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Mathematical Modelling of The East Africa Marine Systems (TEAMS) Fiber OPtic

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Abstract

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Keywords. Fiber optics, frequency, nonlinear, modelling, NLSE, TEAMS
Abbreviations: Nonlinear Schrödinger Equation (NLSE), Electromagnetic Waves (EM), East Africa (EA), The East African Marine System (TEAMS), Information and Communication Technologies (ICT), Computer Communications Review (CCR)..

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Nomenclature

v	Propagation distance along the fiber
t	Propagation distance along the time
V_{gr}	Carrier group velocity
D	Dispersion coefficient.
μ	Envelope components for the real function of z
ν	Envelope components for the real function of τ
λ	Arbitrary parameter
F	Unknown function of t and its derivatives
$\|A\|^2$	Pulse amplitude
α	Fiber losses
β_2	Chromatic dispersion
γ	Fiber nonlinearity
n_2	Kerr nonlinear index coefficient
A_{eff}	Effective core area
c	Light velocity in vacuum
β_1	First order dispersion
k	Wave vector [radians/m] ν
ω	Angular frequency [radians/sec]
$\lambda 0$	Wavelength in vacuum [m]
n	Refractive index
$J_n(x)$	Frequency component of magnitude
Γ	Gamma function
n_1	Refractive index of the medium the light is leaving
θ_1	Incident angle between the light beam and the normal
n_2	Refractive index of the material the light is entering
θ_2	Refractive angle between the light ray and the normal
n	Side frequency number/order of differential equation
x or l	Modulation index
D or $\frac{dy}{dx}$	Differential operator
α	Phase difference
fm	Argument.

1. Introduction

The East African Marine System (TEAMS) is a 5,000-km fibre-optic undersea cable linking the Kenya's coastal town of Mombasa with Fujairah in the UAE was built at a cost of USD 130 million as a joint venture between the government of Kenya and Kenyan operators as follows; 42.5% – Telkom Kenya Ltd, 22.5% – Safaricom Ltd,10% – Kenya Data Networks Ltd, 10% – Econet/Essar Telecom Ltd, 5% – Wananchi Group, 3.75% – Jamii Telecom Ltd, 1.25% – Broadband Access/AccessKenya Ltd, 1.25% – Africa Fibrenet (Uganda) Ltd, 1.25% – InHand Ltd, 1.25% – iQuip Ltd, 1.25% – Flashcom Ltd.
According to Daily Nation, Thursday 30^{th} December 2010.Construction of the cable began in January 2008 on the Emirates' side and arrived in the Kenyan port city of Mombasa on 12th June 2009. Cable construction was completed in August 2009 and the Teams cable went live for commercial service on 1st October 2009. TEAMS cable is connected to the Kenya national fiber backbone network and other major backhaul providers, thus extending the gigabit submarine capacity to the rest of the East African countries: Uganda, Rwanda, Burundi and Tanzania through cross-border connectivity arrangements.TEAMS is strategically positioned in the EA region to spur rapid social, economic and educational development by providing

reliable, worldwide gigabit connectivity to ICT operators, ISPs and large bandwidth users at cost-efficient and competitive rates.

A fiber optic consists of a central core in which light is guided, embedded in an outer cladding of slightly lower refractive index. Light rays incident on the core-cladding boundary angles greater than the critical angle undergo total internal reflection and are guided through the core without refraction.

Recent developments in fibre optics in communication system prove almost zero loss and infinite bandwidth. Indeed, optical fibre communication systems are fulfilling the increased demand on communication links, especially with the proliferation of the internet, Chynoweth (1976). Fiber cables have proven to provide faster and cheaper internet connectivity than the traditional satellites; be it streaming videos live or downloading high-definition videos in a very short time. As a result of recent technological advances in fabrication, light can be guided through fibre optic with very minimal loss and these optical fibers are replacing copper coaxial cables as the preferred transmission medium for electromagnetic waves. The low attenuation and superior signal quality of fibre optic communication systems allow communications signals to be transmitted over much longer distances than metallic-based systems without signal regeneration Ishigure *et al* (1996).

