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**ON PORTFOLIO OPTIMIZATION: A CASE STUDY BASED ON THE
NSE**

KUNGU MICHAEL NDICHU

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Strathmore University
Nairobi, Kenya

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Kungu Michael Ndichu

Signature

Date

This Research Project has been submitted for examination with my approval as the Supervisor.

Mr. Ishmael Maina

Signature

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Strathmore Institute of Mathematical Sciences

Strathmore University

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List of Abbreviations

NSE	Nairobi Securities Exchange
NASI	Nairobi Securities Exchange All Share Index
CAPM	Capital Asset Pricing Model
APT	Arbitrage Pricing Theory
SMB	Small Minus Big
HML	High Minus Low
WML	Winner Minus Loser

Abstract

Diversification and active portfolio management have been suggested as means to enable investors obtain an optimal portfolio return with reduced risk exposure. Factor diversification has been proposed as a means to help explain this portfolio return. Both of this will help form the basis of this research. The aim of this study is to form an optimal portfolio whose return is expected to beat the market index, NSE All Share Index. The study will use the Carhart 4 factor model to classify securities into asset categories from which an optimal portfolio mix will be obtained. The risk-adjusted return to be used will be the Sharpe ratio and the Treynor ratio. This study will use data from the 48 listed securities at the NSE that were tradable during the period of 2007-2015. The study aims to build the knowledge of the portfolio investment in the NSE being a frontier market securities exchange and offer an investment strategy on the same.

1 Introduction

1.0 Background

Portfolio optimization occurs where an investor tries to maximize their portfolio through optimally investing in a pool of diversified portfolios that will give the investor a maximum return for an acceptable level of risk. Prior to Markowitz's "Portfolio Selection" paper, investors used to focus on the risk-reward relationship of individual securities in order to construct their own portfolios. Investment managers used to pick securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these. Markowitz introduced the concept of portfolio selection in place of individual securities selection. Investors would focus on the overall risk-reward characteristics of the portfolio instead of individually picking securities that offered attractive risk-reward characteristics (Markowitz, 1952).

William Sharpe came up with the CAPM model to help investors explain asset returns in their portfolio (Sharpe, 1964). He argued that asset returns would be explained by one market factor, excess market return, $R_m - R_f$. This was a simple method to be used to explain asset returns but attracted some level of criticism. Fama and French helped advance the CAPM model by introducing 2 additional factors; a size factor and value factor (French & Fama, 1993). This formed the Fama-French 3 factor model. This was later advanced by Carhart who introduced momentum as an additional factor to explain asset return as he analyzed the reason behind the persistence in Mutual Fund performance. This led to the formation of the Carhart four factor model which is the subject of this study (Carhart, 1997).

Based on the Carhart 4 factor model, the aim of this study is to classify the securities into subgroups based on their performance. This study will focus to check the performance of the securities traded in the Nairobi Securities Exchange (NSE). This study will focus on the 48 listed securities at the NSE that were tradable during the period of 2007-2015. The aim of the study is to form an optimal

portfolio whose return will be compared against the NASI, NSE All Share index.

Portfolios will be actively rebalanced at the year end to ensure that the global minimum variance portfolio taken up adequately reflects market performance. The return compared to the NASI index will be based on a risk adjusted basis. The risk adjusted measure to be used will be the Sharpe ratio and the Treynor ratio.

1.1 Problem Statement

Portfolio Optimization is keen to any investment manager who wishes to achieve the client's return objective. Portfolio returns are high in emerging markets due to the high cost of capital required to run an investment there (Kaminsky, Lyons, & Schmukler, 2001).

An investor who wishes to maximize their portfolio returns will hold a mean-variance efficient portfolio. This will enable the investor to try and maximize the investment return while reducing their risk (Markowitz, 1952). Through the mean variance optimization an investor will be able to obtain the global minimum variance weights for their portfolio. In this light, this study will aim to obtain optimal portfolio weights using the variance-covariance matrix approach suggested in the mean variance optimization approach.

A factor model is used to help explain asset returns. The Carhart Four factor was preferred to explain asset returns in the Nairobi Securities Exchange (NSE). This is line with the study carried out by Odera (2010) who found that the Fama-French Three factor model was inadequate to explain the asset returns in the NSE and recommended the use of the Carhart Four factor model to explain asset returns (Odera, 2010). This will help classify securities traded in the NSE into sub groups which will form the basis of the portfolio selection process.

1.2 Research Objectives

The aim of this study is to classify asset into asset classes and form an optimal portfolio that can beat the NSE All Share index. This study will aim to achieve the following objectives;

- 1) Use the Carhart Four factor model to form asset classes in the NSE.
- 2) To form an optimal mean variance securities portfolio that can beat the NSE All Share Index in the form of a risk adjusted return.

1.3 Research Questions

Using the 48 listed securities trading in the NSE from the year 2006, this study intend to answer the following questions:

- 1) To investigate the Carhart Four factor model ability to subgroup assets in the NSE?
- 2) Can an optimal mean variance securities portfolio beat the NSE All Share Index in the form of a risk adjusted return?

1.4 Justification

Several studies such as (Kenny, Johnson, & Kunkel, 2013) and (Berger & Fieberg, 2016) have attempted to use the Carhart 4 factor model to explain asset returns and classify asset classes which they used to form optimal portfolios. Berger and Fieberg (2016) found out that the Carhart Four factor model was ideal to explain asset returns listed in the Dow Jones Industrial Index (DJI) from 2000 to 2015. He later used the variance-covariance risk optimization approach to form portfolios and compared their performance across multi-periods in an out-of-sample approach.

Other studies like the (Keppler & Encinosa, 2011) found out that equity markets of emerging markets had been giving a return (19.19%) higher than the MSCI World Index return of 12.67%. Kenya is an example of an emerging market in Africa and is a leading frontier market attracting Foreign Direct Investments (FDI). The aim of this is to try to advice foreign investors that the Kenyan Equity Market

is quite vibrant and can provide higher returns compared to investments in the Developed Markets Equities. There is a knowledge gap where it comes to studies on optimal portfolio investments in the NSE and this paper aims to fill it.

