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Modelling temperature derivatives using Lévy processes

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060078

Submitted in partial fulfillment of the requirements for the Degree of Bachelor of Business Science in Finance at Strathmore University

School of Finance and Applied Economics

Strathmore University

Nairobi, Kenya

December, 2015

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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This Research Proposal has been submitted for examination with my approval as the Supervisor.

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ABBREVIATIONS

HDD	-	Heating Degree Day
GDP	-	Gross Domestic Products
CDD	-	Cooling Degree Day
CAT	-	Cumulative Average Temperature
GBP	-	Great Britain Pound
CME	-	Chicago Mercantile Exchange
DAT	-	Daily Average Temperature
AR	-	Autoregressive
DFT	-	Discrete Fourier Transform

1 Introduction

1.1 Background to the study

Weather derivatives are a new risk management tool which can be widely used in the financial market to avoid the impact of bad weather effects and control the weather risks (Wang et al, 2015). Weather derivatives are different from traditional financial derivatives as their underlying asset such as temperature, humidity, and precipitation, which cannot be traded in the market, so ordinary pricing models such as Black and Scholes formula is not applicable in pricing weather derivatives. The underlying commodity results into one considering the incomplete markets theory when modelling the temperature data.

Weather affects various businesses from the agricultural sector to energy sector. In the agricultural sector, plant growth can be affected by lack of rainfall or temperature among many other factors. Until recently, insurance has been the main tool used by companies' for protection against unexpected weather conditions. But insurance only offers protection against catastrophic or disastrous damage. It does nothing to protect against the reduced demand that businesses experience as a result of weather that is warmer or colder than expected.

The first weather derivatives contract was transacted between Enron and Florida Power and Light in 1996 (Helyette, 2005). In Europe, one of the most famous deals was aimed at protecting the London-based chain of wine bars against bad weather in the year 2001 (Helyette, 2005). As per the contract, the firm would receive 15,000 GBP per day on Thursday and Friday between June and September whenever there was a temperature of 24 degrees Celsius and lower. The maximum payoff was 100,000 GBP.

The use of weather derivatives has been on the rise in security exchanges such as the Chicago Mercantile Exchange (CME). This is because of deregulation of the energy markets which have been observed to have high correlations with changes in weather (Alaton, Djehiche, & Stillberger, 2012). Farmers, energy industries and other parties affected by adverse changes in weather have therefore resorted to hedging against risks due to bad weather by using weather related derivatives.

In Kenya, where agriculture accounts for approximately 23% of Gross Domestic Product (GDP), weather derivatives would therefore be crucial instruments to shield farmers from adverse climatic changes which in turn affect the yield of their crop.

Some of the recent disasters caused by weather changes have impacted areas such as Nepal, which is currently experiencing food insecurity issues given the catastrophic effects of the disastrous quakes followed by oncoming monsoon rains and floods, threatening their livelihood as farmers (Donati, 2015). The Food and Agriculture Organization has estimated that USD 23.4 million will be needed should the weather patterns persist. In Indiana, US, the increase in rainfall has resulted into a crop loss estimated at USD 300 million (Lee, 2015), however, weather changes do have a positive impact in other places. For example, in Melbourne Australia, the Gale-Force winds have caused a rapid drop in temperatures, heavier rainfall and increase of snow which has benefited the ski resorts in Australia (Tan, 2015) as they are able to generate more revenues due to availability of essential resources which in this case is the snow. .

1.2 Statement of the Problem

The list of traded weather derivative contracts is extensive and constantly evolving. For instance, in the Chicago Mercantile Exchange (CME) there are traded weather contracts based on an index of Cumulative Average Temperature (CAT) for European cities for May to September. A CAT index is defined as the sum of the daily average temperatures over the period of the contract. This is however not the case for Kenya because firstly, there is no index to that can be used as a basis for the payoffs and the two main seasons, rainy and dry, have exhibited sporadic and unpredictable behavior in recent years. This unpredictability of the temperature is affecting large-scale investors, especially in Agro-commodity market, with unexpected outputs over the years which calls for an instrument that can be used to hedge against these risks.

This study models the temperature data using Lévy driven continuous-time autoregressive model for the temperature dynamics with seasonal volatility in order to generalize the Brownian motion as the driving noise process to Lévy processes (Huang, 2014). It then proceeds to design a temperature derivative using the various pricing techniques.

1.3 Research objectives

1. To develop the modelling framework for temperature derivatives in the Kenyan market using Levy processes.

1.4 Research Questions

1. What is the suitable modelling framework for Kenyan temperature data?
2. What suitable derivative instrument can be designed given the respective temperature dynamics?

1.5 Justification of the Study

The goal of this research is to develop a modelling framework that can be used in Kenya by analyzing the past temperature data recorded and using it to hedge against weather related losses. A suitable model is necessitated in order to ensure that a comprehensive temperature derivative is designed that reflects the prevalent weather patterns. Levy processes are applied in modelling temperatures with seasonal volatility whereby temperatures fluctuate according to the annual seasons observed in Kenya.

Previous studies (Fred & Jūratė, 2005) have examined weather derivatives while deseasonalizing the temperature data while taking account of the four main seasons, summer, spring, autumn and winter then modelling it using Levy processes in order to form temperature derivatives. The process in this research deseasonalizes the Kenyan temperature data and models using Levy processes while considering for the dry and wet seasonal patterns that are specific to tropical areas like Kenya.

2 Literature review

Temperature is a variable in weather that often adversely affects large scale farmers in their activities due to its volatility and unpredictability. As a result, there should be a way for them to be protected against such risks using financial instruments whereby adverse movements in temperature may not lead to complete losses due to receipt of payments due them. This research uses Lévy processes to model temperature based on the data obtained and as a result develop temperature derivatives that fit this modeled data set.

The following section analyzes some of the existing risk mitigation methods for parties adversely affected by weather in Kenya and thereby recommends suitable alternatives that have been used in various locations and markets and their adaptability in the Kenyan market.