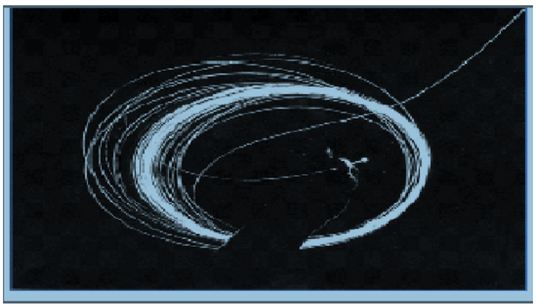

Figure 1. A long, thin optical fiber transmitting a light beam (Photograph courtesy Dr. Chynoweth (1976))

An optical fiber (Figure 1) consists of a central glass core of radius "a" surrounded by an outer cladding made of glass with a slightly lower refractive index. The corresponding refractive index distribution (in the transverse direction) is given by:

$n = n_1$ for $r < a$

$n = n_2$ for $r > a$

The core diameter $d = 2a$ of a typical telecommunication-grade multimode fiber is approximately 62.5 μm with an outer cladding diameter of 125 μm. The cladding index

n_2 is approximately 1.45 (pure silica), and the core index n_1, barely larger, around 1.465μm. The cladding is usually pure silica while the core is usually silica doped with germanium, which increases the refractive index slightly from n_2 to n_1. The core and cladding are fused together during the manufacturing process and typically not separable. An outside plastic buffer is usually added to protect the fiber from environmental contaminants. The diagram below by STEP (2008)

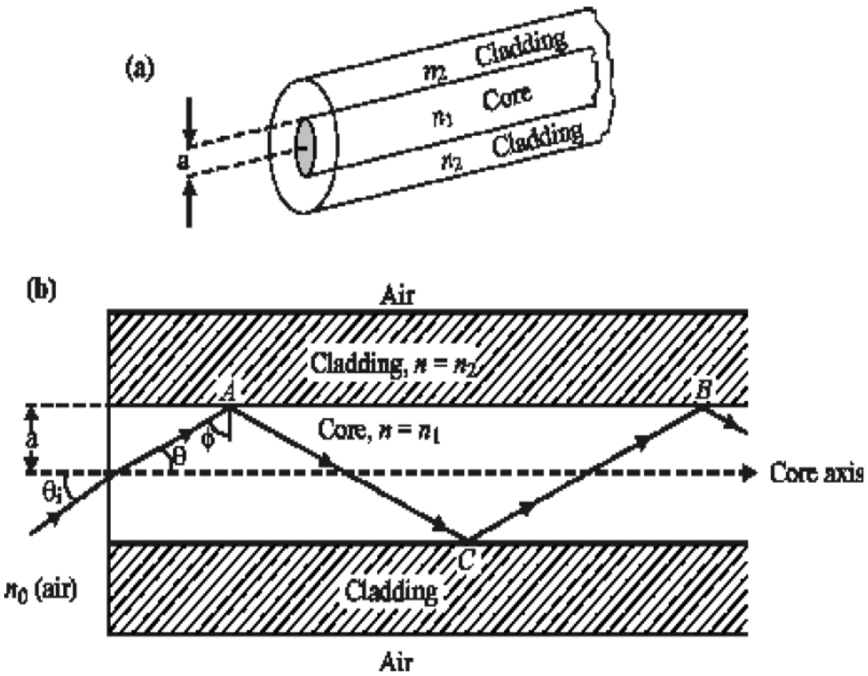

FIGURE 2. *(a) A glass fiber consists of a cylindrical central core surrounded by a cladding material of slightly lower refractive index. (b) Light rays impinging on the core-cladding interface at an angle φ greater than the critical angle φc are trapped inside the core of the fiber and reflected back and forth (A,C, B, etc.) along the core-cladding interface.*

2. Literature Review

Similar research has been done by Kundaeli (2001) analysed an optical fiber communication system using laser rate equations where the effect of electrical pulse shaping on dispersion include pulse distortion was investigated using computer simulation techniques. The results showed that the detrimental effects of the dispersion can be greatly reduced without incurring very high costs. More research has been done on dispersion-dominated nonlinear fiber-optic channels, Sergei *et al* (2012) used used a technique aimed at simplification of the following digital signal processing of nonlinear impairments by using a model describing pulse propagation in the dispersion-dominated nonlinear fiber channel. In the limit of very strong initial pre-dispersion the nonlinear propagation equations for each Fourier mode become local and decoupled.