There are two main shortcomings of this study. First, the study uses secondary data from the NSE. An error with the secondary data will affect results of the study. Secondly, the study focuses only on the Kenyan Equity market and thus may not be a truly representative of the entire Emerging markets.

2 Literature Review

Portfolio optimization will be based on investment in securities that exhibit higher average returns and higher risk-adjusted return over the long trend.

2.1 Portfolio Theory

Diversification came under attack after the 2008/2009 financial crisis. This was after the failure of asset class diversification to protect investors against market failures (Amenc, Ducoulombier, Goltz, Lodh, & Sivagaminathan, 2016). This was a result of the increased correlation between asset classes during the housing market crash. Markowitz had introduced the mathematical concept of diversification in the finance world when he advised investors to invest in a portfolio of securities whose correlations will result in a reduced variance (Markowitz, 1952).

Markowitz assumed that investors were risk averse and would be willing to take less risk for a similar level of return. This formed the basis of minimum variance optimization theory. Investors would focus on forming portfolios that minimized their overall variance. The portfolio with the maximum expected return is not necessarily the one with minimum variance. The investor should choose the portfolio that lie on the efficient frontier of portfolios so as to optimally balance between risk and reward. Markowitz proposed diversification to reduce risk because asset classes were affected by similar systematic factors hence can easily be diversified across asset classes.

Roll (2013) found out that well diversified portfolios are highly correlated within the same asset class and much less correlated across asset classes. This supported Markowitz view of diversification across asset classes. He however disagreed with Markowitz that diversification is an effective tool that minimizes the risk from higher correlations. He instead argues that the true benefit of diversification is enhanced return and not reduced risk because the actual diversification benefit is obtained by the minimum variance portfolio (Roll, 2013).

2.2 Asset Selection

Various models came up after the portfolio theory to explain the relationship between risk and return and how one can maximize return while reducing risk. The Capital Asset Pricing Model (CAPM) was introduced by Sharpe in his 1964 paper. The CAPM argued that investors should hold the market portfolio with positions in the risk-free asset. He argued that asset returns could be explained by the price of time (the risk free rate) and the price of risk (market risk premium). By holding the market portfolio investors could diversify their risk and be left with the systematic risk which he represented using the asset beta. The asset beta would explain the relationship of how the asset moves in relation to the market (Sharpe, 1964).

Various criticism came up over the simplicity of the CAPM model to explain asset return. Asset returns were found to be too complex to be explained by one variable. Stephen Ross introduced the Arbitrage Pricing Theory (APT) which takes into account several factors to explain asset return (Ross, 1976). These factors include macroeconomic, fundamental and statistical factors. Macroeconomic factors use observable economic time series as a measure of the pervasive factors in securities return. Fundamental factors use specific asset attributes that are important in explaining cross-sectional differences in asset returns. Statistical factor uses historical asset returns data to determine factors that influence asset returns (Connor, 1995).

Multifactor models are preferred to explain asset returns in the market as they help reduce the noise since many of the sources of return will be accounted for. According to Connor (1995) asset returns respond linearly to macroeconomic shocks. The macroeconomic factor model is the simplest and most intuitive model of the three. Secondly, the study confirmed that the statistical and fundamental models outperform the macroeconomic model. In addition, the marginal explanatory power of the macroeconomic model is zero when it is added to the fundamental model.

This paper will focus on a fundamental factor model, the Carhart 4 factor model to explain asset returns.

2.3 Fundamental models

Fundamental factor models are a type of multifactor models that use asset specific characteristics to determine the risk factors the asset is exposed to and hence explain its return. There are several fundamental factor models such as the Fama-French three-factor model and the Carhart four factor model.

Fama and French (1993) came up with the Fama and French three factor model to explain asset returns. This was in response to the criticism of the CAPM model. The CAPM model focused on the returns on the market portfolio as the only factor that affected asset returns. It ignored all the intrinsic asset characteristics such as size, leverage, price-earnings ratio and book-to-market equity. They believed that these factors had explanatory power given the relationships between these variables and the average stock returns in a market. They added two additional risk factors, size and value, to the equity risk premium to explain asset returns. This formed the Fama and French 3 factor model. (French & Fama, 1993).

Size factor was to be measured by market capitalization. This would help classify assets into small cap and large cap assets. The difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks formed the SMB_t . Small cap stocks were found to offer a large return than large cap stocks. Value factor was to be measured by book to market equity (BM/ME) where the book value of stocks was compared to their market equity values. The difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks formed the HML_t . High book to market stocks were identified to be undervalued stocks and offered a high return when compared to low book to market stocks which were found to be overvalued stocks (Odera, 2010). This will form basis of the data analysis where the SMB will be

used to classify the stocks into value and growth stocks which will then be divided into various portfolios to compare performance.

Carhart in his 1997 analysis of the Persistence of Mutual Fund Performance included the momentum anomaly to explain the performance of the securities. Momentum was measured by winner minus loser portfolios (WML_t). This modified the Fama and French three factor model to form the Carhart four factor model. Based on the study, persistence in mutual fund performance was mostly driven by the one-year momentum effect. However following the momentum strategy in stocks did not result individual funds earning higher returns. Trimech and Kortas in their study of the Four Factor Carhart model found that the momentum factor, has a significant impact on the expected stock returns within the multiscale framework (Trimech & Kortas, 2009). This will form the basis of this study. This study will test to see if the Carhart Four Factor model can effectively express asset returns in the Nairobi Securities Exchange (NSE).