2.1 Insurance versus index contracts

Index contracts are more advantageous than insurance contracts firstly because their payoffs are based on widely available and objectively measured index hence no need for farm-loss management (Dmitry & Barry, 2004). This greatly reduces the transaction costs compared to unindexed contracts. Another advantage is that there is either no or reduced adverse selection and moral hazard effects because the value of the index doesn't depend on actions of the individual market participants (Dmitry & Barry, 2004). This is unlike insurance contracts which experience both these problems on a wide scale resulting into many losses.

A major disadvantage of index contracts is basis risk. This is because the specific weather variables are measured at specific places and may vary in other locations. This creates a challenge in designing derivatives for a certain group of investors who are geographically separate from the data collection centers set up by meteorological departments.

2.2 Weather derivatives

Various industries are affected by weather which as a result affects their revenues. The weather conditions may be catastrophic or non-catastrophic. It is therefore important that necessary steps are taken to ensure that such effects are mitigated. Financial markets offer two such alternatives, insurance and weather derivatives. The insurance sector allows firms to invest using premiums and receive a payoff in case of catastrophic effects. However, the insurance firms cannot fully determine whether or not the insured will fulfill all necessary obligations such that losses result purely from weather conditions and not acts of negligence on the part of the insured.

In 1999, the Chicago Mercantile Exchange listed weather derivatives, future contracts for heating and cooling degree-days (Calum, 2001). The advantage of such contracts over insurance contracts is that the effects of moral hazard are eliminated because the behavior of the producer is not correlated to the behavior of the weather patterns. A weather derivative would be purchased or sold to hedge against the effects of a specific event[1]. The weather derivative instrument pays the holder an amount contingent upon a specified function of an observable weather index (Linda, Mulong, & Chuanhou, 2007). Weather derivatives differ from other financial derivatives because there is no negotiable underlying asset forming the basis for the contract hence traditional arbitrage methods become difficult to use (Linda, Mulong, & Chuanhou, 2007).

Weather derivatives are used to hedge against demand or quantity risk rather than to hedge price risk (Helyette, 2005). This is because of the difficulty in determining the fair price in order for one to hedge effectively. Some scholars have incorporated techniques of hedging such as delta and gamma hedging but that is not possible given that the underlying is an untraded security and therefore cannot be hedged using normal option hedging techniques. The current research therefore does not include hedging strategies.

Weather derivatives market is viewed as an incomplete market as it is characterized by some if not all of the following: inadequate number of primitive securities, impossibility of continuously trading the underlying, presence of stochastic volatility, or a combination of various sources (Helyette, 2005). In order to define the weather risks in monetary terms, Juliusz (2009) proposes weather conditions require monitoring of two parameters, type of weather exposure indicated using weather indices and the effect of the weather exposure indicated by the tick value or number of standardized weather contracts.

2.3 Weather related risks

There are two main categories of weather related risks, which are catastrophic and non-catastrophic (Juliusz, 2009). Catastrophic weather risk refers to the risk as a result of extreme

[1] Specific event risk refers to those events, which when they occur have a predetermined outcome.

weather conditions such as hail, hurricanes, floods, droughts among others. Non-catastrophic refers to the financial exposure that a business may endure after weather events such as heat, rain, snow, cold or wind.

Non-catastrophic weather risk occurs more often and affects weather sensitive companies. Such companies are in the field of agriculture, brewing, construction, municipal services and in some instances energy related companies. For example, during mild winters, there is less demand to heat up households, and as a result energy companies experience a drop in their revenues (Steven, Barry, & Keith, 2001).

2.4 Modelling temperature

When modelling temperature, the objective is to derive the dynamics of the indices and price the relevant temperature derivatives under some theory of asset pricing. The continuous time model is recommended because it is capable of describing irregularly spaced data and high frequency data (Huang, 2014). Discrete time models may not be able to capture some of this data. Another approach that could be looked at rather than modelling the temperature is modelling the dynamics of the underlying index. This approach is known as the index modelling approach (Huang, 2014). The approach used in this research is continuous time modelling. Lévy processes become more suitable to model dynamics whereby the data set fails to exhibit normality in its variance.

There are however challenges with pricing temperature derivatives. The underlying being temperature and therefore non-tradable makes it difficult to hedge by creating a self-financing portfolio consisting of the money market account and the underlying asset in order to replicate the payoffs (Helyette, 2005). Therefore, the incomplete markets assumption is made when modelling the temperature data and designing suitable derivatives. The following are some of the models that can be used in modeling temperature.

2.4.1 Dornier and Querel model

Dornier & Querel (2000) describe the following Ornstein-Uhlenbeck dynamics for temperature variations:

$$dT(t) = ds(t) + \kappa\big(T(t) - s(t)\big)dt + \sigma(t)dB_t \tag{2.1}$$

Where $s(t) = A + Bt + Csin(\omega t + \phi)$ describes the mean seasonal volatility and the constant κ is the speed temperature reverts to its mean. This model regresses change in deseasonalised temperature against deseasonalised temperature. The volatility function σ *(t)* is assumed to be constant.

2.4.2 Alaton model

Alaton, Djehiche & Stillberger (2012) suggest a model whereby σ *(t)* is a constant function representing a monthly variation in volatility. The rationale for this was because the data set used over a period of 40 years for the area Bromma had constant variance $\sigma^2(t)$ over each month. This may not necessarily be the case for the Kenyan temperature data. A wiener process is used as the driving noise because the temperature differences are close to normally distributed. However, there is no test for normality. This study incorporates the test for normality when modelling the Kenyan data.