Deqiang *et al* (2013) studied performance of concealed optical wireless communication link based on modulated retro-reflector. The simulation results show that wavelength, angle of incidence, and refractive index of CCR are the key factors influencing the performance of link, and the SNR fluctuation is decreased by reflecting of CCR.

Fang D. *et al* (2014) studied the design of a fully-fiber multi-chord interferometer and a new phase-shift demodulation method for field-reversed configuration where the noteworthy feature was mathematically compared to the two divided interference signals, which had the same phase-shift caused by the electron density but possess the different initial phase and low angular frequencies. It was possible to read the plasma density directly on the oscilloscope by the original mathematic demodulation method without a camera.

Buczynski *et al* (2014), who presented experimental and numerical progress in the development of ultrafast solitonic nonlinear directional couplers utilizing multi-component glass dual-core photonic crystal fibers. Due to the fibre birefringence the switching wavelength can be tuned by rotating the polarization of the excitation field.The numerical studies were focused on single fundamental soliton switching that exhibits high extinction ratios. The resultant coupler design was further analysed from the aspect of nonlinear propagation based on coupled nonlinear Schrödinger equations. The simulated nonlinear directional coupler indicates the possibility to realize a high-extinction-ratio switch of sub-nJ 100 fs pulses which are simultaneously compressed below 15 fs.

We therefore fell there is a way we can summarize some important equations needed in the analysis of light propagations in fibers that lead to an increase speed in data transmission.

3. Mathematical Formulation

3.1. Bessel Differential Equations

Bessel differential equations given by;

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right) y = 0, \tag{1}$$

where n is the order of differential equation and it is a given number, real or complex. The point $x = 0$ is a regular singularity, and is the Bessel functions which is a solution of equation (1) which has a solution of the form

$$y = \sum_{K=0}^{8} a_k x^{m+k} \tag{2}$$

Using power series and substituting in equation (2), we get the solution

$$J_n\left(x\right) = \frac{x^n}{2^n \Gamma\left(n+1\right)} \left\{1 - \frac{x^2}{2\left(2n+2\right)} + \frac{x^4}{2.4\left(2n+2\right)\left(2n+4\right)} - \dots\right\}. \tag{3}$$

The Bessel functions $J_n\left(x\right)$ has power series that is convergent, with better convergence than the familiar series for the exponential or trigonometric functions which can also be expressed as the sum for integral values of n, Basmadjian (2002), where n-is a positive integer and not zero. It can be written as an infinite polynomial with terms derived from the gamma function, Γ.

$$J_n\left(x\right) = \sum_{K=0}^{8} \frac{\left(-1\right)^k \left(\frac{x}{2}\right)^{n+2k}}{k!\Gamma\left(n+k+1\right)} \tag{4}$$

We considered only the case where n is an integer. The canonical solutions considered are the Bessel functions of the first kind, $J_n(x)$ non-singular at $x = 0$.

n particular, putting $n = 1$, in the above equation Bessel function of order one is given by

$$J_1\left(x\right) = \frac{x}{2} - \frac{x^3}{2^4.4} + \frac{x^5}{2^2 4^2.6} - \dots \tag{5}$$

The function $J_1\left(x\right)$ is oscillating with a decreasing amplitude and varying period. The roots of these functions are not completely regularly spaced and the amplitude of the wave decreases with the increase value of x, which looks similar to a sine function.

The amplitude of the carrier signal is a function of the modulation index and under some conditions; its amplitude can sometimes converge to zero. This does not mean that the signal disappears, but rather that all of the broadcast energy is redistributed to the side frequencies.

