STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

END OF SEMESTER UNIVERSITY EXAMINATIONS

MASTER OF SCIENCE IN BIOMATHEMATICS

BMA 8101: INTRODUCTION TO MATHEMATICAL COMPUTING

DATE: 15th AUGUST 2023 **TIME: 3 HOURS**

INSTRUCTIONS: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (30 MARKS)

(a) Outline any three benefits and any three challenges of mathematical computing.

[3 marks]

(b) Describe the following as used in **MATLAB** and give an example of each.

(i) if loop. **[2 marks]**

(ii) switch group. **[2 marks]**

(iii) if-elseif ladder. **[2 marks]**

(c) In a bid to understand the pressure, temperature and density as a result of changes in altitudes, scientists came up with models for pressure, temperature and density given by $p = p_0\left(1 - \frac{\lambda z}{T_0}\right)^{g/\lambda R}$, $T = T_0 - \lambda z$ and $\rho = \frac{p}{RT}$ respectively. Where z=altitude(m), T=air temperature(K), p=air pressure (Pa), ρ=air density (Kg/m^3), T_0=288.15 K (Temperature at $z = 0$), p_0=1.01325×10^5 Pa pressure at $z = 0$, R=287 J/(Kg-K) gas constant for air, $g = 9.81 m/s^2$=gravitational constant and $\lambda = 0.0065 K/m$ =lapse rate. Write a **MATLAB** script that calculates p, T and ρ for every $1000m$ from $z = 0$ to $z = 12000m$ and prints the results in a table format and saves the results in a text file.

[5 marks]

(d) Consider the system of algebraic equations given by

$$\begin{aligned} 3x + 4y &= 23, \\ y + 17z &= 10.5, \\ 7x + 2y + 13z &= 19. \end{aligned}$$

Outline the procedure of solving for x, y and z using matrix inversion and by using the backslash operator in **MATLAB** and determine which technique is computationally efficient and why.

[4 marks]

(e) There are several techniques of finding the roots of a given number. Write a **MATLAB** script to compute for the square root of 3 between 1 and 2 using the *Bisection method*.

[5 marks]

(f) In a bid to understand the reduction of error of different algorithms used in optimization of vaccination models. The following data was obtained.

Table 1: Algorithm 1

Iterations	100	200	350	460
Residual	0.06	0.007	0.000004	1.23e-7
Error	1.2e-1	4.8e-2	9.6e-5	1.34e-5

Table 2: Algorithm 2

Iterations	400	700	1560	5200
Residual	0.034	0.0002	5.34e-9	2.9e-10
Error	1.34e-2	5.61e-5	8.32e-8	7.43e-9

Outline various ways this data can be visualized for better deductions. **[4 marks]**

(g) This program determines the best fit polynomial approximating function of orders 2 through 5 for the data set listed below and writes the formatted data in the plot title. List the errors in the code and how to remedy them.

```
function [ ]=curve_fitting_pol()
clear; clc;
x = -10:2:10;
y = [-980 -620 -70 80 100 90 0 -80 -90 10 220];
x2 = -10:0.5:10;
mse = zeros(4);
for n = 2:5

    coef = zeros(n+1);
    coef = polyfitting(x,y,n);
    yc2 = polyval(coef,x2);
    yc = polyval(coef,x);
    MSE(n) = sum((y-yc).^2)/length(x);
    fprintf(' x y yc \n');
    fprintf('------------------------------\n');
    for i = 1:length(x)
        fprintf('%5.1f %5.1f %8.3f \n',x(i),y(i),yc(i));
    end
    fprintf('\n\n');
    subplot(2,2,n-1),plot(x2,yc2,x,y,o),
    xlabel('x'), ylabel('y'), grid, axis([-10 :10 : -1500 : 500]);
    title(sprintf('Degree %d polynomial fit',n));
end
fprintf(' n MSE \n')
fprintf('---------------------\n');
for n = 2:5
```

```
      fprintf(' %d %8.2f \n',n,MSE(n))
end
end
end
```

[3 marks]

QUESTION TWO (15 MARKS)

(a) Outline the procedure of non-linear least-squares approximation using the data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, including the iterative scheme of Gauss-Newton method that is used in the solution.

[7 marks]

(b) Given the data $(1, 4.6)$,$(2, 8.82)$ and $(4, 31.3)$, find the best-fit curve $x = a_0 \exp(a_1 t)$ using the procedure outlined in (a) above. And use it to determine x at $t = 49.3$.

[8 marks]

QUESTION THREE (15 MARKS)

(a) A mathematical model for flu for a county having 4 million people has four population compartments namely, susceptibles, exposed, infected and recovered. Assuming no demographics and there was no acquired immunity. Write a **MATLAB** script that solves the mathematical model considering a period of one year with the initial conditions given as $S = 0.7N, E = 5000, I = 5000, R = 0.3N$, choose arbitrary parameters.

[9 marks]

(b) Considering (a), the daily number of the infected persons is recorded in a *.csv* file, using this information, outline a procedure we can use to estimate the parameters using available data of infected persons.

[6 marks]

QUESTION FOUR (15 MARKS)

(a) Find the line $y = a_o + a_1 x + a_2 x^2$ that best fits the data points $(3, 2.9)$,$(4, 8.2)$,$(6, 7.8)$ and $(9, 10.4)$. **[5 marks]**

(b) Consider the following system of ordinary differential equations

$$\begin{aligned} x'(t) &= 3x(t) + 4y(t) - z(t), \\ y'(t) &= x(t) + 4z(t), \\ z'(t) &= 4x(t) + 4y(t) + 6z(t), \end{aligned}$$

with the initial conditions given as $x(0) = 4.5, y(0) = 2$ and $z(0) = 3$. Using the *dsolve* function, write a **MATLAB** script to obtain the analytical solutions and further how to compare the results when using *ode23* solver. **[6 marks]**

(c) Outline the procedural steps of Monte-Carlo methods. **[4 marks]**