2.4.3 Campbell and Diebold model

Campbell & Diebold (2002) use an autoregressive time series to model temperature variations.

$$T_t = m_t + s_t + \sum_{i=1}^{L} \rho_{t-1} T_{t-1} + \varepsilon_t, \;\; t = 1, 2, \dots \tag{2.2}$$

Where the trend m_t is linear and the seasonality s_t is modeled by a finite number of sines and cosines. This model is different from the Ornstein-Uhlenbeck model as it regresses today's deseasonalised temperature against the temperature observed over the last L days (Fred & Jūratė, 2005). Due to this, it deviates from the Ornstein-Uhlenbeck which is only a first-order autoregression hence unsuitable for this study.

2.4.4 Benth model

Daily temperature data is modeled using an Ornstein-Uhlenbeck process driven by Lévy noise.

$$dT(t) = ds(t) + \kappa(T(t) - s(t))dt + \sigma(t)dL_t \tag{2.3}$$

The only difference with equation 2.1 is the inclusion of the Levy noise $L(t)$. They suggest a Lévy process with marginals following a class of generalized hyperbolic distributions, which is a family of infinitely divisible distributions suitable for modelling skewness and semi heavy tails.

Due to the seasonality of temperature in the USA, they model the annual cycle of temperature using the following simple cosine function.

$$s_t = a_0 + a_1 cos\left(\frac{2\pi}{365}(t - t_0)\right) \tag{2.4}$$

a_0 and a_1 describe average level and amplitude of the mean temperature, and t_o describes the phase angle. The cyclical component is modeled by regressing present day's deseasonalised temperature against the previous day's deseasonalised data.

2.5 Application of Lévy processes in modeling temperature derivatives

Lévy processes are used in the field of science in various ways. In physics, they are used in the study of turbulence, laser cooling and in quantum held theory. In engineering, the processes examine networks, queues and dams. In the field of economics, they examine continuous time-series models while in the actuarial science, for the calculation of insurance and re-insurance risk (Antonis, 2000). They are effective to model assets in financial markets due to the jumps they exhibit.

In the case of modelling mean reverting variables with seasonal mean and volatility, as is the case of temperature, an Ornstein-Uhlenbeck model with residuals generated by a Lévy process rather than Brownian motion is proposed (Fred & Jūratė, 2005). In the case of semi-heavy tails and skewness, a generalized hyperbolic Lévy process[2] are used rather than the general Lévy process. This is because the density and characteristic function is well known making it convenient for derivative pricing.

2.6 Model calibration

Model calibration has been done using different techniques which include the Discrete Fourier Transform, averaging method and the regression method (Zapranis & Alexandridis, 2012). The averaging method computes the average daily temperatures then smoothes them. It is simple but less accurate compared to the others. In DFT, the power spectrum of the variance process is estimated, the peaks are reduced to the level of the background and then the power spectrum is adjusted and inverted back in real time. For the regression model, temperature is regressed on harmonics of 365 days.

[2] Allow an almost perfect fit for financial data (Ernst, 2001)

3 METHODOLOGY

This study first models temperature using continuous time autoregressive processes accounting for seasonal volatility. Lévy processes are considered as the driving noise process rather than Brownian motion because they are able to capture negative skewness and heavy tails in the residuals. It then proceeds to design a suitable temperature derivative based on data obtained from one of the stations as a proxy of a typical temperature

3.1 Research design

The research design used in this study is exploratory. This is because this research aims to observe the behavior of temperature data, which is quantitative in nature, and hence create a suitable modelling framework for temperature derivatives.

3.2 Population and Sampling

The population used in this research is the Kenyan temperature data. The data selected is based on the climatic characteristics of the location from which they are collected, mainly influenced by difference in altitudes in the locations. Out of the 39 meteorological stations in Kenya, this study examines data from 3 stations namely Nakuru, Lamu and Naivasha. The altitudes in these towns are 1901m, 30m, and 6000m respectively. Therefore the sample includes areas with highest, lowest and average altitudes, a key factor that may influence observed temperature levels. The average altitude was arrived at in this study after analyzing the various altitudes of all the 39 weather stations and selecting the stations with the max, median and min altitudes.

3.3 Data collection

3.3.1 Data sources

Daily temperature data was obtained from the Kenya Meteorological Department. Due to lack of an already existing derivatives security market in Kenya and therefore lack of a developed index with which to use when pricing temperature options or futures, simulation is used to generate some of this data, borrowing from the already existing markets and calibrating the data in order to fit the Kenyan market.

3.4 Data Analysis

3.4.1 A Levy based Ornstein-Uhlenbeck Model

This study uses a Levy process with marginals following the class of generalized hyperbolic distributions because these distributions are suitable for modelling skewness and semi-heavy tails.

$$dT(t) = ds(t) + \kappa(T(t) - s(t))dt + \sigma(t)dL(t) \tag{3.1}$$

The solution to the above with application of the Ito formula for semi martingales is given as below.

$$T(t) = s(t) + (T(0) - S(0))e^{\kappa t} + \int_0^t \sigma(u)e^{\kappa(t-u)}\, dL(u) \tag{3.2}$$

The density function of the generalized hyperbolic distribution is given as follows:

$$f_{gh}(x;\ \lambda,\mu,\alpha,\beta,\delta) = c(\delta^2 + (x-\mu)^2)^{\left(\lambda-\frac{1}{2}\right)/2} \exp(\beta(x-\mu)) * K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{(\delta^2 + (x-\mu)^2)}\right) \tag{3.3}$$

Hence K_s is the modified Bessel function of the third kind with index s and the constant c is given by the following:

$$c = \frac{(\alpha^2 - \beta^2)^{\lambda/2}}{\sqrt{(2\pi)}\alpha^{\lambda-\frac{1}{2}}\delta^\lambda K_\lambda\left(\delta\sqrt{(\alpha^2 - \beta^2)}\right)} \tag{3.4}$$

The parameter α represents the steepness of the distribution, μ represents the location of the distribution, β represents the skewness while δ represents the scaling.

