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**THE FINANCIAL IMPLICATION OF LONGEVITY IMPROVEMENTS ON THE
LIABILITIES OF ANNUITY PROVIDERS.**

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ABSTRACT

Different countries around the world have experienced and improvement in mortality over the years. This improvement can be attributed to factors such as technological advancements in the medical field and improved living standards among others. This is generally a good thing, but for life insurance companies and defined benefit pension schemes that provide benefits in form of annuities, there lies the risk of longevity. Which is the risk that people will live longer than expected requiring them to be paid benefits for longer periods. This paper studies the implication of changes in Kenyan mortality on the value of an annuity. It focuses on the mortality of the annuitants in Kenya. These are people who have purchased life annuities from Kenyan insurance companies. Exponential extrapolative methods are used to obtain reduction factors that are later used to extrapolate mortality from the year 2011-2050. An annuity of Ksh.1 is then priced over the extrapolated period for each age range for all years, in order to determine how an annuity changes over the years. From the analysis, the Kenyan mortality has been reducing. This has an inverse effect on the value of annuities as the annuity prices are observed to increase with an increase in mortality over the years. This therefore exposes the Kenyan insurance companies to longevity risk.

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TABLE OF CONTENTS

DECLARATION	i
ABSTRACT	ii
ACKNOWLEDGEMENT	iii
TABLE OF FIGURES	vi
CHAPTER ONE	1
INTRODUCTION.....	1
1.1 Background	1
1.2 Research Objectives	2
1.3 Research Questions	2
1.4 Problem Statement	2
1.5 Importance of the Research.....	3
1.6 Scope of the Research	3
CHAPTER TWO.....	4
LITERATURE REVIEW	4
2.1 Introduction	4
2.2 Implications of Longevity Risk.....	4
2.3 Forecasting Mortality	8
CHAPTER THREE.....	10
METHODOLOGY	10
3.1 Nature of Study	10
3.2 Population and Sampling.....	10
3.3 Data Collection Methods.....	10
3.4 Model Assumptions and Framework.....	11
3.5 Model Assumptions.....	11
3.6 Mortality Forecasting Models	11
3.7 Calculating Change in the Cost of Annuities	21
3.8 Summary	23
CHAPTER FOUR.....	24
RESULTS AND ANALYSIS	24

4.1 Preliminary Analysis	24
4.2 Extrapolating Mortality	26
4.3 Computing Change in the Annuity Prices	31
CHAPTER FIVE.....	33
CONCLUSIONS AND RECOMMENDATIONS.....	33
5.1 Introduction	33
5.2 Summary	33
5.3 Discussion	33
5.4 Recommendations	35
5.5 Conclusion.....	35
REFERENCES.....	37
APPENDICES.....	40
Appendix I: Table showing the 5 year term annuity prices for the years 2011-2050.	40
Appendix II: Table showing the percentage changes in term annuity prices.....	41

TABLE OF FIGURES

TABLE 4.1. TABLE SHOWING PROBABILITIES OF DEATH FOR THE WHOLE KENYAN POPULATION DERIVED FROM WHO WEBSITE.....	24
FIGURE 4.1 -GRAPHS SHOWING PAST TREND OF PROBABILITIES OF DEATH FOR BOTH MALE AND FEMALE MEMBERS OF THE WHOLE POPULATION AGED BETWEEN 55 AND 79,.....	25
FIGURE 4.2 -GRAPHS SHOWING TRENDS IN ANNUAL REDUCTION FACTORS FOR THE PERIOD 2011-2050.	27
FIGURE 4.3- GRAPHS OF EXTRAPOLATED MORTALITY..	28
FIGURE 4.4- GRAPH OF EXTRAPOLATED MORTALITY DRAWN ON A LOGARITHMIC SCALE..	29
FIGURE 4.5- GRAPHS OF EXTRAPOLATED MORTALITY BASED ON 12 YEAR REDUCTION FACTORS PLOTTED ON A LOGARITHMIC SCALE..	30
FIGURE 4.6- GRAPHS SHOWING TREND IN PRICES OF FIVE YEAR TERM ANNUITIES.....	32
TABLE 4.2 - TABLE SHOWING MALE AND FEMALE 5 YEAR TERM ANNUITY PRICES.	40
TABLE 4.3 - TABLE OF ANNUAL PERCENTAGE CHANGES IN TERM ANNUITY PRICES.....	41

CHAPTER ONE

INTRODUCTION

1.1 Background

Over the years there has been an improvement in life expectancy for most countries around the world. Naturally it is a good thing, it is however an issue of concern for pension schemes and annuity providers as well as individuals. Individuals face a risk of outliving their income while annuity providers and pension schemes on the other hand face the risk that they will pay out more benefits than earlier expected (Antolin, 2007). Governments that offer social security benefits also bear the risk of paying out these benefits for longer periods than expected.

The risk of mortality improvements being greater than expected is known as longevity risk and has been recognized as one of the significant risks impacting on the financing of ageing populations around the world. It has implications for insurance companies issuing life annuities and life insurance policies, defined benefit pension funds as well as governments with social security pension obligations (Njenga & Sherris, 2011). Historically, unexpected improvements in mortality rates have led to large, unanticipated increases in life expectancy (Guy, 2013). This as a result leads to higher than expected expenses to various industry participants because as people age there is a strain on the resources available for both government and the private sector. Various measures have been implemented to try and manage longevity risk for an individual these methods include; social security provided by the government, defined benefit pension schemes provided by corporations and whole life annuities provided by life insurers. The law of large numbers was easily used in this case to hedge against longevity. However when aggregate longevity comes in managing longevity proves to be a challenge. Aggregate longevity in this case, being the risk that an average member in a birth cohort will live longer than expected (MacMinn, Brockett & Blake, 2006).

Longevity risk can have serious effects on financial stability and these effects derive from two sources; threats to fiscal sustainability by the government due to large longevity exposures and threat to solvency of private financial and corporate institutions that are exposed to longevity risk (International Monetary Fund, 2012). This paper seeks to determine these financial effects of longevity risk on the providers of annuities specifically.

1.2 Research Objectives

This paper seeks to determine how the improvements in Kenyan mortality over time affect the liabilities of annuity providers, by forecasting mortality into the future assuming that current trends in mortality improvements will continue into the future and thereafter pricing an annuity at specific points in order to see how the price of the annuity changes over time.

1.3 Research Questions

This study seeks to answer the following research questions:

- What has been the current trend in mortality improvements?
- What will be the mortality rate if current improvements continue into the future?
- How do the annuity costs change when mortality improvements are allowed for?

1.4 Problem Statement

With the improvements in mortality around the world, comes the risk that people will live longer than expected and will therefore be paid benefits for longer periods than initially expected. Benefit providers are therefore faced with the challenge of determining annuity prices as the prices of annuities depend on the mortality of annuitants.

Life insurance companies and defined benefit pension schemes are the major providers of benefits to the older generations. Including mortality improvements has a direct impact on cost of survival benefits because, with the improvements in mortality people will live longer and therefore will need to be paid the benefits for longer this therefore has to be factored into the cost of the benefits to be provided. However, not including longevity

risk will risk the financial stability of the annuity provider (Khalaf-Allah, Haberman & Verral, 2006).

In Kenya, defined benefit schemes and life insurance companies use forecast mortality tables to determine the price of annuities. The tables used by defined benefit schemes are mostly the UK annuitant tables and those used by the insurance companies are the Kenyan mortality tables 2010. The problem with these tables is they may fail to properly represent future mortality improvements leading to longevity risk. As a result, the amount that should be charged for a given annuity becomes an issue of concern.

This study therefore seeks to analyze how the costs of benefits provided to annuitants are affected by failure to adequately incorporate the improvements in mortality in the life tables of annuitants in order to determine how significant the risk is.

1.5 Importance of the Research

Failure to incorporate mortality projections into the pricing of annuities could lead to the insolvency of an insurance company and for the defined benefit schemes it could result in the underfunded status of a scheme. This study has the importance of determining the significance of longevity risk on the liabilities of providers of benefits to annuitants. The study will therefore show the need for annuity providers in Kenya to select a forecast method that will help them to adequately recognize the improvements in mortality so as to correctly price the benefits offered. Various industry participants will benefit from this study as it will enable them realize the effect of failing to incorporate mortality improvements in their pricing and in the end the practice of using life tables adjusted for mortality improvements may be implemented.

1.6 Scope of the Research

This study will be based on the providers of annuities in Kenya who are insurance companies and defined benefit pension schemes. However, with the shift of pension schemes from defined benefit to defined contribution, payment of pensions is made by the insurance companies. This study will focus on the annuities offered by the insurance companies.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Longevity risk is the risk that people will live longer than expected or that there will be sudden unexpected mortality decreases (Richards, 2013). The improvements in mortality are due to factors such as the improvements in health care and medical advancements such as organ transplants.

Over the years, there has been an unexpected increase in life expectancy. Bravo, Real & da Silva (2011), note that the improvements in life expectancy mean that individuals need to reconsider their savings in order to maintain a satisfactory standard of living when they retire.

2.2 Implications of Longevity Risk

Longevity risk has been known to be faced by the government, insurance companies and defined benefit pension schemes. For the government it is faced in two ways: through the state pension scheme and since the government is the lender of last resort in case of unexpected mortality improvements. The Kenyan government has very little exposure to longevity risk because it does not offer a state pension to its citizens but it has a non-contributory civil service pension scheme. The scheme exposes the government to longevity risk because it operates on a pay as you go basis and the burden of the same is transferred to the taxpayers.

According to the principles of life cycle finance, longevity risk has an effect on the national savings. An individual faces the task of deciding how much of theory income to consume and how much to save. The savings are determined by the expected lifetimes that are calculated based on the probability of survival to a given age (Gomes & Cocco, 2011). Bloom, Canning & Graham (2003) observe that people generally save while young in order to finance consumption during retirement. Savings tend to rise with the increase

in longevity in order to meet the increased consumption needs on retirement and in order to prevent a scenario where the individual has outlived their income. Improvements in life expectancies however, lead to the postponement of retirement because it means health improvements for the elderly and this has been observed in Kenya as well when the retirement age for civil servants increased from 55 to 60 in order to give the government to pay any outstanding benefits to retirees.