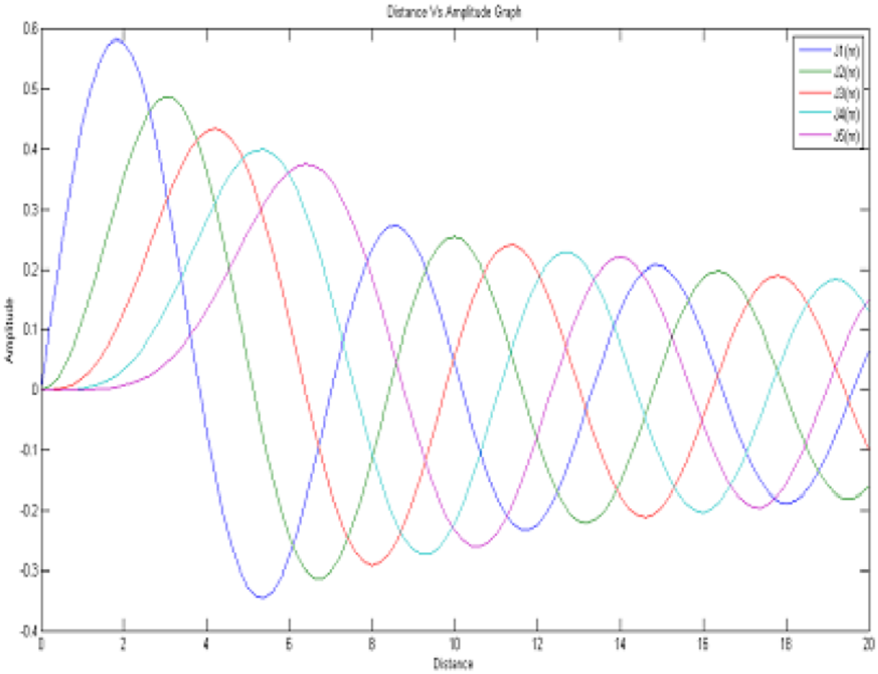

FIGURE 3. Plots of Bessel Functions of the first kind at $0 \leq fm \leq 20$

For computational purposes, we use smaller measures to illustrate the concepts in all the areas in this work.

For instance, if we increase the distance to 80 and 200 units, we obtain the image in Figure 4 and Figure 5.