$L(t)$ is a generalized hyperbolic Lévy process if $L(t)$ is a Lévy process with marginals $L(1)$ being distributed according to the generalized hyperbolic family. The Lévy measure is therefore given by:

$$\ell_{GH}(dz) = |z|^{-1} e^{\beta z} \left\{ \frac{1}{\pi^2} \int_0^{\infty} \frac{exp\left(-\sqrt{(2y + \alpha^2)}\right)|z|}{J_{\lambda}^2\left(\delta\sqrt{(2y)}\right) + Y_{\lambda}^2\left(\delta\sqrt{(2y)}\right)} \frac{dy}{y} + \lambda e^{-\alpha|z|} \right\} dz \tag{3.5}$$

This is when $\lambda \geq 0$ and when $\lambda < 0$, this changes into the following:

$$\ell_{GH}(dz) = |z|^{-1} e^{\beta z} \left\{ \frac{1}{\pi^2} \int_0^{\infty} \frac{exp\left(-\sqrt{(2y + \alpha^2)}\right)|z|}{J_{-\lambda}^2\left(\delta\sqrt{(2y)}\right) + Y_{-\lambda}^2\left(\delta\sqrt{(2y)}\right)} \frac{dy}{y} \right\} dz \tag{3.6}$$

Here, J_λ and Y_λ are the Bessel functions of the first and second kinds respectively with index λ.

3.4.2 Componential analysis of the model

Equation 3.2 can be expressed using the following time series analogue which is a basic discrete time analogue.

$$T_t - S_t = (1 + \kappa)(T_{t-1} - S_{t-1}) + \sigma_t \varepsilon_t, \qquad t = 1,2,\ldots \tag{3.7}$$

The above time series can be written as an additive time series as follows:

$$T_t = s_t + c_t + \tilde{\varepsilon}_t, \quad t = 0, 1, 2, \ldots \tag{3.8}$$

T_t is the average temperature on day t, s_t is the seasonal component, c_t is the cyclical component and $\tilde{\varepsilon}_t$ is the noise.

3.4.2.1 Seasonality

This study uses the Benth (2005) model approach to model the seasonality component of temperature as follows.

$$s_t = a_0 + a_1 \cos\left(\frac{2\pi}{365}(t - t_0)\right) \tag{3.9}$$

a_0 and a_1 are constants describing the average level and amplitude of the mean temperature respectively. t_0 represents a phase angle.

An alternative process used in deseasonalizing the temperature is similar to the one used by US Census X-13-ARIMA-SEATS. The seasonal component of the time series is defined as the intrayear variation that is repeated constantly or in an evolving fashion from year to year (Jackson & Leonard, 2015).

3.4.2.2 Cyclicality and Regression analysis

To obtain the cyclical component, this study regresses today's deseasonalised temperature against the deseasonalised temperature recorded yesterday.

$$c_t = \alpha(T_{t-1} - S_{t-1}) \tag{3.10}$$

Given that $\alpha = 1 + \kappa$, this research obtains the following equation:

$$X_t = \alpha X_{t-1} + \breve{\varepsilon}_t \tag{3.11}$$

This approach is similar to Benth (2005). The approach used by (Campbell & Diebold, 2002) is more complicated as it uses 25 autoregressive lags and 3 sine and cosine terms in s_t in order to reach an R^2 of greater than 90%.

In the case of seasonal heteroscedasticity, Benth (2005) use a deterministic seasonality function when modelling variance as a certain function of time. Campbell and Diebold (2002) propose an ARCH model for the residual dynamics.

Due to variation in the regression parameter α over time due to seasonality or passage of time, this regression is determined monthly and then the average is used.

3.4.3 Seasonality in the residuals

The residuals $\breve{\varepsilon}_t$ exhibit seasonal variations, therefore this study uses a multiplicative time series model for the residuals.

$$\breve{\varepsilon}_t = \sigma_t \varepsilon_t \tag{3.12}$$

This will result into squared residuals from which the average can be obtained to use in the model. If this estimate proves unreliable due to its large fluctuations, smoothing techniques are used such as logarithmic smoothing and moving average technique. As a result, a more regular estimate of the squared residuals will be obtained.

Due to the likelihood of residuals in all selected towns being normally distributed, this study applies the generalized hyperbolic distribution unless the residuals in different cities exhibit normality as indicated by P-values and χ^2 figures.

3.4.4 Correlation between cities

This study tries to identify the correlation between the different cities because this is important for temperature derivatives risk diversification. Positively correlated temperature data from various cities will require different position in the temperature derivatives.

4 FINDINGS

The three locations that were used in this research were Embu, Nakuru and Lamu for average, high and low altitude representative areas respectively. The temperatures data used for these locations was monthly rather than due to its availability. Also, the temperature data runs from 1996 until the first quarter of 2013.

The descriptive statistics of these locations temperature data is indicated below.

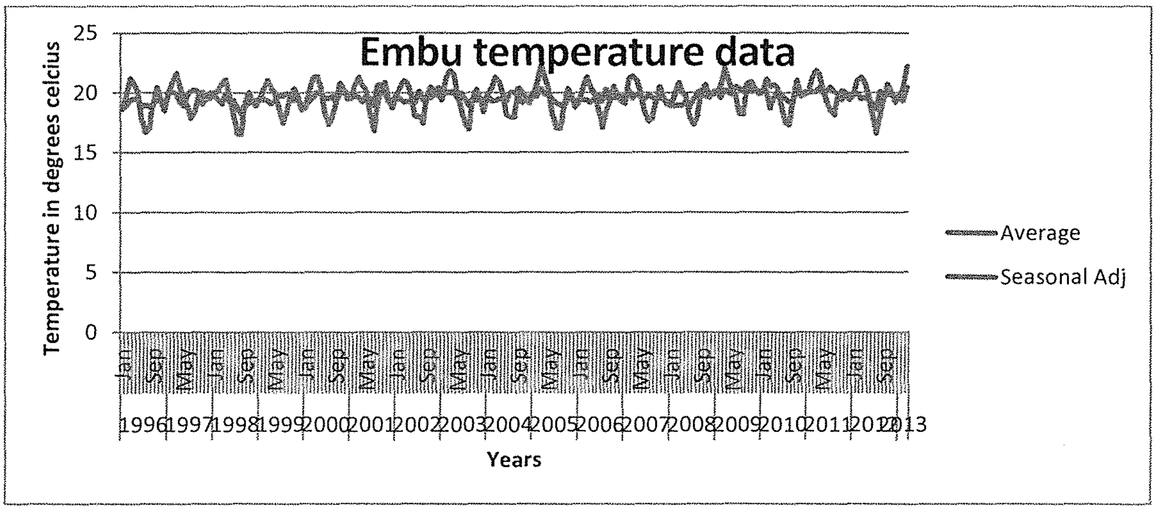
Table 1 Monthly Average Temperature Descriptive Statistics