2.4 Portfolio Optimization

Markowitz in his Portfolio Theory paper argued that for investors to form an optimal portfolio they had to calculate their asset co-variances in order to identify asset classes that will greatly diversify their portfolios (Markowitz, 1952). He suggested the formula for covariance to be;

$$E(R_i, R_j) = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

This was meant to enable investors to identify the relationship across asset classes thus eliminate diversifiable risk by investing in asset classes that are affected by different unsystematic risk. This would reduce the price of risk to the systematic risk that cannot be diversified (Sharpe, 1964).

For a portfolio, Markowitz proposed the use of the variance-covariance matrix to minimize the portfolio variance. This made it easy for investment managers to access the risk-return relationship of their portfolios making Mean-Variance Optimization the mostly widely used form of investment strategy (Niunco, 2011).

Zakamulin (2015) argued that it was difficult to forecast mean returns. This made mean variance optimization to be barely used to make portfolio decisions and argued that risk optimization was what was mainly used in practice, while mean-variance optimization is barely used. This is because Portfolio risk optimization only relies on the forecast of the variance-covariance matrix and not on the mean returns (Zakamulin, 2015). This study will use the variance-covariance matrix approach to determine asset weights and will not forecast asset returns.

Berger and Fieberg (2016) found out that portfolios which are constructed on variance-covariance matrices stemming from multi-scale factor models outperformed portfolio allocations which do not take the multi-scale nature of asset and factor returns into account. They argued that the multi-scale nature of the return distribution should be taken into account when it comes to the forecasting of variance-covariance matrices and their ability to minimize the variance of portfolios. This study will not use a multi-scale factor model in checking for optimal portfolio performance but will rather use an in-sample forecast to identify if a particular portfolio allocation would have produced better returns than the market index-the NASI, NSE All Share Index. Berger and Fieberg (2016) study found out that a multi-scale Carhart Four-factor model is sufficient for minimum variance portfolio selection. This helps confirm the accuracy of the use of the Carhart Four Factor model to form an optimal portfolio (Berger & Fieberg, 2016).

2.5 Information Gap

Kamau (2002) carried out a test in the Nairobi Securities Exchange in which he aimed to establish the effect of size and book to market value on security returns. The findings of his study were that size and book to market value have no relationship with returns of companies quoted at the NSE (Kamau, 2002).

Odera (2010) analyzed the validity of the Fama and French three factor model to explain asset returns in the NSE. She found out that the size effect was found to be not as strong because most stocks are

small. The SMB plays a vital role in explaining portfolio returns for medium and small size portfolios but has no effect on large-scale portfolio returns (Odera, 2010).

The information gap exists in that the authors above focused on identifying the risk factors that affect the Nairobi Securities Exchange (NSE) and did not form portfolios aiming to beat the market index.

The focus of this project is to form an Optimal Portfolio whose risk adjusted return can beat the NSE All Share Index.

3 Methodology

The methodology employed aims at forming an optimal portfolio whose risk adjusted return can beat the market index. The methodology will aim to classify tradable securities into theoretical portfolios based on the Carhart four factor model. Portfolio returns will be obtained through the mean variance approach.

3.1 Research Design

The research design used in this proposal is mainly explanatory. This is because this research will focus on the size factor as the main determining factor to form optimal portfolios.

3.2 Population and Sampling

The variables of interest in this study are the listed securities in the Nairobi Securities Exchange. Therefore, the population of this study will consist of all the companies listed between the 2006 and 2015. Since portfolio re-composition occurs on an annual basis, the data set will keep changing based on the number of new firms listed in the Nairobi Securities Exchange and the number of firms that are delisted.

The market index that will be under study is the NSE All Share Index. This is because NSE All Share Index checks the performance of the entire securities exchange, NSE, and thus is the most diversifiable return an investor can expect to earn.

3.3 Data Collection

The data to be used in this study is primarily quantitative. The study will rely on statistical and historical data relating to the following:

- The Daily Prices of all stocks listed on the NSE from January 2007- December 2014
- The Annual EPS for each stock from the year 2007-2014 for the calculation of the P/E Ratio
- The Book Values of each firm trading in the NSE from the year 2007-2014 for the calculation of the P/B Ratio

The Daily share prices and the EPS data to be used in this study will be sourced from the Nairobi Securities Exchange. The data on Book Values will be collected from the published financial statements of the NSE listed companies.

3.4 Data Analysis

3.4.1 Carhart Four factor model Specification

This study relies on the methodology adopted by Anyssa Trimech and Hedi Kortas in their 2009 paper on the study of Multiscale Carhart Four-Factor Pricing Model: Application to the French Market (Trimech & Kortas, 2009).

The Carhart Four factor asset pricing model (1997) extends the Fama-French Three Factor (French & Fama, 1993) model by adding a fourth factor, the momentum anomaly (Carhart, 1997). The resulting model has four risk factors which include: the Fama-French risk factors, i.e., the market premium Mkt ($R_{m_t} - R_{f_t}$), the size factor SMB (Small Minus Big), and the book-to-market factor HML and the Carhart momentum anomaly factor proxy WML which is short for (Winner Minus Loser).

In the market premium factor Mkt, R_{m_t} stands for market return and R_{f_t} stand for the risk free rate. The market return variable, R_{m_t} , which will be used will be the, NASI, NSE All Share Index. The

Risk free rate factor, R_{ft} , which will be used will be the return on a 91 Day Treasury bill.

The Carhart four-factor model is as follows:

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_i + \gamma_i SMB_i + \delta_i HML_i + \theta_i WML_i$$

The Carhart Four factor model factors will be constructed using six portfolios based on size (SMB) and Value factor (HML) ratios. The size factor will be covered by the Market Capitalization values in order to classify assets into small cap and large cap. The Value factor will be placed based on the Book Value to Market Equity values. Similarly, the Carhart momentum factor will be constructed using six portfolios based on size and momentum.

The portfolios will be updated at the end of every June annually. This yearly rebalancing will lead to the constant revision of stocks present in the portfolio held at every half year. The number of firms will also change during the period of observation as new firms are listed in the NSE and some firms are delisted.