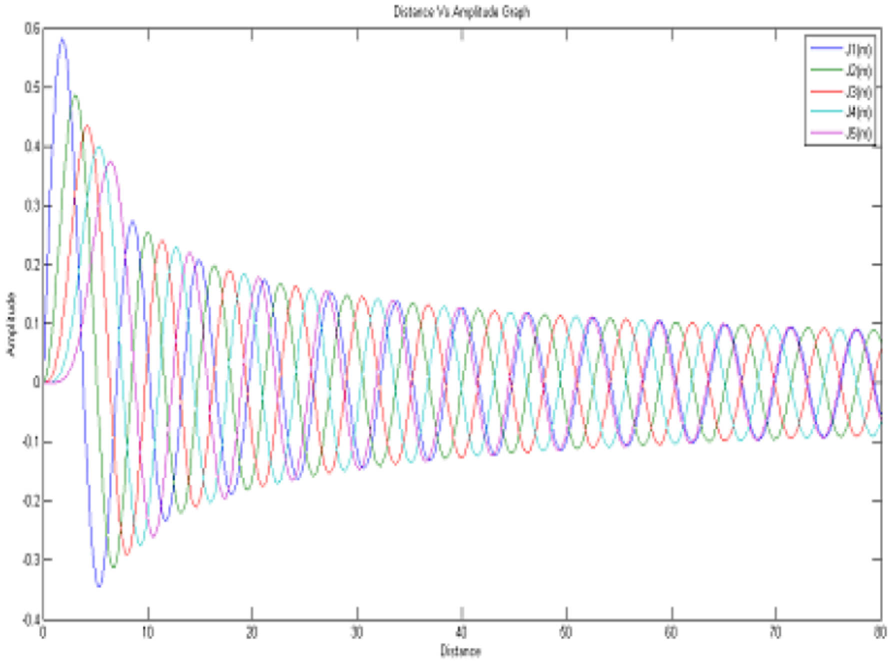

FIGURE 4. Plots of Bessel Functions of the first kind at $0 \leq fm \leq 80$

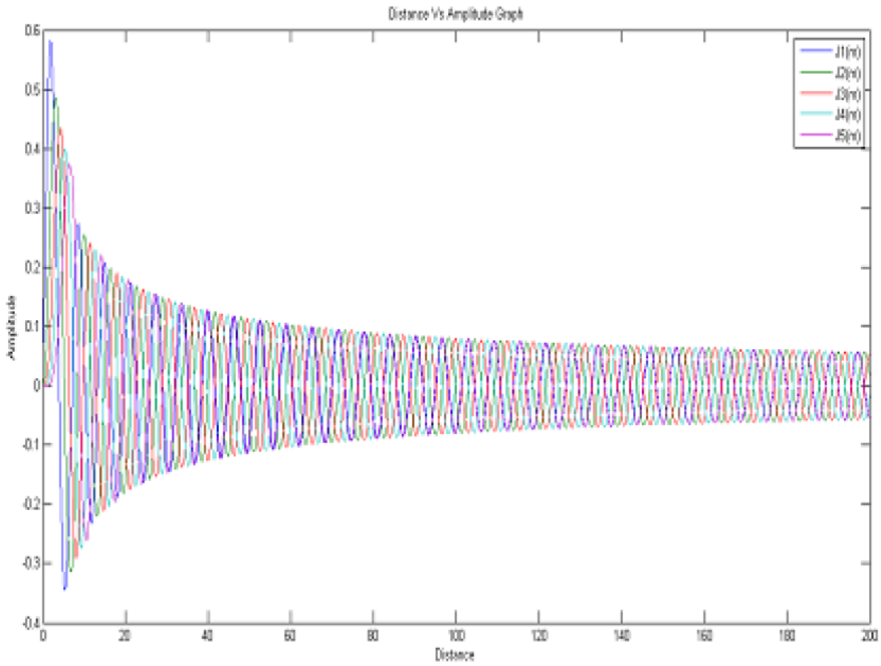

FIGURE 5. Plots of Bessel Functions of the first kind at $0 \leq fm \leq 200$

This modulated signal is consisting of five frequency components added together to give the appearance of a sine wave. In this case, the frequency is varying with time when displayed in the distance domain. From the three graphs of Bessel Functions above, the values of the term $J_n\,(mf)$ which gives amplitude of *n-th* side band with modulation index *mf* are determined using series solution as mentioned in equation (2) and the values of the $J_n\,(mf)$ terms are calculated. Mathematically, the results of the numerical computation of the values of $J_0\,(mf)$, $J_1\,(mf)$, $J_2\,(mf)$, $J_3\,(mf)$, $J_4\,(mf)$ and $J_5\,(mf)$ are plotted. It can be observed from the graph that for small values of *mf*, the only Bessel functions with any significant amplitude are $J_0\,(mf)$ and $J_1\,(mf)$, while the amplitude of the higher-order $(n > 1)$ sideband pairs is very small Saxena *et al* (2009). As *mf* increases, the amplitude of the rest frequency decreases and the amplitude of the higher-order sidebands increase, thus an increasing signal bandwidth. Therefore, the amplitudes of the higher-order sideband pairs eventually approach zero.

3.2. Schrödinger's Equation

Schrödinger's equation is named after Erwin Schröodinger, 1887-1961. This is a second order partial differential equation. These equations will be used where the transmission of multiple channels should address the impacts of dispersion and nonlinear phenomena that occur during transmission. The NLS equation for slowly varying amplitude $\Psi(z,\tau)$ is given by

$$i\Psi_z + \frac{1}{2}D\Psi_{\tau\tau} + \gamma|\Psi|^2\Psi = 0, \tag{6}$$

where $\tau = t - \frac{z}{V_{gr}}$ If we substitute Ψ=μ+iυ equation (6), Malomed (2002), we get the following differential equation for μ and υ

$$-\nu_z + \frac{1}{2}D\mu_{\tau\tau} + \gamma\left(\mu^2 + \nu^2\right)\mu = 0 \tag{7}$$

for the real part of equation (6) and

$$\mu_z + \frac{1}{2}D\nu_{\tau\tau} + \gamma\left(\mu^2 + \nu^2\right)\nu = 0 \tag{8}$$

for the imaginary part of equation (6)

We construct the following trial functional in order to establish variation formulation, He (2004)

$$J\left(\mu,\upsilon\right) = \int \{\lambda\nu\mu_z - (1-\lambda)\,\nu_z\mu - \frac{1}{4}D\mu_\tau^2 + \frac{1}{2}\gamma\left(\mu^4 + 2\mu^2\nu^2\right) + F\}dzd\tau \tag{9}$$