	Monthly Average Temperature		
	Lamu	Nakuru	Embu
Mean	27.61	18.90	19.60
Standard Error	0.08	0.06	0.09
Median	27.60	18.75	19.78
Mode	28.00	18.20	19.70
Standard Deviation	1.10	0.81	1.25
Sample Variance	1.21	0.66	1.56
Kurtosis	-0.58	-0.37	-0.31
Skewness	0.16	0.50	-0.44
Range	5.00	4.05	5.71
Minimum	25.35	17.25	16.54
Maximum	30.35	21.30	22.25
Sum	5880.55	3647.85	4057.71
Count	213.00	193.00	207.00

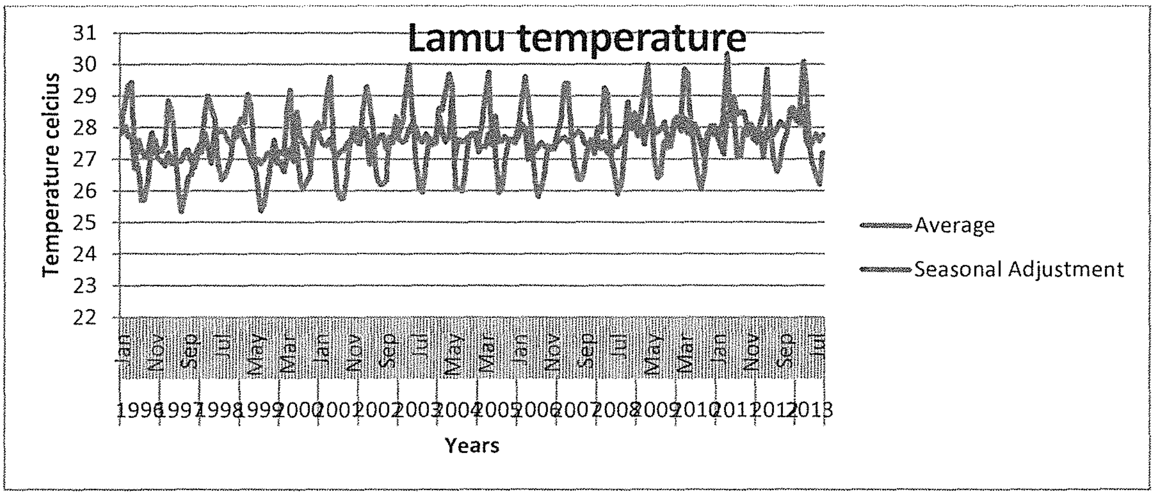
From the above statistics, it can be interpreted that Nakuru experiences the lowest temperatures while Lamu experiences the highest temperatures. The distribution of Nakuru and Lamu temperature is right skewed indicating temperature decrease across the years while Embu temperature is left skewed indicating temperature increase across the period. All the distributions are steep with the steepest distribution being of Lamu. The most volatile temperature data is from Embu.

4.1 Deseasonalizing the data

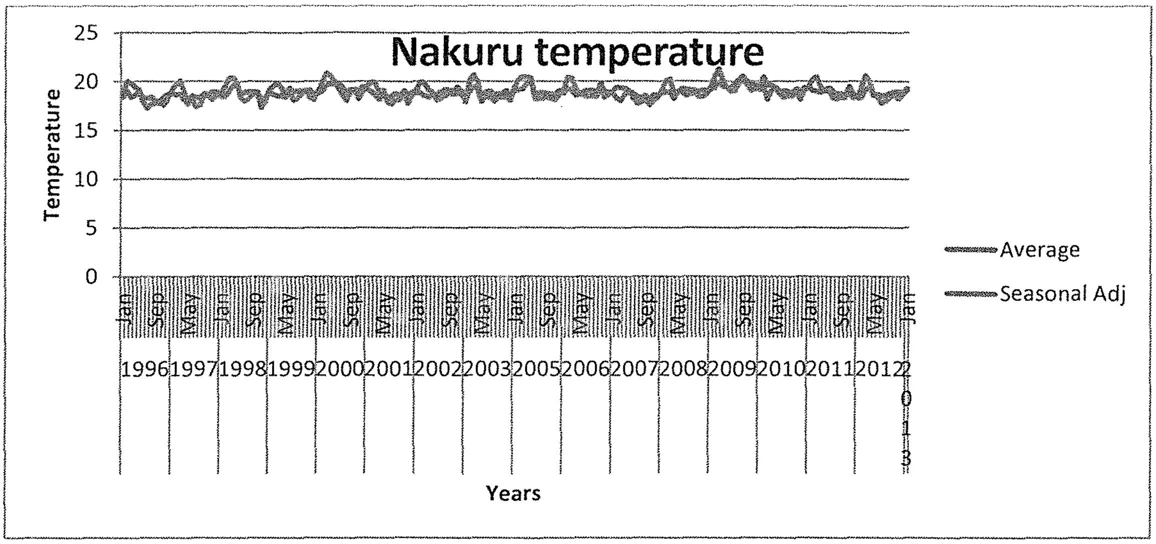
Due to the evident seasonal trends, this research took the approach of deseasonalizing the data using the X-12 model employed by the US government for Census. The graphs below were the observed deseasonalised temperature data for the various locations.