For defined benefit pension schemes longevity comes in terms of the benefit amounts that the scheme will have to pay out on retirement in case the pensioner lives longer than anticipated. Insurance companies are affected through the whole life annuities they offer (International Monetary Fund, 2012).

Gomes and Cocco (2011) note that the underestimation of mortality improvements is the cause of the underfunded status of defined benefit pension schemes and government pay as you go schemes, the underfunding has adverse effects as it will have to be catered for in one of two ways: the contributions can be increased or the benefit lowered. Reducing contribution will mean that the individual will have to reduce their consumption during the accumulation phase and reducing the benefit increases the longevity risk on the part of the individual. In Kenya, the retirement benefits authority regulates the providers of defined benefit schemes. The schemes are required to have a funding level of 100% (Retirement Benefits Authority, 1997). Longevity risk increases the chances that the surplus of the scheme will go below this level which may result in penalties.

Pension schemes generally need to address longevity risk because they make long term commitments. Defined benefit schemes have the greatest exposure to longevity risk as the scheme bears any investment risk and the balance of cost. Underestimation of mortality when making past projections means that defined benefit pension schemes will have to make additional provisions for the increased life expectancies (Thompson & Andersen, 2007). Without considering longevity risk, then the defined benefit pension scheme cannot be effectively managed (Guy, 2013). The significance of longevity risk depends on the specifics of the pension funds with factors such as the funding status, and the demographics of the plan members playing a key role. The younger the membership

of the pension scheme then the more exposed the scheme is to longevity risk (Antolin, 2007).

The effect of longevity on defined benefit pension providers is faced by both the scheme and the sponsor as pension liabilities are payable from the sponsor's financial statements. When the pension risks are too large they may impact on the liquidity of the sponsor because the sponsor will have to chip in to pay out the excess pension obligations. Antolin (2007) while quoting (Friedman et al) includes that the pension risk increases stock volatility of the sponsor, increases equity beta, and raises the weighted average cost of capital. In the Kenyan case, if the liabilities of the sponsor in respect to the defined benefit scheme are too high, the profits of the sponsor will reduce as they will have to pay out more from the sponsor's financial statements. In adverse cases the sponsor will have to issue more debt in order for them to pay off the existing liabilities. This will increase the cost of capital for the company. The beta of the sponsor's stock increases as the company will be considered more risky. This will force the company to pay out more dividends affecting its profitability and hence the stock price becomes volatile.

There has been a shift in the pension schemes from defined benefit to defined contribution. In Kenya, the Retirement Benefits Authority in 2010 required that schemes convert to defined benefit in order to curb the underfunded status of most schemes. Despite the fact that defined benefit schemes have the greatest exposure to longevity risk, defined contribution schemes also face the risk the only difference being that for the defined contribution schemes the individual bears both the investment and longevity risk. Longevity risk is of great consideration when determining the optimal asset allocations for defined contribution pension schemes as individual seek to save enough so as not to face a situation where they reduce their consumption at old age (Yang & Huang, 2009). Defined contribution schemes generally transfer their pension liabilities to insurance companies who are equipped to effectively manage this risk.

Longevity risk is not diversifiable. It therefore cannot be controlled by the law of large numbers, because the policies sold by an insurer are not independent in respect of that source of uncertainty. Guy (2013) notes that traditionally insurers manage longevity risk by: fully funding the liability, hedging unwanted risks and managing a carefully designed,

diversified investment strategy. However with the increase in longevity risk, insurance companies have been unable to correctly price annuity products because of the unexpected improvements in mortality beyond the projected mortality tables used by insurance companies. There is need therefore to effectively recognize the future improvements in mortality, the mortality tables used in pricing should adequately project future mortality improvements, so as to facilitate proper pricing of annuity products.

Wang, Huang, Yang & Tsai (2010) argue that annuity mispricing could lead to significant risk for the insurance companies as it will mean that they may end up paying out more as liabilities and for a longer period of time which will require them to contribute more capital to cater for their liabilities and could ultimately lead to insolvency.

Pricing of life insurance products and annuity providers depends on; stock market returns, interest rates and future mortality rates. In Kenya, life insurance companies have a wide variety of investments, as they try to maximize on the returns on investment in order to be able to pay their liabilities when they fall due and make profit at the same time (Wang et al ., 2010).Guy (2013) finds that there is interdependence between longevity risk and interest rate risk as when interest rates fall longevity risk increases because the liabilities will have to be discounted at a lower interest rate meaning their size will be larger. When longevity increases interest rates increase. Annuity providers will need more prudent interest rate assumptions in order to reduce the significance of longevity risk incase rates decrease. This will be useful in forecasting as an assumption is made that there exists a constant rate of interest.

Lack of developed markets for longevity risk implies that longevity risk cannot be effectively hedged. It is only recently that financial products such as longevity bonds were introduced and in Kenya they are not yet a common instrument to use as a hedge. High returns generally can be seen to be a hedge to the risk because the return on investment would adequately cover the payments of benefits for periods longer than expected. With the decline in the rates of return and poor equity performance however, longevity risk becomes very significant for the providers of benefits (Thompsen & Andersen, 2007).The urgency to recognize and properly hedge longevity risk is brought up in this case therefore and an analysis of how the risk impacts on liabilities in an important consideration.

2.3 Forecasting Mortality

Before the implication of longevity risk can be determined, the question of how significant the risk is should be answered first. According to Guy (2013) this is a two step procedure of first evaluating the size and likelihood of the potential increases in life expectancy and then determining the effect of these increases on pension liabilities.

The size of the potential increases in life expectancy is determined by quantifying the potential range of outcomes for trend of future mortality improvements relative to base mortality. The base mortality in this study will be the mortality of the annuitants that will be projected based on how the mortality has been improving over the years for the whole population in general. Future improvements will be assumed to occur in a similar way and mortality will then be forecast into the future.

There is need to adequately recognize mortality improvements in the pricing of annuity products (Pitacco, Denuit, Haberman & Olivieri, 2009). Recognition of mortality is done by including a proper forecast of mortality improvements in the life tables used for annuity pricing. The concept of forecasting mortality which is a concern for this study can therefore be introduced. Forecasting of mortality is often difficult because there are many drivers of mortality such as age, gender, health conditions among others and they all cannot be included in the mortality forecast and therefore the results obtained may be somehow biased.

Two models need to be considered before a mortality forecast can be made these are the underlying model and the forecast model. The underlying model needs to be in such a way that the parameters can be easily understood. In case the parameters are not easily interpretable then there is need to transform the data into a form that can easily be interpreted. This is possible with the use of logarithms or reduction factors. A forecast method is chosen after the underlying model of mortality has been selected (Booth & Tickle, 2008). The data for this study is based on the probabilities of death and depending on the model used will need to be modified into a different interpretation of mortality.

Khalaf-Allah et al. (2006) note that there is no specific model for all scenarios and the choice of the model depends on the data available, the length of forecasting and the purpose of the forecast. For this paper the forecast is long-term because the mortality being modeled is that of annuitants.

Various methods of forecasting mortality exist and they are discussed in the next chapter.

CHAPTER THREE

METHODOLOGY

3.1 Nature of Study

The study was based on quantitative techniques of data analysis because of the use of numerical data to come up with the findings.

3.2 Population and Sampling

3.2.1 Population

The study is based on the Kenya and was therefore based on the annuitants' mortality table for Kenya as it seeks to establish the effect of mortality improvements on liabilities of annuity providers in Kenya.

3.2.2 Data Sampling

The data was sampled based on non-probability sampling of the mortality of a portion of the Kenyan population that has bought annuities from insurance companies. These people are considered to be having a better standard of living and as a result lower mortality than the rest of the population.

3.3 Data Collection Methods

The data required for this study is mortality for annuitants in Kenya. These are the people that have bought life annuities and the mortality for the whole Kenyan population. The annuitant data that was used was the Kenyan annuitant mortality data. Secondary data collection methods were used. The data was sourced from websites and databases. The mortality data for the whole population was sourced from the WHO website.

The annuitant mortality data was obtained from the Association of Kenya Insurers (AKI) database. The data from WHO is that of the mortality of the general Kenyan population for the years 1990, 2000, 2012, 2013. The annuitant data on the other hand, is for the four year period between 2007 and 2010.

3.4 Model Assumptions and Framework

Generally annuity prices are computed based on the rates of interest and the mortality projections. The uncertainties in mortality therefore have an impact on the cost of these annuities, which is what this study is trying to determine. The study involved two steps the first being the projection of mortality to determine the improvements and the other to determine how the annuity prices change as a result of these improvements by pricing an annuity product.

Longevity risk being a long-term risk means that an extrapolative method of forecasting would be most suitable because it enables us project mortality over longer time periods. An extrapolative method will be simplistic because it will only consider the trend over the years and not factors such as the health condition of the annuitant which also affects mortality.

3.5 Model Assumptions

- A constant rate of interest of 6% was assumed. This is to make it easier to identify the changes in mortality as changes in interest rates affect the annuity prices.
- The only two factors that affect mortality are age and sex.
- Annuities are purchased at age (55)

3.6 Mortality Forecasting Models

Mortality was forecast into the future using the selected forecast method. The method that was used was one that is extrapolative in nature because an extrapolative method of forecasting seeks to forecast mortality based on a data base that contains past mortality experience (Pitacco et al., 2009). An expectation forecasting approach could not be used in this case because it is highly subjective.

Mortality improvements for the general population based on the WHO data were used to determine how mortality changed between 1990-2000, by obtaining reduction factors and an assumption is made that this mortality trend will continue into the future. The factor by which mortality has improved in the past is then applied to the Kenyan annuitants' data to project the mortality improvements for annuitants. The annuitant mortality will be projected from 2011-2050.

The data available to be used for this study is limiting in that only data over a four year period is available and therefore models that have been used commonly, specifically the Lee-Carter model cannot be used in this case. Lee- Carter method requires data over long observation periods for it to give accurate results. In Kenya it is difficult to obtain data for long periods due to the fact that the country does not have developed data storage systems for demographic data.