The advantage of the above trial-functional is that the stationary condition with respect to μ results in equation (7)

Integrating equation (7) with respect to ν, we obtain the Euler–Lagrange equation

$$\mu_z + \gamma\mu^2\nu + \frac{\partial F}{\partial \upsilon} = 0 \tag{10}$$

$$\frac{\delta F}{\delta \nu} = \frac{\partial F}{\partial \upsilon} - \frac{\partial}{\partial z}\frac{\partial F}{\partial \mu_z} + \frac{\partial^2}{\partial \tau^2}\frac{\partial F}{\partial_{\tau\tau}} - \dots \tag{11}$$

Combining equations (10) and (11) and solving for F, we have

$$\frac{\partial F}{\partial \upsilon} = -\mu_z - \gamma\mu^2\nu = \frac{1}{2}D\nu_{\tau\tau} + \gamma\nu^3 \tag{12}$$

Integrating both sides of equation (12) with respect to ν, we get

$$F = -\frac{1}{4}D\nu_\tau^2 + \frac{1}{4}\gamma\nu^4 \tag{13}$$

We, therefore, obtain the following needed variation principle

$$J\left(\mu,\nu\right) = \int L_\lambda dzd\tau \tag{14}$$

where the Lagrange multiplier is defined as

$$L_\lambda = \lambda \nu \mu_z - (1-\lambda)\,\nu_z \mu - \frac{1}{4} D\left(\mu_\tau^2 + \nu_\tau^2\right) + \frac{1}{4}\gamma(\mu^2+\nu^2)^2 + a\mu\mu_z + b\nu\nu_z \tag{15}$$

Zhang *et al*(2005), where a and b are arbitrary constants whose values can be chosen as $a = -i\lambda$.and $b = i(\lambda - 1)$ in order for equation (15) to be written in the form of Ψ

$$L_\lambda(\Psi) = i\left(\frac{1}{2}-\lambda\right)\Psi\Psi_z - i\frac{1}{2}\Psi\Psi_z^* - \frac{1}{4}D|\Psi_z|^2 + \frac{1}{4}\gamma|\Psi|^4 \tag{16}$$

where $\Psi_z^* = (\mu_z - i\nu_z)$.

Selecting the values of $\lambda = 0, \frac{1}{2}$,1 and using the boundary conditions, equation (16) has universality in some sense for nonlinear fiber optics as shown in Fig. 4

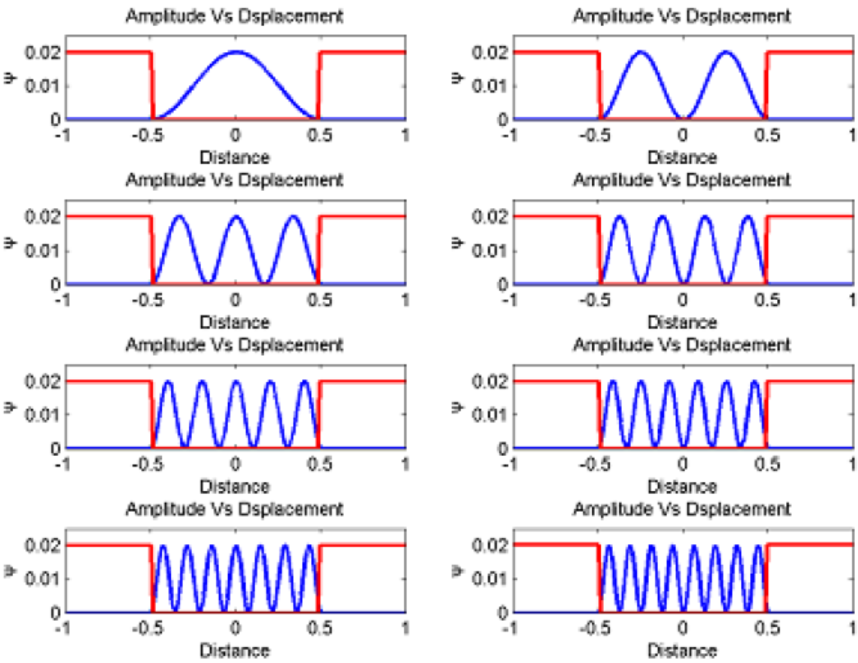

FIGURE 6. Distance Vs Amplitude

3.3. Maxwell Equation

The set of nonlinear differential equations, which is especially suitable for the study of wide-band wavelength-division multiplexed systems of optical communications. The optics of dielectric waveguides is governed by Maxwell's equations, Born (1999).