The Fama-French factor portfolios-Size and Value portfolios

In order to build SMB and HML factors, the following methodology will be repeated at the end of December, every year. Firstly, two groups are formed based on size and labeled Small (S) and Big (B). Here the median market capitalization at the end of June of that year is used as the size breakpoint to group between the two groups.

Secondly, use the BE/BM ratio for the year t, which corresponds to the Book Equity (BE) for the fiscal year ending in (t – 1), divided by the Market Equity (ME) for December of (t – 1) to construct three groups of stocks. These groups, denoted by Low (L), Medium (M) and High (H), result from dividing the cumulative stocks, with respect to the 30th and 70th percentiles, respectively.

Thirdly, let the two factors groups intersect. The intersection of the two size factors; Small(S) and Big (B) and the three value factors; High (H), Medium (M) and Low (L) will form the first six portfolios that explain the Fama-French factors of SMB and HML. The six portfolios will be denoted

by SL, SM, SH, BL, BM and BH, for which one calculates the corresponding returns labeled R_{SL} , R_{SM} , R_{SH} , R_{BL} , R_{BM} and R_{BH} .

Finally compute the SMB risk factor as the difference between the monthly average return on the three small (S) portfolios and the monthly average return on the three big (B) portfolios:

$$SMB = \bar{R}_S - \bar{R}_B = \frac{1}{3}[R_{SL} + R_{SM} + R_{SH}] - \frac{1}{3}[R_{BL} + R_{BM} + R_{BH}]$$

To compute the HML obtain the difference between the monthly average return on the two portfolios, within the high (H) group and the monthly average return on the two portfolios with low (L) BM/BE ratio:

$$HML = \bar{R}_H - \bar{R}_L = \frac{1}{2}[R_{SH} + R_{BH}] - \frac{1}{2}[R_{SL} + R_{BL}]$$

Carhart momentum factor portfolios

To form the six portfolios that express the Carhart momentum factor this study will focus on the interaction between the size and momentum factor. To obtain the WML factor use the following methodology.

Firstly, obtain the 12 cumulative past monthly stock returns and divide them into three groups; Winner (W), Minus (M) and Loser (L) portfolios with respect to the 30th and 70th percentiles, respectively. The 30% of stocks which have the most important cumulative past returns form the Winner (W) group, the next 40% represent the Minus group (M) and the remaining 30% constitute the Loser group (L).

Secondly let the size factor two portfolios; Small(S) and Big(B) intersect with the three momentum factor portfolios; Winner (W), Minus (M) and Loser (L) portfolios. This will form six portfolios denoted by the SW, SM, SL, BW, BM and BL, for which one calculate the corresponding returns labeled R_{SW} , R_{SM} , R_{SL} , R_{BW} , R_{BM} and R_{BL} .

Finally to compute the WML factor obtain the difference between the monthly average return

of the two winner portfolios and the monthly average return of the two loser portfolios:

$$WML = \bar{R}_W - \bar{R}_L = \frac{1}{2}[R_{SW} + R_{BW}] - \frac{1}{2}[R_{SL} + R_{BL}]$$

3.4.2 Mean Variance Portfolio Optimization

This methodology is based on the modified analysis variance-covariance approach suggested by Markowitz in his 1952 paper on Portfolio Selection (Markowitz, 1952)

In the end there will be 12 portfolios; six of the intersection of the Fama-French factors of Size (SMB) and the Value (HML) and six of the intersection of the Size factor (SMB) and the Carhart momentum anomaly factor (WML).

Initially obtain monthly security returns of every individual security. To obtain the weights and allocations of securities in the mean variance portfolio the variance-covariance matrix approach will be used.

To calculate the monthly security returns one can calculate the Holding period return;

$$HPR = R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Based on the asset returns one can calculate the portfolio returns and variance. To obtain the portfolio return assume equal weighting to securities in the individual portfolio. Portfolio return will be calculated by;

$$R_p = \sum_{i=1}^n (W_i R_i)$$

Where W_i represents the weights of the securities in the portfolio

R_i represents the returns of the security.

Securities co-variances have to be calculated in order to calculate the portfolio variance. These can be obtained by the formula;

$$E(R_i, R_j) = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

The variance of the portfolio will then be obtained from the variance-covariance matrix approach in which the securities weights of a portfolio will be multiplied by the variance-covariance matrix and a transpose of itself. The formula of the portfolio will be;

$$Var_p = X^T \Sigma X$$

Where X^T represents the weight matrix transpose

Σ represents the variance-covariance matrix

X represents the weight matrix

The variance-covariance matrix can be expressed as

$$\Sigma = \begin{pmatrix} Var_i & \cdots & Cov(R_n, R_i) \\ \vdots & \ddots & \vdots \\ Cov(R_i, R_n) & \cdots & Var_n \end{pmatrix}$$

The variance-covariance matrix can be done in excel using the COVAR function to calculate the covariances of the portfolio. Through the Data analysis function under the Data tab in excel, one can directly form a variance-covariance matrix using the covariance function.

The Global minimum variance can be obtained in excel using the solver function under the Data tab by placing the Portfolio variance as the subject of a minimization function. The solver will readjust the naïve portfolio weightings to securities portfolio weights that minimize the portfolio variance.

3.4.3 Risk Adjusted Portfolio Returns

Once the optimal portfolio weights have been obtained, monthly returns will be calculated of the portfolios. The monthly returns will be converted into risk adjusted returns through the use of the Sharpe ratio and Treynor ratios;

Sharpe ratio

The Sharpe ratio examines the performance of a portfolio by adjusting for its risk. It is a reward to variability ratio which measures the excess portfolio return been received per unit of standard deviation of returns. It was developed by William Sharpe. The Sharpe ratio is calculated as;

$$S = \frac{E(R_p) - R_f}{\sigma_p}$$

Where $E(R_p)$ = Expected Portfolio return

R_f = Risk free rate of return

σ_p = Portfolio volatility

The 91 day T-bill will be used as the risk free rate. The portfolio volatility will be obtained from the model as the deviation of the portfolio return from the expected return. The Sharpe ratio will be obtained on annual basis.

