Strathmore
UNIVERSITY

Strathmore Institute of Mathematical Sciences
BBS Actuarial Science, BBS Finance & BBS Financial Economics
END OF SEMESTER EXAMINATION

BSA 3108:- THEORY OF FINANCE

Date: 24$^{\text{th}}$ July, 2019   Time: 1415 - 1615 Hours

INSTRUCTIONS

1. There are FIVE questions in this examination

2. Attempt QUESTION ONE and ANY OTHER TWO questions

3. Question one is COMPULSORY and carries 30 marks

4. All the other questions carry 20 marks each

5. Show all *your workings* and clearly define all *notations* used

QUESTION 1   -   COMPULSORY

(a) Discuss the risk aversion attributes of an investor who exhibits the following utility under special circumstances when $\beta = 1$ and $\gamma = -\infty$.

$$u(\omega) = \frac{1 - \gamma}{\gamma} \left(\frac{\alpha \omega}{1 - \gamma} + \beta \right)^{\gamma}$$

[8 marks]

(b) In advancing our knowledge in Finance, it is argued that a theory should achieve three objectives concurrently. It should enable a full cycle of scientific research, formalize theory to tests and yield itself to practical applications.
Using the **State Preference Theory** as an example critically discuss the foregoing comments. Be careful to define all the assumptions and notations used. [10 marks]

(c) At the quarterly meeting of an Investment Club, four members are making proposals for new equity investment for the club. *Alex* wants to buy shares in Amani Ltd, claiming that they have performed poorly in recent weeks and are due for an upturn. *Benta* wants to invest in Bashasha Ltd, a company that recently recruited a new head of marketing, who

has had successes at other companies. Benta feels that this new appointment will have a positive effect on the firm. *Churchill* selects shares at random. This quarter he is recommending that the club buy the shares of Chechema Ltd. *Diana* wants the club to buy shares in Dimba Ltd. His brother works for a major health insurer and has insider information that Dimba's shares will rise sharply in the near future, when it is announced that his company has appointed Dimba as its dentist of choice.

    i. For each club member, describe how their share selection strategy would work in strongly efficient, semi-strongly efficient, weakly efficient and inefficient markets. [8 marks]

    ii. Outline the role that portfolio managers play even if the market is perfect and fully efficient. [4 marks]

Total for Question 1: 30 marks

QUESTION 2   -   OPTIONAL

(a) Consider the following payoffs:

$$X_1 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad X_3 = \begin{bmatrix} 2 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix}$$

The associated price vectors for the payoff matrices X_1, X_2 and X_3 is $p_1 = (1 \ 1)'$, $p_2 = (3 \ 3)'$ and $p_3 = (3 \ 3)'$ respectively.

    i. Which of these pairs is arbitrage free? Explain your response. [2 marks]

    ii. For each of the pairs, calculate the state prices q [4 marks]

    iii. Use your responses to (i) and (ii) above to calculate the equilibrium price for an asset that pays off 4 in state 1 and 6 in state 2. [2 marks]

(b) The optimization problem in Mean Variance Portfolio Optimization (MVO) entails either finding the portfolio ω that has the highest expected return for a given level of risk as measured by portfolio variance or finding the portfolio ω that has the smallest risk that achieves a target return as shown below:

$$\begin{aligned} \min_{\omega} \sigma^2_{p,\omega} &= w'\Sigma w \ s.t. \\ \mu_{p,\omega} &= w'\mu = \mu^0_p = target \ return \\ \mathbf{w'1} &= 1 \end{aligned}$$

    i. Using matrix algebra, work out the optimal portfolio ω that has the smallest risk that achieves the target return [8 marks]

    ii. Explain why most optimization schemes follow the approach in (i) above (i.e. finding the portfolio ω that has the smallest risk that achieves a target return as opposed to finding the portfolio ω that has the highest expected return for a given level of risk) [4 marks]

Total for Question 2: 20 marks

QUESTION 3 - OPTIONAL

(a) One pillar on which Mean Variance Optimization rests is the linkage of investor preferences and asset returns. Specifically, the assumption that investors make their decisions purely on the basis of expected return (mean) and variance of returns. Using suitable workings, explain how the quadratic utility function below leads to expected utility that depends only on the mean and variance if the the investor's wealth is assumed to be normally distributed.

$$u(\omega) = \omega - b\omega^2$$

[8 marks]

(b) Your services have been retained as an investment advisor to Omega Investments Ltd, a Nairobi based company that heavily invests in stocks. Omega invests in the stock of Alpha Telecommunications Ltd currently trading at KES 100. Based on your long term capital markets expectations you strongly believe that the stock market will perform well in the foreseeable future. This strong performance of the stock market together with Alpha's favourable fundamentals has made you to assign a 60% chance of the company's stock to increase by 25% in the next three months. As an astute believer in prudent risk management, you develop a pessimistic scenario assigning a 40% chance of a decline in Alpha's stock by 20% in the next three months. Based on your knowledge recently gathered from your BBS classes you think that using options to take advantage of the market movements will be advantageous to Omega as opposed to taking a gamble with the Alpha's stock itself. You are aware that call and put options on Alpha's stock exist with six-month maturities and an exercise price of KES 90. You have also established that the quarterly compounded risk free rate is estimated at 10%.

   i. What is the maximum amount you would pay for Alpha's *put options* today? [7 marks]

   ii. One of the queries from the Chief Investment Officer (CIO) at Omega is "how sensitive is the *call option* value to changes in the price of the Alpha's stock?" Formulate a well thought response to the CIO based on the Black-Scholes-Merton (BSM) model. Assume that the stock pays dividends at a continuous dividend yield of 1.4%, the continuously compounded interest rate is 8% and the annual volatility of the Alpha's stock returns is 10%. [5 marks]

Total for Question 3: 20 marks

QUESTION 4   -   OPTIONAL

(a) Briefly explain the following characteristics of a coherent risk measure. Explain how Value at Risk (VaR) satisfies each of the attributes.

   i. Translation Invariance   [3 marks]

   ii. Monotonicity   [3 marks]

   iii. Sub-additivity   [3 marks]

   iv. Positive Homogeneity   [3 marks]

(b) Many people believe that wealthy individuals are less risk averse - in absolute terms - than poorer individuals. Prove that this is only true if people exhibit Decreasing Absolute Risk Aversion.   [4 marks]

(c) An investor exhibits negative exponential utility of the form below:

$$u(\omega) = -e^{-\alpha\omega}$$

Work out the expression for the investor's certainty equivalent level of wealth if $\alpha > 0$ and the investor's wealth is normally distributed as $\omega \sim N(\mu, \sigma_\omega^2)$.   [4 marks]

Total for Question 4: 20 marks

QUESTION 5   -   OPTIONAL

(a) Briefly discuss the Joint Hypothesis problem as postulated in Asset Pricing.   [4 marks]

(b) In the author's concluding remarks on the Arbitrage Pricing Theory (APT), Bailey (2005) observes that "In a sense, the APT is more general than the Capital Asset Pricing Model (CAPM) because, if the CAPM is true, the predictions of the two are indistinguishable (observationally equivalent). However, there are circumstances for which the APT predictions could hold when those of the CAPM do not." Using suitable workings, discuss the foregoing statement under the following subheadings:

   i. The APT is a factor model under the arbitrage principle   [4 marks]

   ii. If the CAPM is true, the predictions of the two models, APT and CAPM, are indistinguishable   [6 marks]

   iii. The APT is definitely advantageous over the CAPM its limitations notwithstanding?   [6 marks]

Total for Question 5: 20 marks

END!