Due to the material nonlinear effects, a fiber-optic transmission line is a nonlinear channel Haiqing and David (2010).

According to Koshiba (1973), we understand that "a waveguide mode is a transverse field pattern whose amplitude and polarization profile remains constant along the longitudinal direction". The electric and magnetic fields of a mode can be written as

$$E(r,t) = E_m(x,y)\, e^{(i\beta z - i\omega t)} \tag{17}$$

$$H(r,t) = H_m(x,y)\, e^{(i\beta z - i\omega t)}, \tag{18}$$

where m is mode index, $E_m(x,y)$ and $H_m(x,y)$are the mode field patterns and β is the propagation constant of the mode. These equations are differentiated twice to give the TE and TM modes respectively as follows

$$\frac{\partial^2 E_y}{\partial x^2} + \left(k_0^2 - \beta^2\right) E_y = 0 \tag{19}$$

for TE modes,
and

$$\frac{\partial^2 H_y}{\partial x^2} + \left(k_0^2 - \beta^2\right) H_y = \frac{1\partial\varepsilon\partial H_y}{\varepsilon\partial x\partial x} \tag{20}$$

for TM modes where $k_0 = \frac{\omega^2}{c^2} n^2(x)$.
A guided mode can exist only if it satisfies a transverse resonance condition, such that the reflected wave has constructive interference with itself. The transverse component (x-component) of the wave vector inside the core is $h_1 = k_{01}cos\theta$, where θ is the angle of incidence, and $k_{0i} = \frac{2\pi ni}{\lambda}$, $i = 1, 2, 3$ and the longitudinal component $\beta = k_{01}cos\theta$.Verma *et al* (2011)
The transverse for the substrate and cover regions can be defined as $h_2 = k_{02}cos\theta$, and $h_3 = k_{03}sin\theta$, where k_{01}, k_{02} and k_{03} are the propagation constants in the respective regions. Applying boundary conditions $\frac{n_1}{n_2}$ and $\frac{n_1}{n_3}$ the interface and using equations (19) and (20) we obtain the eigen value equation for TE and TM mode as follows.

$$tan\left(\frac{h_1 d}{2} - \frac{m\pi}{2}\right) = \frac{\sqrt{V^2 - h_1^2 d^2}}{h_1 d},\ m = 0,\ 1,\ 2\ldots \tag{21}$$

for TE modes

$$tan\left(\frac{h_1 d}{2} - \frac{m\pi}{2}\right) = \frac{n_1^2}{n_2^2}\frac{\sqrt{V^2 - h_1^2 d^2}}{h_1 d},\ m = 0,\ 1,\ 2\ldots \tag{22}$$

for TM modes.