The Sharpe ratio is preferred as a measure of risk adjusted return since it is a simple measure and adjust for both systematic and idiosyncratic risks. It thus tells an investors the return the portfolio has earned in respect to the entire portfolio risk.

The Sharpe ratio however fails in that it treats all volatility the same. Because of this it penalizes strategies that have upside volatility (big positive returns).

Treynor ratio

This is a risk adjusted ratio that adjusts excess portfolio return for market risk. It is also used to measure and compare the performance of a portfolio manager. The Treynor ratio was developed by Jack Treynor. The Treynor ratio is calculated as;

$$T = \frac{E(R_p) - R_f}{\beta_p}$$

Where $E(R_p)$ = Expected Portfolio return

R_f = Risk free rate of return

β_p = Portfolio beta

Portfolio beta is a measure of how the asset (portfolio) is volatile to the market. It is a measure of the risk arising from exposure to general market movements as opposed to idiosyncratic factors. The portfolio betas will be obtained from the regression model of the Carhart four factor model;

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_i + \gamma_i SMB_i + \delta_i HML_i + \theta_i WML_i$$

β_p will be taken in comparison to how the portfolio return behaves in respect to the excess market return; Mkt ($R_{m_t} - R_{f_t}$).

The 91 day T-bill rate will be also be used as the risk free rate. The Treynor ratio will be obtained on annual basis.

The Treynor ratio is preferred because it attempts to measure the successful of an investment manager in providing investors' compensation for the risk inherent in the market which cannot be diversified away.

The Treynor ratio fails in that is a ranking ratio only. The Treynor ratio assumes portfolios are affected by the same systematic risk and ignores the total risk affecting portfolios which may defer across portfolios.

4. Results and analysis

4.1 Common factors in Excess Returns

4.1.1 Excess asset return on all portfolios

The Carhart Four-factor model was used to help explain the portfolio returns:

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_i + \gamma_i SMB_i + \delta_i HML_i + \theta_i WML_i$$

Results from the regression include;

```

Fixed-effects (within) regression      Number of obs      =      84
Group variable: codeportfolio~o      Number of groups   =      12

R-sq:  within = 0.8892                Obs per group: min =      7
      between = 0.6806                avg =              7.0
      overall = 0.7319                max =              7

corr(u_i, Xb) = 0.1026                F(3,11)            =      .
                                          Prob > F            =      .
  
```

(Std. Err. adjusted for 12 clusters in codeportfolio)

RiRf	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
RmRf	.8422624	.0707968	11.90	0.000	.6864397	.9980851
SMB	1.31e-10	1.71e-10	0.76	0.460	-2.46e-10	5.09e-10
HML	.0009045	.0005507	1.64	0.129	-.0003076	.0021165
WML	.1615295	.0612486	2.64	0.023	.0267224	.2963367
_cons	-.0254996	.0055818	-4.57	0.001	-.0377851	-.013214
sigma_u	.01786075					
sigma_e	.01236284					
rho	.67608116	(fraction of variance due to u_i)				

The Carhart Four Factor model is able to explain 88.92% of the variations in portfolio returns. The within transformation R-squared is used because the Fixed Effect model was finally picked after running the Hausman test. The Excess market return, Value and Momentum are key variables to explain securities returns return at a 95% confidence interval. Size is not a significant variable to explain securities return given that most securities trading in the NSE are small stocks with very few

large stocks dominating the Securities market activities. The 5 largest market capitalization securities contribute more than 50% of the entire capitalization of the NSE securities (63 securities as per December 2014).

Use of Panel Data

Given that our data is panel in nature, both the Random Effects model and the Fixed Effect model were run.

The Fixed Effect model assumes that the unobserved effects of the explanatory variables on the dependent variable are the same for all units. To eliminate the unobserved effect, the time mean of an explanatory variable is subtracted from the individual values of the explanatory variable.

The Random Effect model assumes that the unobserved effect is uncorrelated with all the explanatory variables. Thus no transformation will be done because of the lack of the un-correlation with the explanatory variables.

The Hausman Test was carried to identify which of the two models would best explain the relationship in the regression with the explanatory variables. The Hausman test compares the Random Effects and the Fixed Effects estimates. Under the null hypothesis, individual effects are random, so both Random Effects and the Fixed Effects are consistent. The Random Effects model is better. Under the alternative, the Random Effects and the Fixed Effects estimators diverge. The Fixed Effect is better.

$H_0 = RE$ and FE estimates are consistent. Random Effects model is picked.

$H_1 = RE$ and FE estimates diverge. Fixed Effects model is picked.

	Coefficients			
	(b) fixed	(B) random	(b-B) Difference	$\sqrt{\text{diag}(V_b - V_B)}$ S.E.
RmRf	.8422624	.6740164	.168246	.
SMB	1.31e-10	-1.62e-11	1.47e-10	8.15e-11
HML	.0009045	.0001292	.0007752	.0006707
WML	.1615295	.6162519	-.4547223	.0437471

b = consistent under Ho and Ha; obtained from xtreg
 B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
 = 107.04
 Prob>chi2 = 0.0000
 (V_b-V_B is not positive definite)

Reject null hypothesis that the Random Effects provides consistent estimates since the p-value is at zero. The Fixed Effect Model is picked.

4.2 Portfolio Formation

4.1 Intersection of the Size and Value Factor (First 6 portfolios)

The First six portfolios are formed from the intersection of the HML factor and the SMB factor. This was gotten and arranged on a yearly basis focusing on the end year figures of the Book-to-Market values and the Market capitalization.

