Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

BBS (Actuarial Science, Finance and Financial Economics)

END OF SEMESTER EXAMINATION

BSA 2206 FINANCIAL MATHEMATICS

DATE: 21st November 2018 Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question One

A. A university student receives a 3-year sponsorship grant. The payments under the grant are as follows:

Year 1: £ 5,000 per annum paid continuously.

Year 2: £ 5,000 per annum paid monthly in advance.

Year 3: £ 5,000 per annum paid half-yearly in advance

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly. [6]

B.

a. Explain what is meant by the expectations theory for the shape of the yield curve. [2]

i. Short-term, one-year annual effective interest rates are currently 8%; they are expected to be 7% in one years time, 6% in two years time and 5% in three years time.

i. Calculate the gross redemption yields (spot rates of interest) from 1-year, 2-year, 3-year and 4-year zero coupon bonds assuming the expectations theory explanation of the yield curve holds. [4]

ii. The price of a coupon paying bond is calculated by discounting individual payments from the bond at the zero-coupon bond yields in (i). Calculate the gross redemption yield of a bond that is redeemed at par in exactly four years and pays a coupon of 5 per annum annually in arrear. [4]

C. A property development company has just purchased a retail outlet for $4,000,000. A further $900,000 will be spent refurbishing the outlet in six months' time. An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for $6,800,000. The initial rent will be $360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance. Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective. [9]

D. £1,000 is invested for 10 years. In any year the yield on the investment will be 4% with probability 0.4, 6% with probability 0.2 and 8% with probability 0.4 and is independent of the yield in any other year. Calculate the standard deviation of the accumulation at the end of 10 years. [5]

Question Two

a. Explain what is meant by the following terms:

i. Payback period [2]

ii. Discounted payback period [2]

An insurance company is considering setting up a branch in a country in which it has previously not operated. The company is aware that access to capital may become difficult in twelve years time. It therefore has two decision criteria. The cashflows from the project must provide an internal rate of return greater than 9% per annum effective and the discounted payback period at a rate of interest of 7% per annum effective must be less than twelve years.

The following cashflows are generated in the development and operation of the branch.

Cash Outflows

Between the present time and the opening of the branch in three years time the insurance company will spend £1.5m per annum on research, development and the marketing of products. This outlay is assumed to be a constant continuous payment stream. The rent on the branch building will be £0.3m per annum paid quarterly in advance for twelve years starting in three years time. Staff costs are assumed to be £1m in the first year, £1.05m in the second year, rising by 5% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in three years time and assumed to be incurred for 12 years.

Cash Inflows

The company expects the sale of products to produce a net income at a rate of £1m per annum for the first three years after the branch opens rising to £1.9m per annum in the next three years and to £2.5m for the following six years. This net income is assumed to be received continuously throughout each year. The company expects to be able to sell the branch operation 15 years from the present time for £8m.

Determine which, if any, of the decision criteria the project fulfils. [16]

Question Three

A pension fund has the following liabilities: annuity payments of £160,000 per annum to be paid annually in arrears for the next 15 years and a lump sum of £200,000 to be paid in ten years. It wishes to invest in two fixed-interest securities in order to immunise its liabilities. Security A has a coupon rate of 8% per annum and a term to redemption of eight years. Security B has a coupon rate of 3% per annum and a term to redemption of 25 years. Both securities are redeemable at par and pay coupons annually in arrear.

i. Calculate the present value of the liabilities at a rate of interest of 7% per annum effective. [3]

ii. Calculate the discounted mean term of the liabilities at a rate of interest of 7% per annum effective. [4]

iii. Calculate the nominal amount of each security that should be purchased so that both the present value and discounted mean terms of assets and liabilities are equal. [9]

iv. Without further calculation, comment on whether, if the conditions in (iii) are fulfilled, the pension fund is likely to be immunised against small, uniform changes in the rate of interest. [4]

Question Four

An insurance company holds a large amount of capital and wishes to distribute some of it to policyholders by way of two possible options.

Option A

£100 for each policyholder will be put into a fund from which the expected annual effective rate of return from the investments will be 5.5% and the standard deviation of annual returns 7%. The annual effective rates of return will be independent and $(1+ i_t)$ is lognormally distributed, where i_t is the rate of return in year t. The

policyholder will receive the accumulated investment at the end of ten years.

Option B

£100 will be invested for each policyholder for five years at a rate of return of 6% per annum effective. After five years, the accumulated sum will be invested for a further five years at the prevailing five-year spot rate. This spot rate will be 1% per annum effective with probability 0.2, 3% per annum effective with probability 0.3, 6% per annum effective with probability 0.2, and 8% per annum effective with probability 0.3. The policyholder will receive the accumulated investment at the end of ten years.

Deriving any necessary formulae:

a. Calculate the expected value and the standard deviation of the sum the policyholders will receive at the end of the ten years for each of options A and B. [13]

b. Determine the probability that the sum the policyholders will receive at the end of ten years will be less than £115 for each of options A and B. [5]

c. Comment on the relative risk of the two options from the policyholders' perspective.

[2]

Question Five

a. Derive the formula for obtaining delivery price of a forward contract under each of the following circumstances (Define clearly all symbols used): [9]

- A forward contract on a stock that pays no dividend.
- A forward contract on a stock that pays fixed amount of dividend
- A forward contract on a stock that pays dividend yield

b. A bond is priced at £95 per £100 nominal, has a coupon rate of 5% per annum payable half-yearly, and has an outstanding term of five years. An investor holds a short position in a forward contract on £1 million nominal of this bond, with a delivery price of £98 per £100 nominal and maturity in exactly one year, immediately following the coupon payment then due. The continuously compounded risk-free rates of interest for terms of six months and one year are 4.6% per annum and 5.2% per annum, respectively. Calculate the value of this forward contract to the investor assuming no arbitrage. [5]

c. A three-year forward contract is to be issued on a particular company share. The current market value of the share is £4.50 and a dividend of £0.20 per share has just been paid. The parties to the contract assume that the future quarterly dividends will increase by 1% per quarter-year compound for the first two years and by 1.5% per quarter-year compound for the final year. Assuming a risk-free force of interest of 5% per annum, and no arbitrage, calculate the forward price. [6]