Various models have been used in the past to try and project mortality rates and hence predict the trend in mortality improvements (International Monetary Fund, 2012). Some of these were very useful in this research. The models used to forecast mortality rates can be of three types: models that use the underlying drivers of mortality (process based methods), extrapolative methods that use historical trends only to predict future mortality and explanatory methods that use causal relationships for example econometric models (Antolin , 2007). Explanatory methods are however not fully developed because of the imperfect understanding of the relationships between risk factors this makes the forecasts unreliable (Booth & Tickle, 2008). An additional forecast method introduced by Booth and Tickle is expectation models that rely on expert opinion to determine the mortality. The disadvantage of expectation models is their subjective nature and for it to be accurate opinions of many experts will need to be considered, this study will require a model that uses past trends to forecast mortality because of the nature of the data available.

The data used in this study is the mortality of annuitants both male and female and as a result models such as those that are explanatory in nature could not be used because of lack of data on other drivers of mortality. This study will focus on extrapolative forecast methods.

Extrapolative methods can either be stochastic or deterministic. Deterministic models are those that forecast life expectancies without the use of forecast probabilities while stochastic models use forecast probabilities to forecast into the future.

Different researchers have used different extrapolative methods to forecast mortality. The various forecast methods include: models that use time series for example the lee-carter model, those that use econometric analysis such as spline models and finally those that

are based on interdependent projection of age specific mortality such as graduation models and CMI. Stoeldraijer, van Duin, van Wissen & Janssen (2013) further split the extrapolative projection into parametric for example the Lee carter model where data is fit using a statistical method into a desired curve and Targeting where a target for long term improvements is set and the model tries to achieve these targets.

(Khalaf-Allah et al ., 2006) include four extrapolative methods of forecasting: models based on the independent projection of age-specific mortality rates or forces of mortality, including mortality reduction factor models; related models based on the logit transformation; models based on graduating mortality rates with respect to age for specific time period and then projecting the parameters; models based on graduating mortality rates with respect to age and time simultaneously; e) the Lee-carter method. A discussion of these models can be found in Khalaf-Allah (2007).

Methods that use linear continuations of the mortality trend to forecast future mortality are a viable option for mortality forecast. For this study however, these methods will not be appropriate because they do not forecast mortality over the long term (Booth & Tickle, 2008).

The most commonly used model is the two factor Lee-Carter model that uses time series to extrapolate the mortality (Booth & Tickle, 2008). The main assumption of this model which is also a drawback is that future mortality will follow the trend of past mortality which may not be the case.

Lee-Carter Method:

The structure of the lee carter model is given by

$$\ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}, \quad (3.1)$$

a_x Is the age pattern of log mortality rates averaged across years.

b_x Is the first principal component reflecting change in the log mortality rate at each age.

k_t Is the first set of principal component scores at year t and measures the general level of the log mortality rate.

$\epsilon_{x,t}$, Is the residual at age x in year t.

The model adjusts k_t by refitting the total number of deaths. The adjustment gives more weight to high mortality rates and thus counteracts the effect of log transformation. Adjusted k_t is forecasted using random walk with a drift. Forecasted mortality rates are then obtained by multiplying the forecasted principal component scores with mean and principal context (Booth & Tickle, 2008).

Lee –Carter model does not include the cohort effect in forecasting mortality it only takes into account the age and period effects .There have been some improvements in this model to try and cater for the cohort effect by using the Lee-Carter APC model introduced by Renshaw and Haberman (2003). This model however still has some draw backs such as the lack of smoothness (Bruce et al ., 2013) quoting (Life Office Mortality Committe, 2007).

The problem of smoothness can be solved by use of P-spline model which includes a penalty that can be adjusted to strike balance between the fitness of data and smoothness. Where a high penalty means greater smoothness and poor fit and a low penalty indicated a good fit but rough data.

The Lee-Carter model requires that data over a long period of time for it to yield accurate results. For this research however, the data available is for four years only and because of this the Lee-Carter model cannot be used.

Parametric Formula:

The oldest means of forecasting mortality has been to fit a parametric formula and the formula used most has been the Gompertz formula of mortality forecasting. In this method, the parameter estimates are treated as independent time series projected into the future in order to project the life tables used in pricing. The major limitation to this model is that the results are highly dependent on the parametric formula chosen (Bravo, Real & da Silva, 2009). Gompertz and Makeham’s laws of mortality are parametric survival models.

Renshaw, Haberman & Hatzoupoulos (1996) proposed a two-factor model based on an over-dispersed Poisson distribution of deaths and comprising a Gompertz-Makeham’s graduation term in combination with a multiplicative age-specific trend adjustment term (Khalaf-Allah et al, 2006).

Sithole, Haberman & Verrall (2000) used the same model to forecast the mortality of annuitants and pensioners in the UK. The model was used by Khalaf-Allah et al. (2006) to project the annuitant mortality for the period between 1983 and 1996 and 20-year projections were made. This is a clear indication that the model does not require large data sets to forecast mortality into the future.

The equation for the force of mortality is therefore

$$\mu_{x,t} = [\exp(\beta_0 + \sum \beta_j L_j(x'))] \exp[\sum [\alpha_i + \sum_{j=1}^s \gamma_j L_j(x')] t^i] \tag{3.2}$$

L_j is a Legendre polynomial that are generated by

$$L_0(x) = 1, L_1(x) = x, (n+1)L_{n+1}(x) = (2n+1)xL_n(x) - nL_{n-1}(x), \quad \text{When } n \text{ is an integer and } n > 1$$

X' and t' are age and time variables respectively transformed linearly.

The equation can be re-written as

$$\mu_{xt} = \exp\left\{ \sum_{j=0}^s \beta_j L_j(x') \right\} \exp\left\{ \sum_{i=1}^r (\alpha_i + \sum_{j=1}^s \gamma_{ij} L_j(x')) t^{i,j} \right\} \quad (3.3)$$

The first multiplicative term in equation (1.0) takes the form of a GM (0, s) formula. GM (0, S) is the initial for Gompertz-Makeham's formula. The second term is the age specific adjustment term provided that at least one of the γ_{ij} terms is not preset to zero.

The unknown parameters can be estimated using the maximum likelihood estimation method. A model that provides the best fit is first established and projections made based on the 40-year period. Using the model and the projections obtained reduction factors that can be used subsequently are determined by revising the model (Khalaf-Allah et al., 2006).

To estimate unknown parameters the actual number of deaths a_{xt} are modelled as independent poisson response variables A_{xt} of a generalized linear model. the mean and variance for this model are:

$$E[A_{xt}] = m_{xt} = R_{xt}^c \mu_{xt} \quad (3.4)$$

$$\text{var}(A_{xt}) = \phi m_{xt} \quad (3.5)$$

$R_{xt}^c \mu_{x+t}$ is the central exposed to risk

ϕ is the scale parameter.

The function that links the unknown parameters is a log function.

$$\log m_{xt} = \log R_{xt}^c + \log \mu_{xt} = \eta_x t$$

So that

$$\log m_{xt} = \log R_{xt}^c + \beta_0 + \sum_{j=1}^s \beta_j L_j(x') + \sum_{i=1}^r \alpha_i t'^i + \sum_{i=1}^r \sum_{j=1}^s \gamma_{ij} L_j(x') t'^i \quad (3.6)$$

Estimation of the parameters is carried out using the quasi log-likelihood approach which involves maximizing the expression

$$\frac{1}{\phi_{x,t}} \sum -m_{xt} + a_{xt} \log m_{xt} \quad (3.7)$$

To determine the optimum values of r and s , improvement in the scaled deviance for successive increases in the values of r and s is compared with the χ^2 random variable with 1 degree of freedom as an approximation. The optimal values chosen are the minimum values of r and s beyond which improvement in deviance is not statistically significant.

Deviance and Pearson residuals are used to carry out diagnostic tests on the model.

Reduction factors that are used are those that are determined as the ratio of the forces of mortality rather than mortality rates. These reduction factors are applied to lives attaining age 55 t years after the base year which is 2010 for this study.

$$\mu_{x,t} = \mu_{x,0} RF_{(x,t)} \quad (3.8)$$

Where $\mu_{x,0}$ is the value of $\mu_{x,0}$ in the Kenyan mortality table (2010).

Reduction factors will be determined for both male and female annuitants. Using the formula:

$$RF(x,t) = \exp\left[\frac{1}{w_i} (\alpha_i + \gamma_{11}(x')) \right] \quad (3.9)$$

Where w_i denotes half the calendar year range for the investigation period.

Simpler extrapolative methods were discussed in Pitacco et al. (2009). The methods are discussed below and due to their simple nature are used in this study.

Reduction Factors

To get the future mortality, the relationship between probabilities of death at (x) that refers to a given calendar year t' and a generic year t where $t' >$ can be represented as

$$q_x(t) = q_x(t')R_x(t - t') \quad (3.10)$$

$R_x(t - t')$ Is referred to as the reduction factor because it is expected to be less than 1 to indicate the prevailing downward trend in probabilities of death. A simplification to this is making an assumption that the reduction factors do not depend on age. Therefore for all t and x

$$R_x(t - t') = R(t - t') \quad (3.11)$$

Making an assumption that the trend in the observed reduction factors will continue into the future, mortality forecast is obtained through the appropriate modeling procedure being applied to the reduction factor. The projected mortality is obtained through (3.10)

The approach to extrapolation based on reduction factors can be based on either a mathematical formula or a graphical method. The mathematical formula commonly used is the exponential formula

Exponential Formula

The exponential formula is commonly used in practice. These methods have been implemented in USA, Great Britain, Germany and Austria.

Here we are using a simplification of cohort tracking. Mortality for a specific age is observed over a period to determine the change in mortality over a given time period instead of following the cohort along different ages.

