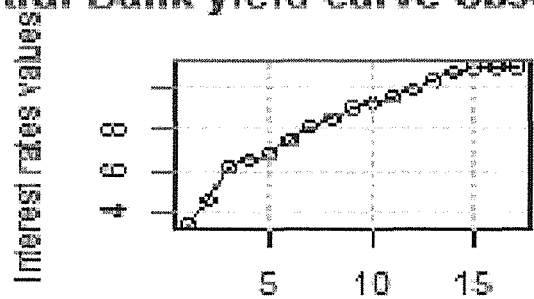


Maturities structure in years

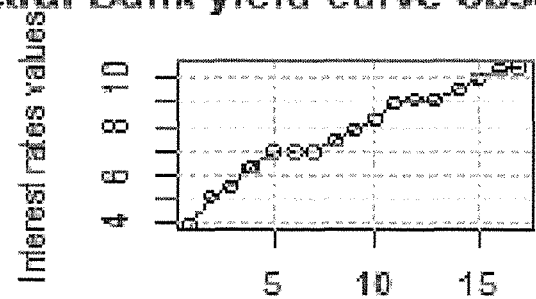
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Maturities structure in years



Maturities structure in years



Maturities structure in years

Appendix 3: Correlation-Matrix of the Different Bond Maturities

Years	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30
1	0.003640	0.002396	0.001982	0.001774	0.001650	0.001567	0.001508	0.001463	0.001429	0.001401	0.001378	0.001360	0.001318	0.001277	0.001252	0.001235
2	0.002396	0.001618	0.001358	0.001228	0.001151	0.001099	0.001062	0.001034	0.001012	0.000995	0.000981	0.000969	0.000943	0.000917	0.000901	0.000891
3	0.001982	0.001358	0.001150	0.001047	0.000984	0.000943	0.000913	0.000891	0.000873	0.000860	0.000848	0.000839	0.000818	0.000797	0.000785	0.000776
4	0.001774	0.001228	0.001047	0.000956	0.000901	0.000865	0.000839	0.000819	0.000804	0.000792	0.000782	0.000774	0.000755	0.000737	0.000726	0.000719
5	0.001650	0.001151	0.000984	0.000901	0.000851	0.000818	0.000794	0.000776	0.000762	0.000751	0.000742	0.000735	0.000718	0.000701	0.000691	0.000685
6	0.001567	0.001099	0.000943	0.000865	0.000818	0.000787	0.000764	0.000748	0.000735	0.000724	0.000716	0.000709	0.000693	0.000677	0.000668	0.000662
7	0.001508	0.001062	0.000913	0.000839	0.000794	0.000764	0.000743	0.000727	0.000715	0.000705	0.000697	0.000690	0.000675	0.000660	0.000651	0.000645
8	0.001463	0.001034	0.000891	0.000819	0.000776	0.000748	0.000727	0.000712	0.000700	0.000690	0.000682	0.000676	0.000662	0.000647	0.000639	0.000633
9	0.001429	0.001012	0.000873	0.000804	0.000762	0.000735	0.000715	0.000700	0.000688	0.000679	0.000671	0.000665	0.000651	0.000637	0.000629	0.000623
10	0.001401	0.000995	0.000860	0.000792	0.000751	0.000724	0.000705	0.000690	0.000679	0.000670	0.000663	0.000656	0.000643	0.000629	0.000621	0.000616
11	0.001378	0.000981	0.000848	0.000782	0.000742	0.000716	0.000697	0.000682	0.000671	0.000663	0.000655	0.000649	0.000636	0.000623	0.000615	0.000610
12	0.001360	0.000969	0.000839	0.000774	0.000735	0.000709	0.000690	0.000676	0.000665	0.000656	0.000649	0.000643	0.000630	0.000617	0.000610	0.000604
15	0.001318	0.000943	0.000818	0.000755	0.000718	0.000693	0.000675	0.000662	0.000651	0.000643	0.000636	0.000630	0.000618	0.000605	0.000598	0.000593
20	0.001277	0.000917	0.000797	0.000737	0.000701	0.000677	0.000660	0.000647	0.000637	0.000629	0.000623	0.000617	0.000605	0.000593	0.000586	0.000581
25	0.001252	0.000901	0.000785	0.000726	0.000691	0.000668	0.000651	0.000639	0.000629	0.000621	0.000615	0.000610	0.000598	0.000586	0.000579	0.000575
30	0.001235	0.000891	0.000776	0.000719	0.000685	0.000662	0.000645	0.000633	0.000623	0.000616	0.000610	0.000604	0.000593	0.000581	0.000575	0.000570

Appendix 4: Weights of Different Bond Maturities as at October 2014

Maturity length(Years)	Weights
1	0.00%
2	15.67%
3	0.00%
4	3.24%
5	19.03%
6	4.41%
7	0.94%
8	4.44%
9	1.97%
10	15.91%
11	0.43%
12	6.66%
15	15.50%
20	6.56%
25	2.19%
30	3.05%
Total Weights	100.00%

Source: Central Bank of Kenya

Appendix 5: Mean and Standard Deviation of Different Bond Maturities from March 2010 to October 2014

Treasury Bonds at different maturities		
Maturity	Mean	Standard Deviation
1_YR	0.101051	0.060335
2_YR	0.113262	0.040222
3_YR	0.117332	0.033918
4_YR	0.119366	0.030913
5_YR	0.120587	0.029173
6_YR	0.121401	0.028046
7_YR	0.121982	0.027259
8_YR	0.122419	0.026679
9_YR	0.12276	0.026233
10_YR	0.12303	0.025884
11_YR	0.123436	0.025366
12_YR	0.123252	0.0256
15_YR	0.123844	0.024856
20_YR	0.124251	0.024359
25_YR	0.124496	0.024067
30_YR	0.124658	0.023873