Results from the SMB factor and the HML Factor are stated below;

```

Random-effects GLS regression           Number of obs   =       42
Group variable: codeportfo~o          Number of groups =        6

R-sq:  within = 0.8742                  Obs per group: min =        7
      between = 0.6543                      avg =       7.0
      overall = 0.8645                      max =        7

                                         Wald chi2(3)     =        .
corr(u_i, X) = 0 (assumed)              Prob > chi2      =        .

```

(Std. Err. adjusted for 6 clusters in codeportfolio)

RiRf	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
RmRf	.7903991	.069895	11.31	0.000	.6534075	.9273908
SMB	4.52e-11	7.36e-11	0.61	0.539	-9.90e-11	1.89e-10
HML	-.0009145	.0008486	-1.08	0.281	-.0025777	.0007487
WML	.3092609	.0316199	9.78	0.000	.2472871	.3712348
_cons	-.0245262	.0043424	-5.65	0.000	-.0330372	-.0160152
sigma_u	0					
sigma_e	.01379503					
rho	0	(fraction of variance due to u_i)				

The six initial portfolios are focus on the intersection of the Size and Value factors. This is based on the Fama-French 3 Factor model which advocated for the inclusion of Size and Value factor to help capture more variations in the securities returns. The Random Effect model is picked since the Hausman test result was fail to reject null hypothesis given a p-value of 0.3155, which is above any level of significance.

A regression of the six portfolio returns shows that the Carhart Four Factor model is able to explain up to 86.45% of the variations. The excess market return and the momentum factor variables are significant in explaining the portfolio returns at a 95% confidence interval.

The SMB risk Factor is obtained from;

$$SMB = \bar{R}_S - \bar{R}_B = \frac{1}{3}[R_{SL} + R_{SM} + R_{SH}] - \frac{1}{3}[R_{BL} + R_{BM} + R_{BH}]$$

The HML risk factor is obtained from;

$$HML = \bar{R}_H - \bar{R}_L = \frac{1}{2}[R_{SH} + R_{BH}] - \frac{1}{2}[R_{SL} + R_{BL}]$$

Results of the SMB and the HML risk factors can be observed in the appendix 1 and 2.

SMB-HML excess securities returns

We regress excess asset returns against the market risk premium, size factor and BE/ME and momentum in order to be able to test the explanatory power of the model;

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_i + \gamma_i SMB_i + \delta_i HML_i + \theta_i WML_i$$

	α_i	β_i	γ_i	δ_i	θ_i	<i>Mean</i>	<i>Median</i>	<i>Max</i>	<i>Min</i>
<i>B/H</i>	-0.0890441	.4381315	9.03e-10	.0015812	.4456505	-9.46%	-8.96%	-1.06%	-15.62%
<i>B/M</i>	-.012675	.6039463	-8.80e-10	-.0067107	.567455	-8.71%	-8.87%	-4.57%	-12.46%
<i>B/L</i>	-.0067053	.8114726	-7.63e-11	-.0264983	.3364548	-8.02%	-9.78%	-0.30%	-11.96%
<i>S/H</i>	-.0000571	1.217166	-1.35e-08	.0127644	.0003221	-8.30%	-7.90%	-1.03%	-12.55%
<i>S/M</i>	-.1045679	.7786692	3.63e-08	-.0159592	-.1438063	-9.37%	-9.15%	-3.19%	-14.86%
<i>S/L</i>	-.0689701	.5135822	7.12e-09	.0366127	-.2579074	-7.98%	-7.98%	-3.83%	-10.56%

The Carhart Four Factor model is able to explain variations in stock returns up to 96.93% of the Small and Minus portfolio. Small and High is the least explained with an explained variation up to 90.12%. The Small and High portfolio has all the variables significant at a 95% confidence interval. The excess market return is the most significant variable to explain excess securities return across the portfolios at a 95% confidence interval, with the SMB being significant in the Small and High portfolio at the 95% confidence interval while being significant at a 90% confidence interval for the Big and High and the Big and Minus Portfolios.

4.2 Intersection of the Size and Momentum Factor (Last 6 portfolios)

The last six portfolios are formed from the intersection of the SMB factor and the WML factor. This was gotten and arranged on a yearly basis focusing on the end year figures of the Market capitalization and the cumulative 12 month securities returns.

Results from the SMB factor and the WML Factor are stated below;

```
Fixed-effects (within) regression          Number of obs   =       42
Group variable: codeportfolio             Number of groups =        6

R-sq:  within = 0.9088                    Obs per group:  min =        7
        between = 0.9387                  avg =       7.0
        overall = 0.5816                  max =        7

corr(u_i, Xb) = 0.0752                    F(3, 5)         =        .
                                                Prob > F        =        .
```

(Std. Err. adjusted for 6 clusters in codeportfolio)

RiRf	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
RmRf	.8721942	.1386633	6.29	0.001	.5157487	1.22864
SMB	4.55e-11	1.20e-10	0.38	0.721	-2.64e-10	3.55e-10
HML	.0008123	.0012543	0.65	0.546	-.0024119	.0040364
WML	.0763296	.1333926	0.57	0.592	-.266567	.4192262
_cons	-.0221698	.0095289	-2.33	0.067	-.0466646	.002325
sigma_u	.02866941					
sigma_e	.01117542					
rho	.86809595	(fraction of variance due to u_i)				

The last six portfolios are formed from the intersection of the SMB factor and the WML factor. This is based on the Carhart Four Factor model who advocated for the inclusion of the momentum anomaly to explain the performance of the securities. Carhart advocated for the inclusion of momentum in his 1997 analysis of the Persistence of Mutual Fund Performance.

The Fixed Effect model is picked since the Hausman test result was to reject null hypothesis given a p-value of 0.0000, which is at zero.

A regression of the six portfolio returns shows that the Carhart Four Factor model is able to explain

up to 90.88% of the variations. The excess market return is the only significant variable in explaining the portfolio returns at a 95% confidence interval.