Assume the mortality profiles are such that the behavior over time of the logarithms of q_x 's is approximately linear. We can find the value δ_x such that for $h=1, 2, \dots, n-1$, we have:

$$\ln q_x(t_{h+1}) - \ln q_x(t_h) \approx \delta_x (t_{h+1} - t_h) \quad (3.12)$$

Hence

$$\frac{q_x(t_{h+1})}{q_x(t_h)} \approx e^{-\delta_x(t_{h+1}-t_h)} \quad (3.13)$$

Alternatively $r_x = e^{-\delta_x}$ can be defined leading to:

$$\frac{q_x(t_{h+1})}{q_x(t_h)} \approx r_x^{t_{h+1}-t_h} \quad (3.14)$$

Assume that for each age x the parameter δ_x or r_x can be estimated through least squares procedure. The graduated probabilities $\hat{q}_x(t)$ can be calculated. The constraint $\hat{q}_x(t_n) = \hat{q}_x(t_n)$ can be introduced.

Relation (3.13) suggests a natural extrapolation formula. Set $t' = t_n$ and assume for $t' < t$:

$$q_x(t) = q_x(t') r_x^{t-t'} \quad (3.15)$$

The formula above originates from analysis of mortality profiles and hence constitutes an example of the horizontal approach. The horizontal approach is where extrapolations are performed independently for each q_x .

The reduction factor can be expressed as:

$$R_x(t - t') = r_x^{t-t'} = e^{-\delta_x(t-t')} \quad (3.16)$$

The exponential formula can be generalized. From above it was seen that $r_x < 1$, then

$$q_x(\infty) = 0 \quad (3.17)$$

Where

$$q_x(\infty) = \lim_{t \rightarrow \infty} q_x(t) \quad (3.18)$$

It may be more realistic to assign a positive limit to the mortality at any age x .

$$q_x(t) = q_x(t')[\alpha_x + (1 - \alpha_x)r_x^{t-t'}] \quad (3.19)$$

The reduction factor is therefore

$$R_x(t - t') = [\alpha_x + (1 - \alpha_x)r_x^{t-t'}] \quad (3.20)$$

From (3.10) we have:

$$q_x(\infty) = \alpha_x q_x(t') \quad (3.21)$$

(3.10) can be simplified by assuming that $r_x = r$ for all x , which we obtain

$$R_x(t - t') = [\alpha_x + (1 - \alpha_x)r^{t-t'}] \quad (3.22)$$

This is a valid assumption despite the fact that mortality decline is not necessarily uniform across a wide range of ages. The assumption is valid when there is a limited set of ages involved in the mortality forecast, like in the case of this study where the projections for annuitant mortality is needed.

The exponential formulae can be placed in a more general context. The annual probability of death below can be the annual probability of death:

$$q_x(t) = a_x + b_x c_x^t \quad (3.23)$$

This is the general exponential formula for the projections via extrapolation. Which parameters a_x, b_x, c_x depend on age x and are independent of the calendar year t . Thus, $q_x(t)$ is an exponential function of t .

3.7 Calculating Change in the Cost of Annuities

The second part of the study will be to determine the cost of annuities when the mortality improvements are incorporated and there after the increase due to mortality improvements will be determined. Annuity factors will be computed based on a constant rate of interest and the new mortality forecasts and an annuity of ksh.1 are priced. To determine the effect of mortality improvements, the price of an annuity sold at age 55 will be determined for all age 55 over the forecast period and the new cost payable by a person turning age 55 at a given time t will indicate the change in the annuity prices.

An alternative method would be to determine how the price of an annuity changes for a given cohort by determining the present value of the future benefits for a person that bought an annuity at age 55 every year they survive.

Due to the limiting nature of the data, term annuities were computed instead of whole life annuities.

3.7.1 Using Simulation to Determine the Difference in Cost of Annuities

The use of simulation methods as a way for representing actuarial problems has increased over the years, due to the technological improvements and the affordability of the hardware and software required (Khalaf-Allah et.al ., 2006), (Currie, 2004) . The difference in the cost of annuities using a stochastic simulation method can be determined, as we are able to model a particular path that a group of persons can follow in their lifetime. In this model the time of death of an annuitant is assumed to be a random variable and valuations similar to those in a deterministic model are performed. For this study simulation method of forecasting will be an appropriate method because, the age at which

an annuitant dies is a stochastic random variable and the mean of the distribution of simulated present values can be compared. Simulation will be useful because the annuities being modeled are not for an individual but for a group of policyholders and therefore simulation would be a simplistic way of getting the present values.

Khalaf-Allah et al. (2006) suggested a model where a single life annuity with payments of ksh.1 payable in arrears a constant interest rate is used for the computation. The annuity cost is calculated to be the average present value of payments made to the policyholders, both male and female. A random number is generated from a uniform (0, 1) distribution for each annuitant for each year. If the q-type probability of the policyholder is smaller than this number, we consider that the policyholder survives and the relevant survival payment for that year is recorded. If the policyholder survives for that year another random number is generated and if again this is bigger than the q-type probability for that interval it is assumed that the policyholder survives again and the relevant payment is recorded. This continues until the policyholder dies. All the payments made to this policyholder during his/her lifetime are then recorded and the present value of the annuity payments received at policy inception, for each annuitant is calculated. The same is recorded for all annuitants.

An assumption is made that all policy holders are of the same age and the simulations can be assumed to apply to one annuitant rather than having many simulations for each annuitant. The number of simulations in each case is determined in such that the results obtained are representative, as the distribution of the results is found to be sensitive to the number of simulations performed.

3.8 Summary

The study determines the effect of mortality improvements on the cost of liabilities of Kenyan annuity providers. It does this by projecting the Kenyan annuitant mortality by making an assumption that past trends in mortality changes will continue into the future. The changes in mortality are then incorporated into the mortality tables used for pricing annuities. An annuity is then priced at different points in the future for both male and female annuitants using the projected mortality to determine how the price of the annuity changes for a specific age with the incorporation of mortality changes. Thereby, answering the research questions.

CHAPTER FOUR

RESULTS AND ANALYSIS

4.1 Preliminary Analysis

The data from WHO that represented the mortality for the whole population was in an abridged life table format. This therefore meant the annuitant mortality rates table had to be minimized to an abridged life table format. This limited the number of age ranges used in the analysis. For the analysis, R software and excel were the software used to come up with the results.

4.1.1 Mortality Trend for the Whole Population

Table 4.1 below shows how the mortality for the whole population has been from the years 1990-2013 for both male and female.

Ages	Female				Male			
	1990	2000	2012	2013	1990	2000	2012	2013
<1	0.0589	0.0630	0.0444	0.0432	0.0681	0.0736	0.0529	0.0516
0 -4	0.0361	0.0440	0.0247	0.0236	0.0379	0.0460	0.0262	0.0251
5 -9	0.0171	0.0193	0.0155	0.0145	0.0201	0.0222	0.0191	0.0183
10 -14	0.0097	0.0102	0.0123	0.0115	0.0108	0.0112	0.0137	0.0131
15 -19	0.0122	0.0131	0.0139	0.0134	0.0147	0.0148	0.0171	0.0167
20 -24	0.0168	0.0422	0.0178	0.0169	0.0227	0.0321	0.0217	0.0213
25 -29	0.0196	0.0823	0.0258	0.0238	0.0253	0.0601	0.0266	0.0254
30 -34	0.0212	0.0765	0.0317	0.0285	0.0273	0.0778	0.0325	0.0304
35 -39	0.0235	0.0720	0.0389	0.0356	0.0301	0.0894	0.0403	0.0375
40 -44	0.0283	0.0552	0.0365	0.0351	0.0350	0.0767	0.0435	0.0415
45 -49	0.0336	0.0663	0.0376	0.0365	0.0414	0.0711	0.0473	0.0463
50 -54	0.0430	0.0596	0.0423	0.0415	0.0575	0.0654	0.0565	0.0559
55 -59	0.0566	0.0583	0.0526	0.0517	0.0765	0.1015	0.0724	0.0714
60 -64	0.0849	0.0846	0.0780	0.0767	0.1076	0.1195	0.1021	0.1012
65 -69	0.1334	0.1330	0.1228	0.1210	0.1599	0.1598	0.1509	0.1497
70 -74	0.2108	0.2089	0.1984	0.1957	0.2414	0.2393	0.2309	0.2289
75 -79	0.3224	0.3188	0.3071	0.3042	0.3603	0.3566	0.3471	0.3454
80 -84	0.5017	0.4979	0.4891	0.4872	0.5461	0.5370	0.5270	0.5256
85 -89	0.6632	0.6610	0.6566	0.6549	0.7143	0.7059	0.6987	0.6975
90 -94	0.8009	0.8004	0.8016	0.8005	0.8501	0.8440	0.8416	0.8409
95 -99	0.8979	0.8989	0.9041	0.9035	0.9345	0.9307	0.9329	0.9327
>100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.1 Table showing probabilities of death for the whole Kenyan population derived from WHO website. From this table it can be seen that probabilities of death between the year 1990 and 2000 increased for all ages. However from 2000 to 2013 for both male and female the probabilities of death have been reducing for all age ranges.

From the analysis of the mortality of the whole population, it is observed that between 1990 and 2000 there was an increase in the mortality for all age ranges except the ages between 70 and 89. From 2000 to 2013 mortality has been decreasing for all ages. This shows that people are dying at a lower rate in recent years. This then brings us to the issue of longevity when applied to the annuitants who are of better health.

From table 4.1 above, the mortality of the population, both male and female aged between 55 and 79 is plotted in figure 4.1 below on a logarithmic scale.