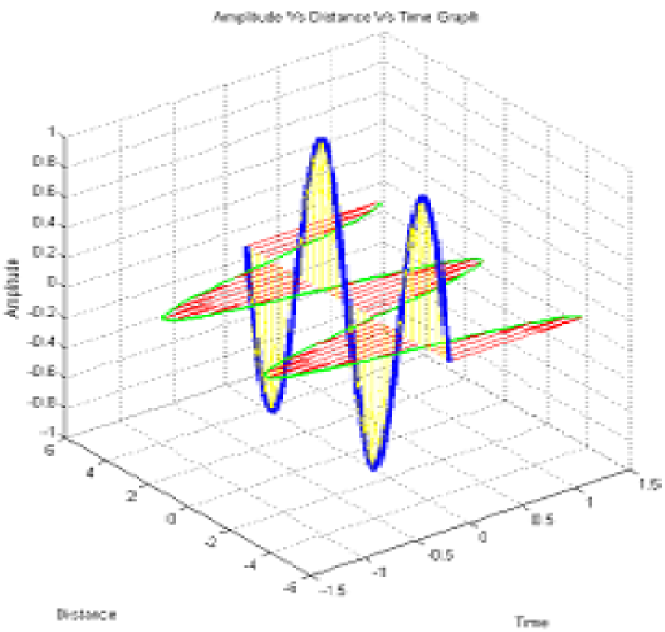

Figure 7. Maxwell Equation Graph in 3D

From Fig. 4, the left and right hand side of equations (21) and (22) is a function of h_1d and effective refractive index Neff of a guided mode which will give the value of β and thus effective index. Applying the boundary conditions; we show the propagation constant for the graph in three dimensions structure, having components for the TE like mode or effective indices are considered in the horizontal structure to get a final structure. The TM field of the vertical structure and the Neff obtained from the analytical calculations.
These results show the changes in shape of these pulses by propagating in optical fiber. From our analysis, we have therefore asserted that sine waves describe many oscillating phenomena. When the wave is damped, each successive peak decreases as time goes on.

Conclusion

This method is applied in ultrahigh-speed all-optical signal processing systems and has the potential to be expanded for more complicated computing functionality. From the graphs, the behaviour of solutions to the PDE's, ODE's and Bessel functions together with their interactions and applications in the fiber optics solutions theory in communication engineering. We model these ODE's and PDE's to describe the impact of dispersion in data transmission. The Bessel functions have a decaying amplitude and therefore recommended for short transmission of data. The Maxwell and Schrodinger equation plots maintain the shapes and therefore recommended for long distance transmission of data.

References

[1] Agrawal G. P, (2007). *Nonlinear Fiber Optics, fourth edition*, Academic Press is an imprint of Elsevier, 84 Theobald's Road, London UK.

[2] Basmadjian D. (2003). *An introduction to Mathematical Modeling of Physical Systems*, Oxford University Press, New York.

[3] Born M. and E.Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 7th ed. Cambridge University Press, 1999.

[4] Buczynski, R. Andriukaitis, G. ; Balciunas, T.(2014); *Ultrafast solitonic nonlinear directional couplers utilizing multicomponent glass dual-core photonic crystal fibres, Transparent Optical Networks (*ICTON).

[5] Chynoweth, A.G (, 1976). *Lightwave Communications: The fiber lightguide*, *Physics Today*, **29** (5), 28.

[6] Deqiang D, Balin Tian, Wei L ,Ying L, Peian R.(2013); Communication Technology (ICCT), 15th IEEE International Conference on DOI: 10.1109/ICCT.2013.6820392 ,pp 315 - 318 Elsevier Science B.V; 2002.

[7] Fang D, Sun, Qizhi ; Zhao, Xiaoming ; Jia, Yuesong (2014); Design of a fully-fiber multi- chord interferometer and a new phase-shift demodulation method for field-reversed configuration. Review of Scientific Instruments **85**, 053510 - 053510-6.

[8] Haiqing W. and David V. (2010): *Fibers*, *McGill University, Montreal, Canada.*

[9] He JH (2004); *Variational principle for some nonlinear partial differential equations with variable coefficients.* Chaos, Solitons Fractals; **19**, 847–851.

[10] Ishigure, T., E. Nihei, and Y. Koike(1996). "*Optimum Refractive Index Profile of the Graded Index Polymer Optical Fiber, Toward Gigabit Data Links*," *Applied Optics*, 35; 2048–2053.

[11] Koshiba, M. (1973). *Optical waveguide analysis*, New York, McGraw-Hill.

[12] Kundaeli H.N (2004); *Analysis of an optical fiber communication system using laser rate equations,* Tanzania Journal of Science, pp 73–74

[13] Malomed BA (2002); *Variational methods in nonlinear fiber optics and related fields.* In: Wolf E, editor. Progress in optics; **43**.

[14] STEP (Scientific and Technological Education in Photonics), an *NSF ATE Project (2008) CORD Communications* 601 Lake Air Drive, Suite E Waco, Texas 76710-5841, pp 14.

[15] Verma A, Prajapati Y, Ayub S, Saini J, Singh V. (2011); *International Journal of Engineering, Science and Technolog*y Vol. 3, No. 3, 2011, pp. 36-40.

[16] Zhang J, Yong J, Yu, Pan N. (2005); *Variational principles for nonlinear fiber optics.* Elsevier Ltd.

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