Embu deseasonalised temperature data



Lamu deseasonalised temperature



Nakuru deseasonalised temperature

It was observed that the Lamu monthly temperature data exhibited the most seasonality as compared to the other two locations.

In order to obtain the cyclicity of the temperature data, the average deseasonalised temperatures of the current month are regressed against the average deseasonalised temperatures of the previous month. The results obtained are as follows.

Table 2: Regression variables of Lamu, Embu and Nakuru

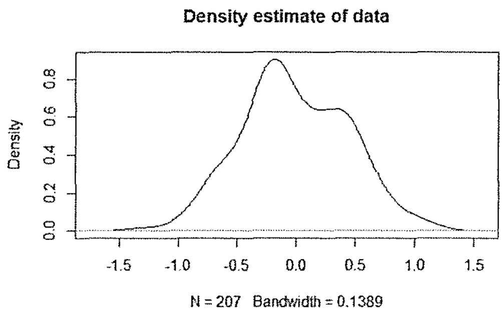
	Lamu	Embu	Nakuru
Intercept	13.71	11.52	8.51
X1	0.50	0.41	0.55
R Square	25.3%	17.5%	30.4%
Adjusted R Square	25.0%	16.7%	30.0%

4.2 Modelling the residuals

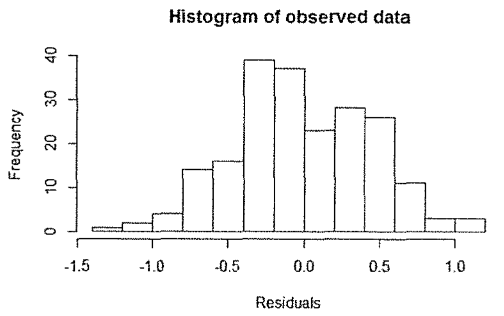
In order to determine the distributions which h the residuals followed, this research carried out a chi-square goodness of fit to establish that the residuals as shown in section concerning Seasonality in the residuals follow a Poisson process and are not normally distributed. The distributions of the residuals for the various data sets were modeled in R software.

These are the frequency density and histogram figures of the residuals in the various locations.

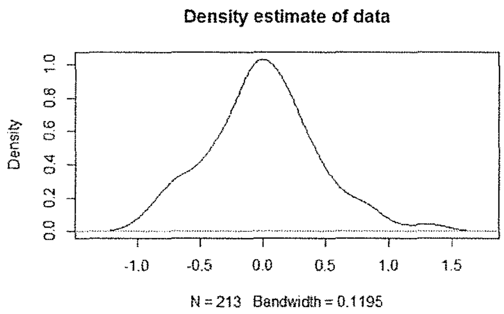
Table 3 Frequency density and histograms of deseasonalised temperature residuals



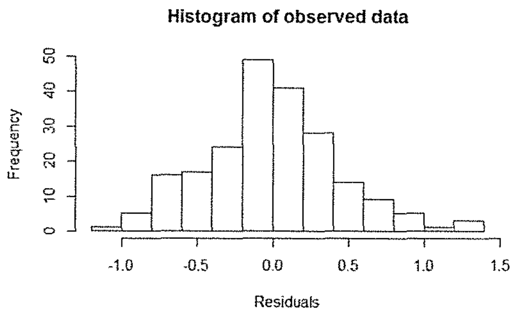
Frequency density of Embu deseasonalised temperature residuals



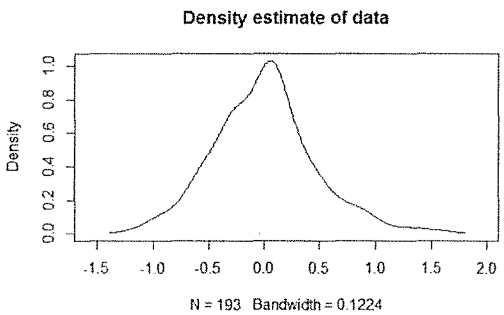
Histogram of Embu deseasonalised temperature residuals



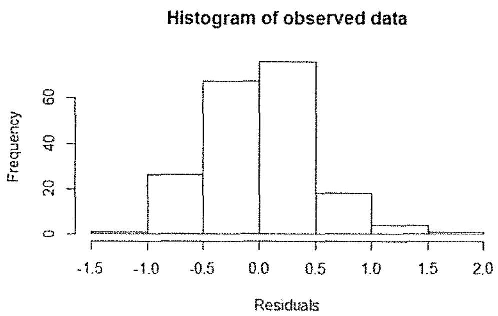
Frequency density of Lamu deseasonalised temperature residuals



Histogram of Lamu deseasonalised temperature residuals



Frequency density of Nakuru deseasonalised temperature residuals



Histogram of Nakuru deseasonalised temperature residuals

Table 4 Residual descriptive statistics

Descriptive data for residuals	Lamu	Embu	Nakuru
Mean	0.0811	-0.0069	-0.0007
Standard Error	0.0092	0.0144	0.0151
Median	0.0259	-0.0277	0.0119
Mode	0.00008	0.1132	-0.1336
Standard Deviation	0.1285	0.2011	0.2108
Sample Variance	0.0165	0.0404	0.0444
Kurtosis	11.8710	-0.3059	0.7817
Skewness	2.9823	0.0715	0.3923
Range	0.8375	1.1098	1.2806
Minimum	0	-0.5916	-0.5487
Maximum	0.8375	0.5182	0.7319
Sum	15.6648	-1.3367	-0.1418
Count	193	193	193
Largest(1)	0.8375	0.5182	0.7319
Smallest(1)	0	-0.5916	-0.5487
Confidence Level(95.0%)	0.0182	0.0285	0.0299

Using the goodness of fit results for the temperature residuals, this study doesn't reject the null that the residuals follows a Poisson process. This is because of a high p value.