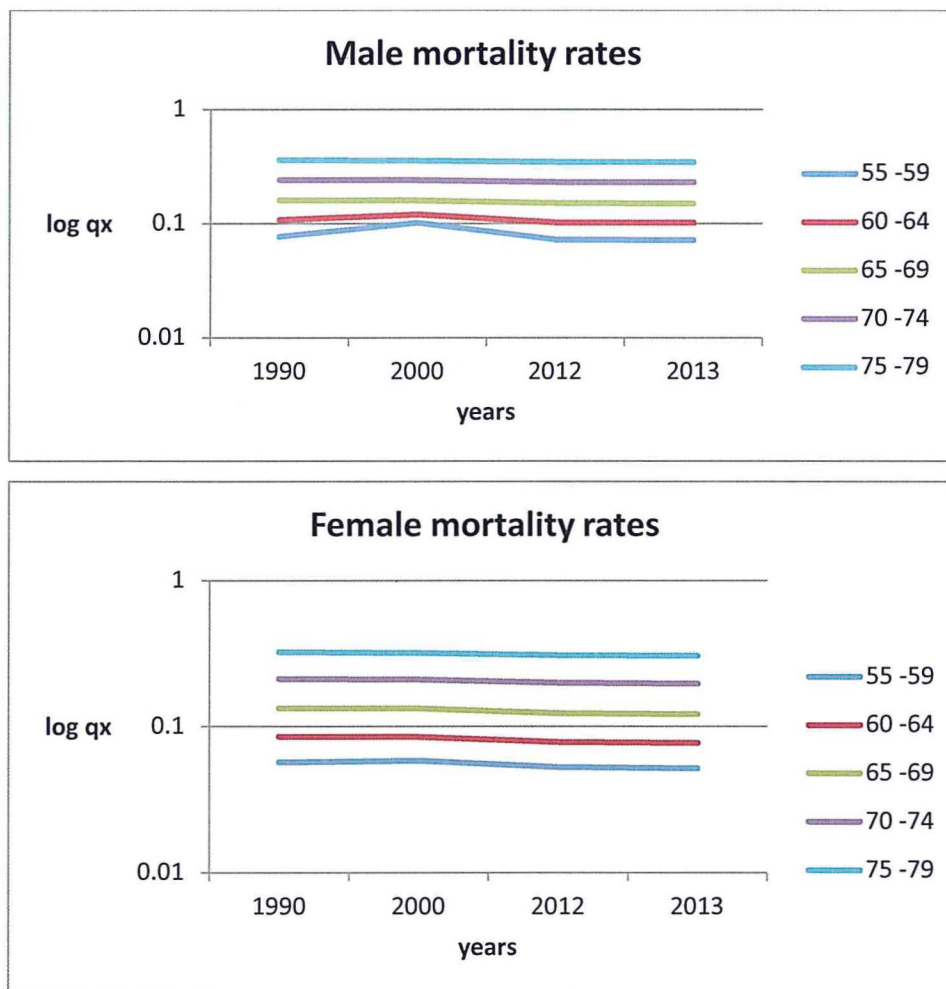


Figure 4.1 -graphs showing past trend of probabilities of death for both male and female members of the whole population aged between 55 and 79, plotted on a logarithmic scale. The upward slopes of the lines on the graphs show an increase in probabilities of death while a downward sloping graph shows a decrease in the probabilities of death.

From the graphs in figure 4.1 above there has been a decline in the probabilities of death from 2000-2013. Despite the increase that is observed between 1990 and 2000. For both male and female members of the population. This decreasing trend is assumed to continue into the future and is the basis for projecting mortality in later sections of this paper.

4.2 Extrapolating Mortality.

From the WHO data, the annual reduction factors and 12 year reduction factors were obtained using the exponential extrapolation formula (3.14). These were computed using the R software and was done for both male and female over all years.

Due to the limiting nature of the data after converting it into an abridged life table format, the only age ranges over which mortality was extrapolated were (55-59, 60-64 and 65-69).

4.2.1 Reduction Factors

Both the yearly and a 12 year reduction factors were computed for comparison purposes. The reduction factors are generally less than one showing that mortality has been reducing over the years across all age ranges. In addition to this the reduction factors over the years has been decreasing for all ages as seen in the graphs in figure 4.2 below. This shows that over the years there is a greater decline in the probabilities of death for the given age ranges.

The results obtained in figure 4.2 show that the older the individual the greater the reduction factor, or the closer the reduction factor is to one. Meaning for older ages relative to younger ages the rate at which mortality changes is lower. The reduction factors also decrease over the years indicating mortality is set to improve even more over the years.

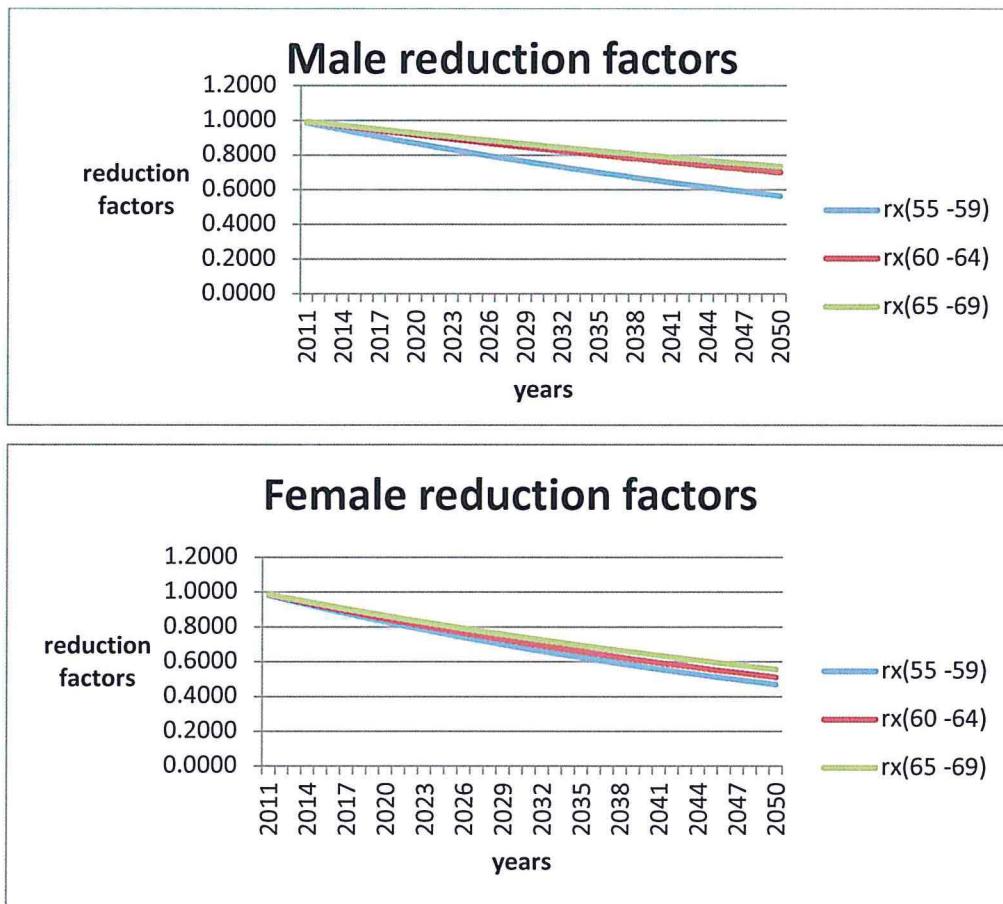


Figure 4.2 Graphs showing trends in annual reduction factors for the period 2011-2050. For both male and female. The downward slope of the lines for the different age ranges, indicate that the reduction factors fall over the 40 year period. From the line graphs, it can be seen that the reduction factors increase with age as the highest green line belongs to the greatest of the age ranges. The steepness of the slopes shows the rate at which reduction factors fall. The age range 55-59, has a steeper slope indicating a greater fall in reduction factors.

4.2.2 Extrapolated Mortality Rates

Using the one year reduction factors and the formula in (3.15). Mortality for the specified age ranges was extrapolated for the period 2011-2050. The results were as seen in the graphs in figure 4.3 below. The graphs were plotted on a log scale so as to magnify the intensity of the mortality decline because for the graphs in figure 4.2 above it is difficult to tell how significant the change in mortality from one year to another is. The other

reason why the log is used is to undo the exponential function that was applied to forecast mortality over the years.

