Strathmore UNIVERSITY

School of Computing and Engineering Sciences
Bachelor of Science in Electrical and Eletronics Engineering
End of Semester Examination
MAT 1101 - Mathematics I

Date: 7th November 2022 Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

a) Compute the absolute value and the conjugate of:

i. $z = (1 + i)^6$. [2 Marks]

ii. $w = i^{17}$. [2 Marks]

b) If $y = Sin(3x)$, show that $y'' + 9y = 0$. [2 Marks]

c) Find the domain of the following functions:

i. $T(m) = \frac{5m}{m^2-3m-4}$. [1 Mark]

ii. $S(w) = \sqrt{2w+3} - 1$. [1 Mark]

d) Prove by induction that

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \in \mathbb{N}$. [4 Marks]

e) Use integration by parts to evaluate $\int (1 + x^2)e^{-x}dx$. [3 Marks]

f) Express the sum $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots + \frac{1}{100}$ in sigma notation. [1 Mark]

g) Use the limit comparison test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^5 + 2^n}{3^n}$$

diverges or converges. [2 Marks]

h) Find the first 3 terms in the Maclaurins series expansion of:

$$\frac{x}{\sqrt{1-x^2}}.$$

[3 Marks]

i) Evaluate the limit: $Lim_{h \longrightarrow a} \frac{\frac{1}{h} - \frac{1}{a}}{h-a}$. [3 Marks]

j) Find y' given that $x^{Cos(y)} = Cos^x(y)$. [3 Marks]

k) Solve by the method of integrating factor $\frac{dy}{dx} - \frac{3y}{1+x} = (x+1)^4$.
. [3 Marks]

QUESTION TWO [15 Marks]

a) Simplify $2i(i-1) + (\overline{\sqrt{3}+i})^3 + (1+i)(\overline{i+1})$. [3 Marks]

b) Prove by induction that: [5 Marks]

$$1^2 + 4^2 + 7^2 + \cdots + (3n-2)^2 = \frac{n}{2}(6n^2 - 3n - 1).$$

c) Evaluate $\left(Sin\left[\frac{\pi}{6}\right] + iCos\left[\frac{\pi}{6}\right]\right)^{18}$. [4 Marks]

d) Given that $y = \frac{1-x^2}{1+x^2}$, show that: [3 Marks]

$$y' = \frac{-4x}{1-x^4}.$$

QUESTION THREE [15 Marks]

a) Evaluate $Lim_{x\to 4}\frac{x-4}{\sqrt{x}-\sqrt{8-x}}$. [3 Marks]

b) Given that $x = a\,(Cos(\beta) + \beta Sin(\beta))$ and $y = a\,(Sin(\beta) - \beta Cos(\beta))$, find $\frac{d^2y}{dx^2}$. [5 Marks]

c) Find the first 4 terms of the Maclaurins series for the function by first finding the partial fraction decomposition of the function:

$$\frac{x+1}{x^2-5x+6}.$$

[7 marks]

QUESTION FOUR [15 Marks]

a) Use an appropriate substitution to evaluate the following integrals:

   i. $\int \frac{Cos(ln[x])}{x}dx$. [2 Marks]

   ii. $\int 3Cos(x)e^{Sin(x)}dx$. [2 Marks]

b) Evaluate the following integrals:

   i. $\int Sin^5(x)Cos^2(x)dx$. [3 Marks]

   ii. $\int tan^6(x)Sec^4(x)dx$. [3 Marks]

c) Integrate by partial fractions: [5 Marks]

$$\int \frac{x^3+3x+1}{(x+1)^2(x-2)^2}dx.$$

QUESTION FIVE [15 Marks]

a) Find $y^{'}$ given the following functions:

i. $y = Cos(x)$. [3 Marks]

ii. $y = 3Sin(5x)Cos(4x)$. [3 Marks]

iii. $y = \frac{Sin(x)}{1+Cos(x)}$. [3 Marks]

iv. $Sin(xy) = 5xy^2 + Cos(y^3)$. [3 Marks]

b) Use reduction of order to find the second solution for

$$x^2y^{''} - 3xy^{'} + 4y = 0$$

given that $y_1 = x^2$. Give the general solution. [3 Marks]