Table 5 Goodness of fit Chi-square test for temperature residuals

	Chi-Square	p-value
Nakuru	2.5938	0.9572
Lamu	3.6144	0.9894
Embu	3.3761	0.9847

4.2.1 Residuals modeled using generalized hyperbolic distribution

The parameters of this family were fitted using maximum likelihood estimation in R software.

Table 6: Parameters obtained

	μ (mu)	α (alpha)	β (beta)	γ (gamma)	λ (lambda)
Nakuru	-0.0007	10.6780	8.8245	0.3923	0.7817
Lamu	0.0811	184.5253	180.5877	2.9823	11.8710
Embu	-0.0069	2.7405	1.7676	0.0715	-0.3059

Mu, represents the mean, alpha and beta are shape parameters of the distribution while lambda and gamma are representing kurtosis and skewness respectively. Alpha represents the steepness or fatness of the tails and beta represents the scaling.

4.3 Temperature derivative

A suitable temperature derivative is the Heating Degree Day which this study shall use the Nakuru temperature for the year 2012 to model. The strike temperature is 18 degrees Celsius. The hypothetical multiplier is KES 200 for every additional degrees Celsius up and above 18.

Table 7 HDD Payoffs of Nakuru data

2012	Temperature	T-strike	Payoffs (KES)
January	27.6028	15.6029	3120.58
February	27.5117	15.5117	3102.34
March	27.8511	15.8511	3170.22
April	27.0483	15.0483	3009.66
May	27.9616	15.9616	3192.32
June	27.9433	15.9433	3188.66
July	27.7621	15.7621	3152.42
August	27.9365	15.9365	3187.30
September	28.1902	16.1903	3238.06
October	28.0734	16.0735	3214.70
November	28.0570	16.0570	3211.40
December	28.3182	16.3183	3263.66

5 Conclusions and Recommendations

The findings above depict the underlying temperature process used to model temperature derivatives suitable for the Kenyan market.

5.1 Deseasonalizing the temperature

The process used in deseasonalizing the temperature is similar to the one used by US Census X-13-ARIMA-SEATS. The seasonal component of the time series is defined as the intrayear variation that is repeated constantly or in an evolving fashion from year to year (Jackson & Leonard, 2015).

5.2 Trend analysis

When analyzing the trend component of the temperature data, it was observed that the regression parameters had very low values of R squared. This was lower than those obtained by Benth in his research of Norwegian temperature data. However, this study still uses the same regression formula to obtain the trend in the data and attributes the low R squared to use of regressing monthly deseasonalised temperature data rather than daily deseasonalised temperature data.

5.3 Modelling the residuals

The residuals were modeled using Levy processes as they were assumed to follow a generalized hyperbolic distribution. This was confirmed using a Chi-square goodness of fit test that indicated the residuals don't follow a normal distribution and therefore cannot be modeled using a Gaussian process. Therefore the Brownian motion is substituted with a Levy process for each of the cities as this assumption holds for them all.

5.4 Limitation of study

The major limitation to this study is that the temperature data is monthly rather than daily hence it leads to less significant results than if the temperature data was daily.

5.5 Conclusion

This study was able to obtain a suitable call option, a Heating Degree Day option based on the Nakuru temperature data. The temperature data used was the one modeled by the prior postulated mean reverting model that involves deseasonalizing the temperature then modelling the residuals following a Levy process. The results obtained for the payoffs indicate that the Heating Degree

Day is a suitable option for hot areas which exhibit continuously high temperatures greater than 18 degrees Celsius.

A cooling degree day option on the other hand would be suitable to areas experiencing low temperatures such as Nakuru.

5.6 Recommendations

This study fails to study various location temperatures in order to develop suitable temperature derivatives in areas where large scale farming and manufacturing companies whose profitability is influenced by temperature such as soft drink manufacturers. In order to capture this, more meteorological station temperature data should be analyzed and suitable derivative instruments developed.

Also, a future derivative should be designed using this modelling framework, among other different financial derivative products.

Recommended areas of further research are including Monte Carlo simulation in order to obtain daily temperature data for the various regions. Also, an ARCH model could be used to deseasonalize the temperature data so as to obtain higher values of R_squared and adjusted R_squared for the values.

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Positive λ

In the case $\lambda > 0$ we get a simpler Lévy-Khintchine representation which does not translate to negative λ, since a integral representation similar to Theorem B.19 is not valid.

Lemma *Denote by $\xi(u) = \int_0^\infty e^{-uz}\,\mathrm{gig}(z)dz$ the Laplace transform of the GIG distribution. Then we have $\varphi_0(u) = \xi\big(\frac{u^2}{2} - i\beta u\big)$ where φ_0 is the characteristic function of the GH distribution with $\mu = 0$.*

Proof. The GH distribution has the following mixture representation $\mathrm{gh}(dx) = \int_0^\infty N(\beta w, w)(dx)\,\mathrm{gig}(w)dw$ where N is the normal distribution. With Lukasz (1970, Theorem 12.1.1) follows

$$\begin{aligned}\varphi_0(u) &= \int e^{iux}\,\mathrm{gh}(dx) = \int_0^\infty \int e^{iux} N(\beta w, w)(dx)\,\mathrm{gig}(w)dw \\ &= \int_0^\infty \exp\left(-\left(\frac{u^2}{2} - i\beta u\right) w\right) \mathrm{gig}(w)dw \\ &= \xi\left(\frac{u^2}{2} - i\beta u\right).\end{aligned}$$

□

Lemma *For the symmetric centered case $\beta = \mu = 0$ we obtain the following representation for the Laplace transform of the GIG distribution*

$$\xi(u) = \left(\frac{\alpha^2}{\alpha^2 + 2u}\right)^{\lambda/2} \frac{\mathrm{K}_\lambda\big(\delta\sqrt{\alpha^2 + 2u}\,\big)}{\mathrm{K}_\lambda(\alpha\delta)}.$$