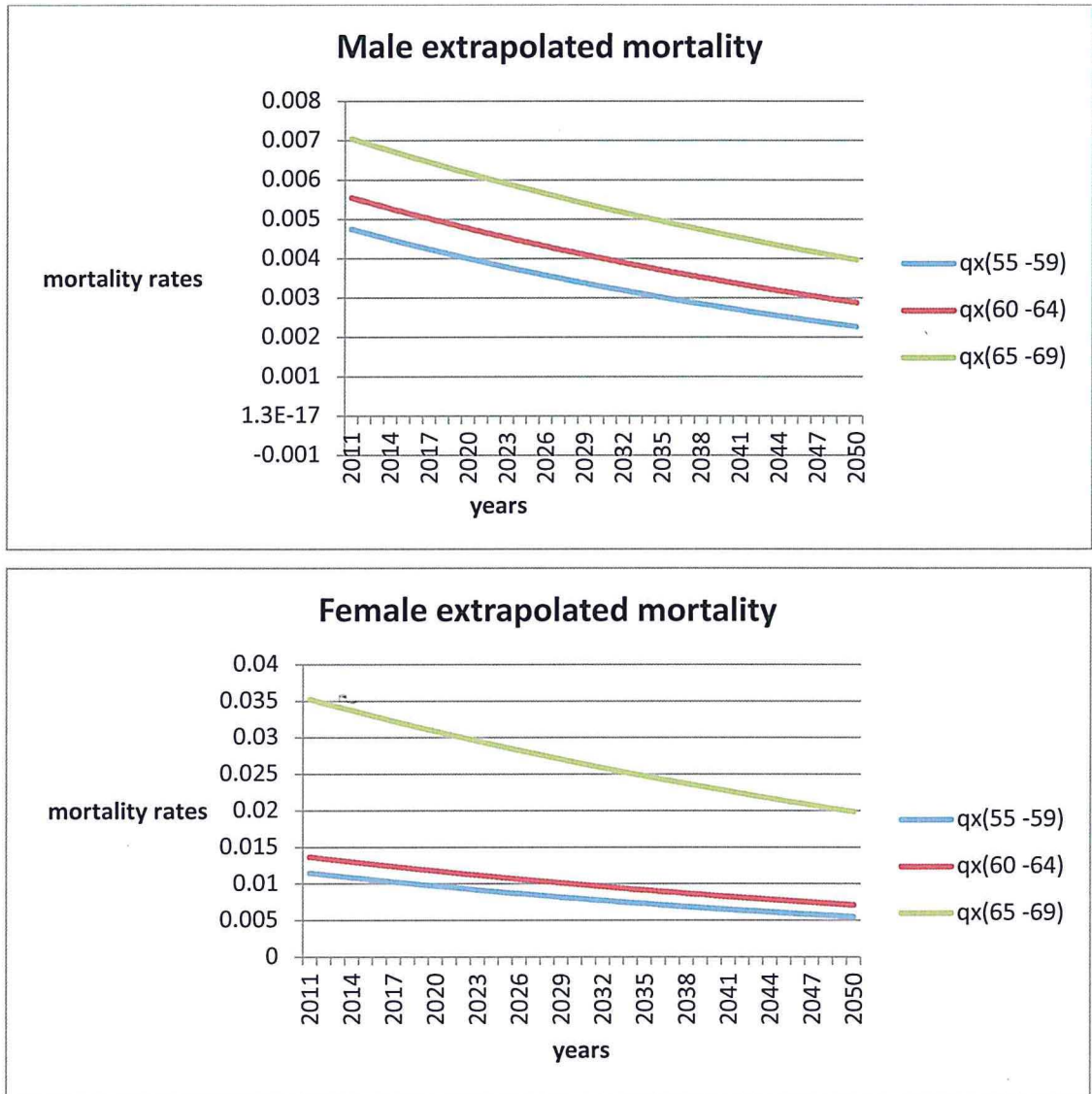


Figure 4.3- Graphs of extrapolated mortality. The downward sloping graphs show that mortality is expected to reduce over the years. The steeper the slope the greater the decrease in mortality over the years. From the graphs the higher the graph is on the y axis, the higher the probability of death for that age range relative to the rest.

From figure 4.3 above, the mortality can be seen to be decreasing over the years. In addition to this, male mortality can be seen to decrease more than that of females as shown by the greater slope in the graph.

Mortality rates are generally higher for older ages relative to younger ones. The graphs in figure 4.4 below further magnify the intensity of the mortality decline.

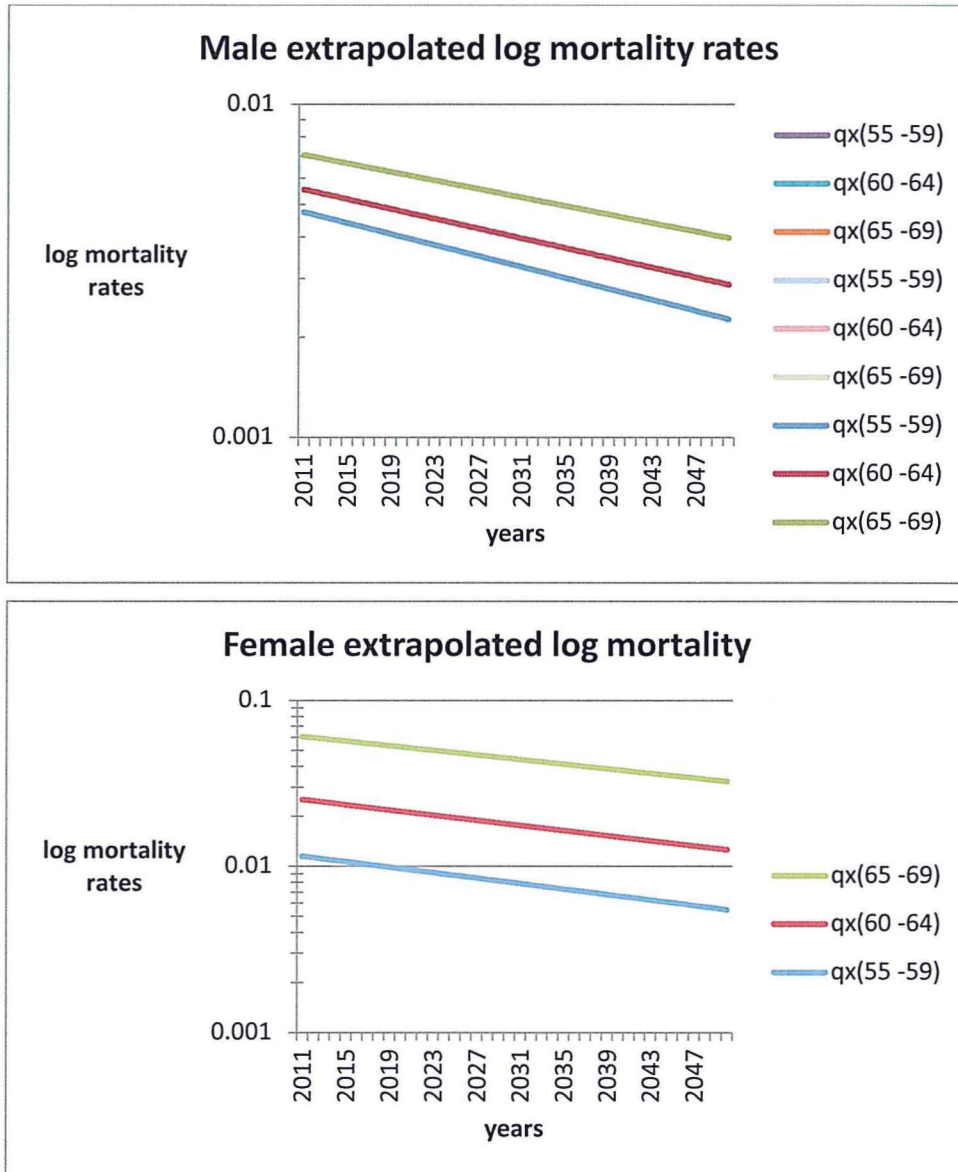


Figure 4.4 Graphs of extrapolated mortality drawn on a logarithmic scale. The downward slope of the line graphs shows a decrease in the probabilities of death for the whole population. The steepness of the slopes indicates a greater decrease in the log probabilities of death.

The log mortalities show the same trend as the initial extrapolated mortalities. The probabilities of death for both male and female annuitants decrease over the years for all ages.

The log mortalities were plotted in order to magnify the intensity of the reduction in death probabilities.

Male mortalities can be seen to decrease to a greater extent compared to female mortalities from the steep nature of the graph.

Probabilities of death were also extrapolated based on 12 year reduction factors to illustrate the 12 year effects. The intention was to use 10 year periods but the data used was limiting and 12 year periods were the closest that could be used.

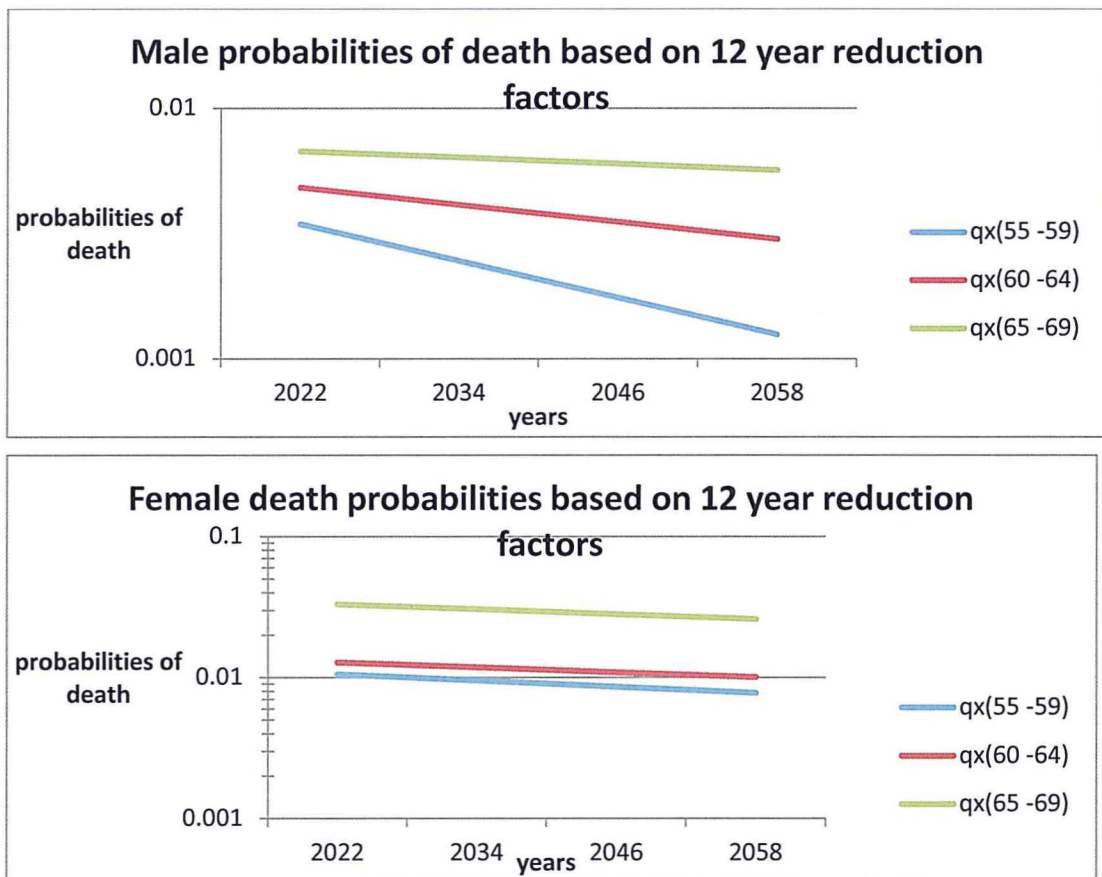


Figure 4.5 Graphs of extrapolated mortality based on 12 year reduction factors plotted on a logarithmic scale. The slope of the graphs show a decrease in the probabilities of death and the greater the slope the greater the decrease in death probabilities

4.3 Computing Change in the Annuity Prices.

An assumption was made that the annuity being priced is of ksh.1 and the interest rate used is 6% similar to that used in actuarial tables. Due to the limiting nature of the data, it was difficult to compute the present values of whole life annuities because there were no death rates for all age ranges. A 5 year term annuity was calculated instead to show the relationship between mortality changes and the prices of annuities. From the computation it was observed that over the years, the price of a 5year term annuity increases. This means that as the probability of death decreases over the years, the cost of purchasing a term annuity goes up because the probability of a policyholder dying during the policy term is lower. This means the annuity provider will pay out the benefits for the term period. This is illustrated by the tables in appendix I and II respectively and the graphs in figure 4.6 below.