The WML risk factor is obtained from;

$$WML = \bar{R}_W - \bar{R}_L = \frac{1}{2}[R_{SW} + R_{BW}] - \frac{1}{2}[R_{SL} + R_{BL}]$$

Results of the WML risk factor can be observed in the appendix 3.

SMB-WML excess securities returns

We regress excess asset returns against the market risk premium, size factor and BE/ME and momentum in order to be able to test the explanatory power of the model;

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_i + \gamma_i SMB_i + \delta_i HML_i + \theta_i WML_i$$

	α_i	β_i	γ_i	δ_i	θ_i	Mean	Median	Max	Min
<i>B/W</i>	.0091428	.9413122	-6.17e-11	.0091483	.0910206	-5.73%	-5.77%	1.19%	-9.42%
<i>B/M</i>	-.0045795	1.094674	3.61e-11	.0003496	-.3484426	-8.40%	-7.76%	-1.84%	-12.31%
<i>B/L</i>	-.1490081	.3325435	9.51e-10	-.0082825	-.5622726	-13.20%	-12.73%	-10.92%	-16.37%
<i>S/W</i>	.0404633	1.173349	1.81e-09	.0056901	-.4605176	-5.01%	-4.39%	1.81%	-8.61%
<i>S/M</i>	-.005953	1.334466	-3.93e-09	.0200908	-.5466364	-8.49%	-7.76%	-1.46%	-12.52%
<i>S/L</i>	-.0103093	.9998132	-1.59e-08	.0040808	.1544366	-11.13%	-10.32%	-4.54%	-15.92%

The Small and Minus Portfolio is most explained by the Carhart Four factor with explained variations up to 99.04% while the Big and Loser Portfolio is the least explained by the Carhart Four factor model with an explained variation up to 92.45%.

The excess market return is the most significant variable to explain excess securities return across the

portfolios at a 95% confidence interval, with the SMB being significant in the Big and Loser portfolio at the 95% confidence interval.

4.3 Portfolio Selection and Optimization

The size factor is the main factor used to select the securities to be used to form the optimal portfolio. The optimal portfolio was picked by taking a long position in the Small sized stocks and shorting the Big sized stocks. Both the Value factor and the momentum factor was used to further group the Size ranked securities. The portfolio weights was then based on the Markowitz portfolio optimization rules, where the portfolio weights were came up based on the maximization of the Sharpe ratio of the portfolio. The stocks picked were revised on annual basis based on the December Market capitalization values.

The table below presents the results of the creation of portfolios based on the Size Factor, the composition of the portfolio weights can be observed in the appendix 4.

	2008	2009	2010	2011	2012	2013
<i>Portfolio Excess</i>						
<i>Return (Rp-Rf)</i>	7.64%	22.51%	0.90%	5.41%	11.90%	22.97%
<i>Portfolio Excess</i>						
<i>Return (Rp-Rm)</i>	0.18%	-0.50%	0.09%	1.29%	3.51%	8.10%
<i>Portfolio</i>						
<i>Variance</i>	4.64%	6.09%	19.68%	2.82%	0.69%	7.36%
<i>Standard</i>						
<i>Deviation</i>	21.54%	24.67%	44.36%	16.80%	8.33%	27.14%
<i>Sharpe ratio</i>	35.47%	91.26%	2.02%	32.23%	142.90%	84.64%
<i>Treynor ratio</i>	9.07%	26.73%	1.06%	6.43%	14.12%	27.27%

The importance of the intermediate portfolios is observed since on average portfolios that have the intermediate portfolios within them earn a higher return. This is because the intermediate portfolios increase the asset classes for the investor to diversify his portfolio. This reduces his risk enabling him to earn a higher risk adjusted return on a year to year basis.

The table below presents the results of the optimum portfolios without the intermediate portfolios within them. The composition of the portfolio weights can be observed in the appendix 5.

	2008	2009	2010	2011	2012	2013
Portfolio Excess Return (Rp-Rf)	7.64%	3.95%	0.90%	88.06%	55.65%	26.77%
Portfolio Excess Return (Rp-Rm)	0.18%	-0.09%	0.09%	21.05%	16.40%	9.44%
Portfolio Variance	4.06%	3.34%	18.39%	9.77%	4.42%	10.12%
Standard Deviation	20.14%	18.26%	42.88%	31.260%	21.03%	31.81%
Sharpe ratio	37.93%	21.64%	2.09%	28.69%	154.69%	84.17%
Treynor ratio	9.07%	4.69%	1.06%	10.55%	45.07%	26.79%

Portfolio returns are expressed as in the form of the excess risk free return and excess market return. The investor is keen to earn a return higher than the NSE All Share Index, market return. The investor is able to earn a return above the market return and thus is able to beat the market return in all the years other than the year 2009.

The Sharpe ratio gives the investor a risk adjusted return per unit of risk. This enables the investor to access the return the portfolio has earned in respect to the entire portfolio risk. In this case, the investor

has an earned a positive Sharpe ratio on a year to year basis with the year 2012 being the year that the investor earned the largest return.

The Treynor Ratio help the investor to compare the performance of the portfolio manager through his management of market risk. The portfolio manager is able to manage the portfolio exposure to the undiversifiable market risk though rebalancing his portfolio on a year to year basis granting the investor a positive Treynor ratio over the years.

5. Conclusion

This study enabled us to evaluate whether the Carhart Four factor model has an explanatory power over the Kenyan securities market. The Carhart Four Factor model explanatory power was checked on how it is able to explain the individual portfolio returns. A cross-sectional study of the various portfolios across time also helped in assessing how the Carhart Four Factor model is able to capture the variations across different portfolios. The Carhart Four Factor model proved useful to capture variations across individual portfolios. This formed basis in the formation of the portfolios and subsequent weight allocation.

We also focused on the formation of optimal portfolios. The Size factor enabled us to break the tradable securities in the NSE into the Small and Big stocks. This was further broken based on the intersection of the Size factor and the Value factor and the Momentum factor enabling us to obtain 12 portfolios on which our portfolio weights distribution will be based on.