The cost of term annuities is generally higher for males than females because females generally have a lower mortality than their male counterparts according to this analysis. It is therefore only fair that a male annuitant should be charged more for the same annuity, since their likelihood of dying before the end of the policy term is higher.

The term annuity prices increase over the years. A percentage of the increase in annuity prices over one year periods for all years is shown in the table 4.3 below indicates that over time annuity prices increase at a greater percentage indicating that mortality improvements are greater over the years.

From the percentage change in annuity prices per annum, the annuities prices increase at a decreasing rate. Annuity prices for female annuitant increases at a greater rate than that of male annuitants.

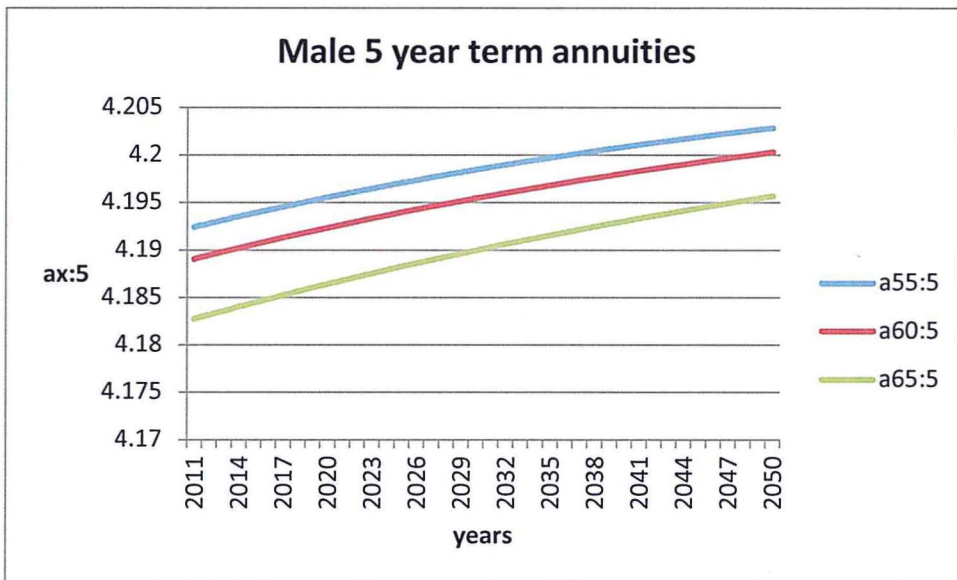
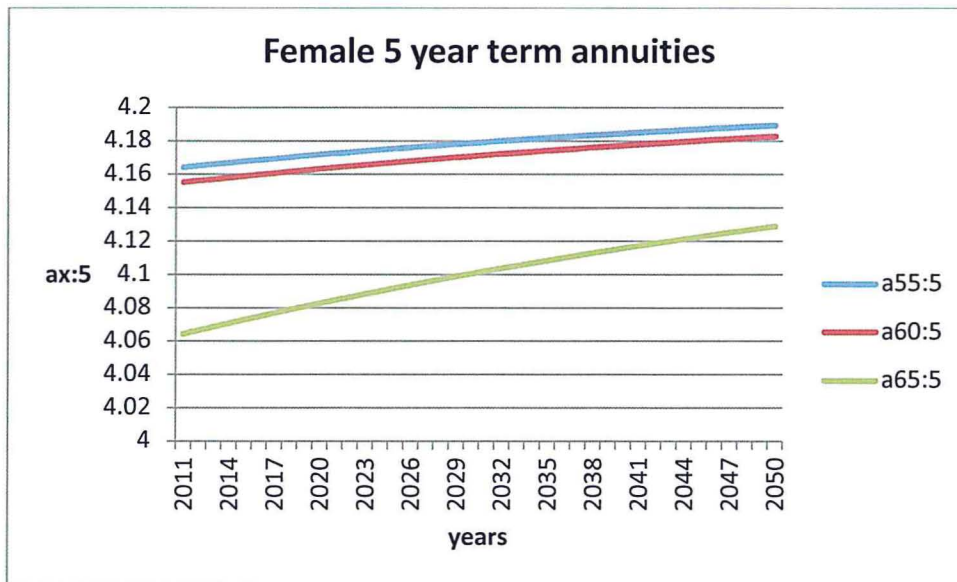


Figure 4.6 Graphs showing trend in prices of five year term annuities. The upward sloping curves show an increase in the term annuity prices over the years. The steeper the curve the greater the increase in the price of an annuity.

The graphs above show an increase in term annuity prices over the years. A comparison of the two graphs shows that male annuities are generally more expensive than female ones. Female annuities however increase at a greater rate than male annuities.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

From the analysis and results obtained, it is evident that the mortality rates for the Kenyan population have been decreasing over the years. And this trend is expected to continue into the future. Consideration of the changes in mortality rates over the years is important because it affects the cost of annuities and if not considered, may lead to significant losses to the providers of annuity benefits who are mostly insurance companies and defined benefit pension schemes.

5.2 Summary

The study sought to determine how Kenyan mortality has been changing over the years and how this affects the cost of an annuity. The main purpose of doing this is to determine the impact of longevity risk on the liabilities of annuity providers. From the literature in chapter two, longevity risk is a significant risk for annuity providers and should therefore be considered. The focus was on the mortality of annuitants but the projections were made based on the trend of mortality for the general population. The analysis and results show a decrease in Kenyan mortality over the years. This affects the annuity prices as a decrease in mortality increases the cost of a term annuity.

5.3 Discussion

The annuitant mortality data obtained had a number of limitations that affected the analysis and results of this study. The first limitation was the lack of data for older ages greater than 70 years of age, and those of ages less than 55. This hindered the computation of whole life annuities and instead term annuities were considered. In addition to this, the WHO data was in an abridged life table format. This necessitated the conversion of the annuitant mortality data to an abridged life table format based on WHO standards, this led to a limitation in the number of age ranges to three.

From the data obtained, the Kenyan population as a whole has exhibited a decrease in the mortality rates over the years it can be seen that the mortality at all ages from the year

2000-2013, has been decreasing. This when applied to mortality rates of the annuitants in Kenya produces a similar effect over the forecasted period 2011-2050, for both male and female annuitants.

Contrary to theory, males according to the annuitant mortality data and extrapolated mortality have lower probabilities of death. Most researchers find that female annuitants tend to have lower mortality rates this is attributed to both biological and social structural traits. The result from this analysis also goes against the WHO report that indicated Kenyan women tend to live longer than men. This inconsistency could be an error as a result of the quality of data used.

In the long term however, the mortality of the female population decreases at a faster rate than that of males. This shows that in a few years time, Kenyan mortality will converge to the theoretical concepts that women tend to have lower probabilities of death than men.

The term annuities computed to show how mortality improvements affect the cost of annuities, show that over the years with a decrease in probabilities of death for both men and women, the cost of a term annuity increases. This is because an annuity pays out benefits for as long as an individual is alive. If mortality improves, it means people will live for longer and the annuity provider will be required to pay out the annuity for the term period.

It can therefore be inferred that this will also be the case if a whole life annuity is to be priced as the benefits will be paid for longer if one survives beyond the expected ages.

Due to the fact that from the analysis men have a lower mortality than women, the annuity price for the male population is greater than that of females. The annuity prices however, increase at a decreasing rate. This is because the rate at which mortality rates reduce is expected to decrease over time.

Annuity pricing should therefore be done using life tables that incorporate the changes in mortality over the years so as to enable the annuity provider charge an adequate amount for a given level of benefit.

5.4 Recommendations

From the results and analysis and the relation to theory, a number of things should be taken into account by the providers of annuities. The first and major thing that should be considered is mortality improvements need to be included in the life tables used for pricing. This is because if they are not included, then the provider is exposed to the risk of longevity as lower premiums will be charged for an annuity that will be paid for longer.

In addition to this, proper records on the mortalities need to be kept so as to make mortality forecasting accurate and hence more efficient for the Kenyan scenario. A number of data problems were encountered in the course of this study that made the results obtained not as accurate as they should be.

Mortality improvements should however be monitored overtime. This is because there may be an occurrence in the future that could lead to mortality fluctuations relative to the forecast method used.

5.5 Conclusion

From this study, we come to a conclusion that the mortality of the Kenyan population just like that of many other nations is decreasing, a trend that is expected to continue into the future. The decrease in mortality rate has an inverse relationship to the prices of annuities. When mortality rates are low, the annuity factors are high meaning the price of purchasing an annuity increases. With this therefore annuity providers in the country who currently used the Kenyan life tables and the UK annuitant mortality tables, need to consider the changing mortality rates in their pricing so as to enable them charge the right premiums for each benefit provided.

With the consideration of possible future mortality improvements, annuity providers will be able to prevent losses as a result of longevity risk and the risk of insolvency will be reduced as a result. This will enable them to earn greater profits and will make them sustainable at the same time.

The government through the respective regulatory bodies should oversee the introduction and implementation of the incorporation of mortality improvements to ensure stability of the industry and hence economic growth.

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APPENDICES

Appendix I: Table showing the 5 year term annuity prices for the years 2011-2050.