Taking long position in the Small stocks and short positions in the Big stocks enabled the investor to earn a positive return. The investor is able to rebalance his portfolio on a year to year basis enabling him to be able to continuous earn a high return. The investor checks the return of his portfolio based on the excess return earned on the market return-NSE All Share Index. The investor is also to evaluate the portfolio performance based on the risk adjusted returns of Treynor ratio and Sharpe ratio.

The investor is bale to evaluate the importance of holding positions in the intermediate portfolios. By

comparing the performance of an Optimal weighted portfolio with holdings in the intermediate portfolios against that without, the investor is able to see the importance in having positions in the intermediate portfolios since they enable him to diversify his holdings and thus reduce his risk exposure.

6. Recommendation

A recommendation for this study would be a similar hypothesis testing in a more developed market in Africa over a longer range of years. The Nairobi Securities Exchange has few listed stocks. A study on a larger Securities exchange will bring more conclusive results especially if the developed exchange has far more data availability.

The limitations outlined provide more opportunities for further research on the topic. Areas of expansion include the use of the Fama and French Five factor model to capture addition of other fundamental factors; the use of a larger sample size and a longer time horizon.

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Appendixes

Appendix 1

	SMB factor	2009	2010	2011	2012
1	January	7.43%	-1.59%	5.32%	3.82%
2	February	5.65%	-0.79%	1.00%	-1.85%
3	March	-1.87%	6.73%	-4.88%	-0.03%
4	April	-0.41%	-4.74%	-6.91%	2.10%
5	May	-11.09%	-7.73%	2.08%	-0.37%
6	June	6.50%	7.44%	5.18%	-2.01%
7	July	5.80%	-4.98%	0.39%	1.64%
8	August	-2.32%	0.15%	6.30%	-2.79%
9	September	-4.00%	-4.27%	1.14%	-5.37%
10	October	1.38%	-3.33%	6.81%	3.00%
11	November	-1.97%	1.62%	-1.24%	3.80%
12	December	0.46%	3.99%	1.38%	0.18%

Appendix 2

	HML Factor	2009	2010	2011	2012
1	January	-6.60%	-2.89%	-3.82%	-10.61%
2	February	16.29%	5.40%	-4.96%	-8.55%
3	March	3.73%	3.59%	2.61%	-2.41%
4	April	3.74%	4.45%	-1.27%	-0.64%
5	May	6.61%	0.50%	-3.25%	-7.12%
6	June	4.65%	4.25%	-9.01%	-2.30%
7	July	-6.60%	4.12%	-0.48%	-6.30%
8	August	-1.77%	-4.84%	2.52%	-1.83%
9	September	5.54%	0.55%	2.20%	-4.80%

10	October	5.81%	-1.25%	-9.88%	-4.99%
11	November	-4.46%	0.86%	-1.57%	6.49%
12	December	2.45%	-4.71%	-2.45%	-3.92%

Appendix 3

	WML factor	2009	2010	2011	2012
1	January	-2.80%	-0.88%	2.89%	7.96%
2	February	17.96%	4.18%	7.44%	9.18%
3	March	6.21%	23.52%	-0.86%	0.57%
4	April	0.59%	2.46%	13.53%	5.93%
5	May	9.42%	3.09%	7.07%	9.12%
6	June	11.07%	35.14%	9.27%	0.06%
7	July	7.45%	1.78%	7.37%	5.71%
8	August	2.67%	5.32%	4.09%	6.18%
9	September	5.74%	0.40%	10.54%	7.27%
10	October	9.28%	4.43%	7.92%	6.12%
11	November	1.41%	26.00%	5.76%	14.76%
12	December	6.27%	5.30%	7.00%	6.62%

Appendix 4

	2008	2009	2010	2011	2012	2013	2014
Big&High	-0.706	-3.158	-0.842	-0.347	-0.844	-2.562	-1.239
Big&Minus	0.407	-0.843	0.157	0.445	-2.274	0.183	-0.018
Big&Loser	2.795	0.164	1.493	2.428	-0.758	-3.385	1.872
Small&High	1.966	1.946	-2.989	0.375	0.202	-4.609	1.021
Small&Minus	2.174	2.624	-3.592	-0.477	0.662	-2.282	3.539
Small&Loser	-2.516	0.145	-1.079	-0.300	-0.645	1.310	-0.158
Big&Winner	-0.575	2.106	0.850	-1.377	2.524	4.934	0.665
Big&Minus	-0.293	1.911	0.832	0.220	1.404	-0.281	-0.464
Big&Loser	-0.142	0.612	-1.349	-0.348	-0.449	1.362	-0.979

Small&Winner	1.689	-0.621	2.902	-0.401	1.013	0.807	-0.632
Small&Minus	-2.239	-0.932	1.742	0.829	0.122	1.411	-3.500
Small&Loser	-1.560	-2.953	2.874	-0.047	0.041	4.111	0.894
SUM	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Appendix 5

	2008	2009	2010	2011	2012	2013	2014
Big&High	-0.706	-0.555	-0.842	-5.639	-3.947	-2.987	-1.453
Big&Loser	-0.213	0.291	1.074	3.732	1.753	-3.718	1.415
Small&High	1.299	0.375	-0.272	6.255	4.671	-3.241	0.268
Small&Loser	-1.752	-0.038	-0.912	-2.376	-1.274	0.709	-0.309
Big&Winner	1.695	0.549	1.241	0.458	1.379	5.663	0.072
Big&Loser	0.570	-0.240	-1.262	-0.277	-0.476	1.592	-1.020
Small&Winner	1.091	1.517	0.669	-2.996	-1.794	0.647	0.824
Small&Loser	-0.984	-0.899	1.304	1.844	0.688	2.335	1.203
SUM	1.000	1.000	1.000	1.000	1.000	1.000	1.000