	MALE			MALE			FEMALE			FEMALE		
	qx(55-59)	qx(60-64)	qx(65-69)	a55x:5	a60:5	a65:5	qx(55-59)	qx(60-64)	qx(65-69)	a55x:5	a60:5	a65:5
2011	0.00474	0.00554	0.00704	4.19239	4.18902	4.18271	0.01145	0.01364	0.03519	4.16412	4.15490	4.06412
2012	0.00465	0.00545	0.00694	4.19276	4.18941	4.18315	0.01124	0.01341	0.03468	4.16502	4.15586	4.06629
2013	0.00457	0.00536	0.00683	4.19313	4.18980	4.18357	0.01103	0.01319	0.03417	4.16591	4.15681	4.06842
2014	0.00448	0.00527	0.00673	4.19349	4.19017	4.18400	0.01082	0.01297	0.03367	4.16678	4.15774	4.07052
2015	0.00440	0.00518	0.00664	4.19385	4.19055	4.18441	0.01062	0.01275	0.03318	4.16764	4.15865	4.07259
2016	0.00431	0.00509	0.00654	4.19420	4.19091	4.18482	0.01042	0.01254	0.03270	4.16848	4.15955	4.07463
2017	0.00423	0.00501	0.00644	4.19454	4.19127	4.18522	0.01022	0.01233	0.03222	4.16930	4.16043	4.07665
2018	0.00415	0.00492	0.00635	4.19487	4.19162	4.18562	0.01003	0.01212	0.03175	4.17011	4.16130	4.07863
2019	0.00407	0.00484	0.00626	4.19520	4.19197	4.18601	0.00984	0.01192	0.03128	4.17090	4.16216	4.08058
2020	0.00400	0.00476	0.00617	4.19552	4.19231	4.18639	0.00966	0.01172	0.03083	4.17168	4.16300	4.08251
2021	0.00392	0.00468	0.00608	4.19584	4.19265	4.18677	0.00948	0.01152	0.03038	4.17244	4.16382	4.08440
2022	0.00385	0.00460	0.00599	4.19615	4.19298	4.18715	0.00930	0.01133	0.02993	4.17319	4.16463	4.08627
2023	0.00378	0.00453	0.00590	4.19645	4.19330	4.18751	0.00912	0.01114	0.02950	4.17393	4.16543	4.08811
2024	0.00371	0.00445	0.00581	4.19675	4.19362	4.18788	0.00895	0.01095	0.02907	4.17465	4.16622	4.08993
2025	0.00364	0.00438	0.00573	4.19704	4.19393	4.18823	0.00879	0.01077	0.02864	4.17536	4.16699	4.09172
2026	0.00357	0.00430	0.00564	4.19733	4.19424	4.18859	0.00862	0.01059	0.02822	4.17605	4.16775	4.09348
2027	0.00350	0.00423	0.00556	4.19761	4.19454	4.18893	0.00846	0.01041	0.02781	4.17673	4.16849	4.09522
2028	0.00344	0.00416	0.00548	4.19789	4.19484	4.18928	0.00830	0.01024	0.02740	4.17740	4.16923	4.09693
2029	0.00337	0.00409	0.00540	4.19816	4.19514	4.18961	0.00814	0.01007	0.02700	4.17806	4.16995	4.09861
2030	0.00331	0.00402	0.00532	4.19843	4.19542	4.18995	0.00799	0.00990	0.02661	4.17870	4.17066	4.10028
2031	0.00325	0.00395	0.00524	4.19869	4.19571	4.19027	0.00784	0.00973	0.02622	4.17933	4.17136	4.10191
2032	0.00319	0.00389	0.00517	4.19895	4.19599	4.19060	0.00769	0.00957	0.02584	4.17995	4.17204	4.10353
2033	0.00313	0.00382	0.00509	4.19920	4.19626	4.19091	0.00755	0.00941	0.02546	4.18056	4.17272	4.10512
2034	0.00307	0.00376	0.00502	4.19945	4.19653	4.19123	0.00741	0.00925	0.02509	4.18116	4.17338	4.10668
2035	0.00301	0.00370	0.00494	4.19969	4.19679	4.19154	0.00727	0.00910	0.02472	4.18174	4.17403	4.10823
2036	0.00295	0.00363	0.00487	4.19993	4.19705	4.19184	0.00713	0.00895	0.02436	4.18232	4.17467	4.10975
2037	0.00290	0.00357	0.00480	4.20016	4.19731	4.19214	0.00700	0.00880	0.02400	4.18288	4.17530	4.11125
2038	0.00284	0.00351	0.00473	4.20039	4.19756	4.19244	0.00687	0.00865	0.02365	4.18344	4.17592	4.11272
2039	0.00279	0.00346	0.00466	4.20061	4.19781	4.19273	0.00674	0.00851	0.02331	4.18398	4.17653	4.11418
2040	0.00274	0.00340	0.00459	4.20083	4.19805	4.19301	0.00661	0.00836	0.02297	4.18451	4.17713	4.11561
2041	0.00269	0.00334	0.00453	4.20105	4.19829	4.19330	0.00649	0.00822	0.02263	4.18503	4.17772	4.11703
2042	0.00264	0.00328	0.00446	4.20126	4.19853	4.19357	0.00637	0.00809	0.02230	4.18555	4.17830	4.11842
2043	0.00259	0.00323	0.00440	4.20147	4.19876	4.19385	0.00625	0.00795	0.02198	4.18605	4.17887	4.11979
2044	0.00254	0.00318	0.00433	4.20168	4.19899	4.19412	0.00613	0.00782	0.02166	4.18654	4.17943	4.12114
2045	0.00249	0.00312	0.00427	4.20188	4.19921	4.19439	0.00601	0.00769	0.02134	4.18703	4.17998	4.12248
2046	0.00244	0.00307	0.00421	4.20207	4.19943	4.19465	0.00590	0.00756	0.02103	4.18750	4.18052	4.12379
2047	0.00240	0.00302	0.00414	4.20227	4.19965	4.19491	0.00579	0.00743	0.02072	4.18797	4.18106	4.12508
2048	0.00235	0.00297	0.00408	4.20246	4.19986	4.19516	0.00568	0.00731	0.02042	4.18843	4.18158	4.12636
2049	0.00231	0.00292	0.00402	4.20264	4.20007	4.19541	0.00558	0.00719	0.02012	4.18888	4.18210	4.12761
2050	0.00226	0.00287	0.00397	4.20282	4.20027	4.19566	0.00547	0.00707	0.01983	4.18932	4.18260	4.12885

Table 4.2 Table showing male and female 5 year term annuity prices. From the table the term annuities increase over the years for both male and female annuitants for all ages ranges. The term annuity prices are also greater for male annuitants compared to female annuitants.

Appendix II: Table showing the percentage changes in term annuity prices.

PERCENTAGE CHANGES IN ANNUITY PRICES						
Years	Female			Male		
	a55x:5	a60:5	a65:5	a55x:5	a60:5	a65:5
2011						
2012	0.0533%	0.0231%	0.0533%	0.0089%	0.0093%	0.0104%
2013	0.0525%	0.0227%	0.0525%	0.0088%	0.0092%	0.0102%
2014	0.0517%	0.0224%	0.0517%	0.0086%	0.0090%	0.0101%
2015	0.0509%	0.0220%	0.0509%	0.0084%	0.0089%	0.0099%
2016	0.0501%	0.0216%	0.0501%	0.0083%	0.0087%	0.0098%
2017	0.0494%	0.0212%	0.0494%	0.0081%	0.0086%	0.0096%
2018	0.0486%	0.0209%	0.0486%	0.0080%	0.0084%	0.0095%
2019	0.0479%	0.0205%	0.0479%	0.0078%	0.0083%	0.0093%
2020	0.0472%	0.0202%	0.0472%	0.0077%	0.0081%	0.0092%
2021	0.0465%	0.0198%	0.0465%	0.0075%	0.0080%	0.0091%
2022	0.0458%	0.0195%	0.0458%	0.0074%	0.0079%	0.0089%
2023	0.0451%	0.0192%	0.0451%	0.0073%	0.0077%	0.0088%
2024	0.0444%	0.0188%	0.0444%	0.0071%	0.0076%	0.0087%
2025	0.0437%	0.0185%	0.0437%	0.0070%	0.0075%	0.0085%
2026	0.0431%	0.0182%	0.0431%	0.0069%	0.0074%	0.0084%
2027	0.0424%	0.0179%	0.0424%	0.0067%	0.0072%	0.0083%
2028	0.0418%	0.0176%	0.0418%	0.0066%	0.0071%	0.0082%
2029	0.0412%	0.0173%	0.0412%	0.0065%	0.0070%	0.0080%
2030	0.0405%	0.0170%	0.0405%	0.0063%	0.0069%	0.0079%
2031	0.0399%	0.0167%	0.0399%	0.0062%	0.0068%	0.0078%
2032	0.0393%	0.0164%	0.0393%	0.0061%	0.0066%	0.0077%
2033	0.0387%	0.0162%	0.0387%	0.0060%	0.0065%	0.0076%
2034	0.0382%	0.0159%	0.0382%	0.0059%	0.0064%	0.0075%
2035	0.0376%	0.0156%	0.0376%	0.0058%	0.0063%	0.0074%
2036	0.0370%	0.0154%	0.0370%	0.0057%	0.0062%	0.0073%
2037	0.0365%	0.0151%	0.0365%	0.0056%	0.0061%	0.0072%
2038	0.0359%	0.0148%	0.0359%	0.0055%	0.0060%	0.0070%
2039	0.0354%	0.0146%	0.0354%	0.0054%	0.0059%	0.0069%
2040	0.0349%	0.0143%	0.0349%	0.0053%	0.0058%	0.0068%
2041	0.0343%	0.0141%	0.0343%	0.0052%	0.0057%	0.0067%
2042	0.0338%	0.0139%	0.0338%	0.0051%	0.0056%	0.0066%
2043	0.0333%	0.0136%	0.0333%	0.0050%	0.0055%	0.0065%
2044	0.0328%	0.0134%	0.0328%	0.0049%	0.0054%	0.0064%
2045	0.0323%	0.0132%	0.0323%	0.0048%	0.0053%	0.0064%
2046	0.0318%	0.0130%	0.0318%	0.0047%	0.0052%	0.0063%
2047	0.0314%	0.0127%	0.0314%	0.0046%	0.0052%	0.0062%
2048	0.0309%	0.0125%	0.0309%	0.0045%	0.0051%	0.0061%
2049	0.0304%	0.0123%	0.0304%	0.0044%	0.0050%	0.0060%
2050	0.0300%	0.0121%	0.0300%	0.0043%	0.0049%	0.0059%

Table 4.3 - Table of annual percentage changes in term annuity prices. From the table female annuity prices decrease at a greater percentage than male annuity prices over the years. The annuity prices increase at a decreasing rate.