Proof. Note that $\xi(u) = \varphi\big(\sqrt{2u}\,\big)$ holds for $\beta = 0$. The result follows with Lemma 1.37. □

Lemma *The Laplace transform of the* $\mathrm{GIG}(\lambda, \delta^2, \alpha^2 - \beta^2)$ *distribution is given by*

$$\xi(u) = \left(\frac{\alpha^2 - \beta^2}{\alpha^2 - \beta^2 + 2u}\right)^{\lambda/2} \frac{\mathrm{K}_\lambda\big(\delta\sqrt{\alpha^2 - \beta^2 + 2u}\,\big)}{\mathrm{K}_\lambda\big(\delta\sqrt{\alpha^2 - \beta^2}\,\big)}.$$

Proof. We abbreviate the norming constant of the generalized inverse Gaussian distribution by

$$b(\lambda, \chi, \psi) := \frac{(\psi/\chi)^{\lambda/2}}{2\,\mathrm{K}(\sqrt{\psi\chi})},$$

where $\chi = \delta^2$ and $\psi = \alpha^2 - \beta^2$. Hence, the Laplace transform is given by

$$\begin{aligned}
\xi(u) &= b(\lambda,\chi,\psi)\int_0^\infty e^{-ux}\exp\left(-\frac{1}{2}\left(\frac{\chi}{x}+\psi x\right)\right)dx \\
&= \frac{b(\lambda,\chi,\psi)}{b(\lambda,\chi,\psi+2u)} \\
&= \frac{(\psi/\chi)^{\lambda/2}}{2\,\mathrm{K}_\lambda(\sqrt{\chi(\psi+2u)})}\frac{2\,\mathrm{K}_\lambda(\sqrt{\chi\psi})}{\left(\frac{\psi+2u}{\chi}\right)^{\lambda/2}} \\
&= \left(\frac{\psi}{\psi+2u}\right)^{\lambda/2}\frac{\mathrm{K}_\lambda(\sqrt{\chi(\psi+2u)})}{\mathrm{K}_\lambda(\sqrt{\chi\psi})}.
\end{aligned}$$

Replacing ψ and χ completes the proof. □

For the proof of the following theorem we need some integral representations.

Lemma

$$-\ln\left(1+\frac{u^2}{2z}\right) = \int\frac{e^{iux}-1-iux}{|x|}e^{-\sqrt{2z}|x|}dx \tag{1.41}$$

$$-\ln\left(1+\frac{u^2}{\alpha^2}\right) = \int\frac{e^{iux}-1-iux}{|x|}e^{-\alpha|x|}dx \tag{1.42}$$

Proof. See Keller (1997, Lemma 50c and p. 83) □

Theorem *The Lévy-Khintchine representation of the characteristic function of the generalized hyperbolic distribution for* $\lambda \geq 0$ *is*

$$\begin{aligned}
\ln\varphi(u) &= iu\mu + \int(e^{iux}-1-iux)g(x)dx \\
g(x) &= \frac{e^{\beta x}}{|x|}\left(\int_0^\infty\frac{\exp(-\sqrt{2y+\alpha^2}|x|)}{\pi^2 y(J_\lambda^2(\delta\sqrt{2y})+Y_\lambda^2(\delta\sqrt{2y}))}dy+\lambda e^{-\alpha|x|}\right)
\end{aligned}$$

Proof. First of all, we assume that $\mu = \beta = 0$ and define $w_u := \delta\sqrt{\alpha^2+2u}$. Hence, we can follow

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}u}\ln\xi(u) &= \frac{\mathrm{d}}{\mathrm{d}u}\ln\mathrm{K}_\lambda(w_u) - \frac{\mathrm{d}}{\mathrm{d}u}\ln(\alpha^2+2u)^{\lambda/2} \\
&= \frac{\mathrm{K}'_\lambda(w_u)\delta^2/w_u}{\mathrm{K}_\lambda(w_u)} - \frac{\lambda}{2}\frac{2}{\alpha^2+2u} \\
&= \frac{\delta^2}{w_u\,\mathrm{K}_\lambda(w_u)}\left(-\frac{\lambda}{w_u}\mathrm{K}_\lambda(w_u)-\mathrm{K}_{\lambda-1}(w_u)\right)-\frac{\lambda\delta^2}{w_u^2} \\
&= -\frac{\delta^2\lambda}{w_u^2}-\frac{\delta^2\,\mathrm{K}_{\lambda-1}(w_u)}{w_u\,\mathrm{K}_\lambda(w_u)}-\frac{\lambda\delta^2}{w_u^2} \\
&= -\frac{2\delta^2\lambda}{w_u^2}-\frac{\delta^2\,\mathrm{K}_{\lambda-1}(w_u)}{w_u\,\mathrm{K}_\lambda(w_u)}
\end{aligned}$$

Since $\lambda \geq 0$, we can use Theorem B.19 to derive the following by integrating the latter identity

$$\begin{aligned}
\ln \xi(u) &= \int_0^u \left(-\delta^2 \frac{\mathrm{K}_{\lambda-1}(w_v)}{w_v \mathrm{K}_{\lambda-1}(w_v)} - \frac{2\delta^2 \lambda}{w_v^2} \right) dv \\
&= -\delta^2 \int_0^\infty \int_0^u \frac{g_\lambda(y)}{y + w_v^2} dv\, dy - \lambda \int \frac{2}{\alpha^2 + 2v} dv \\
&= -\delta^2 \int_0^\infty \int_0^u \frac{dv}{y + \delta^2(\alpha^2 + 2v)} g_\lambda(y) dy - \lambda \ln\Big(1 + \frac{2u}{\alpha^2}\Big);
\end{aligned}$$

here we use that $\int_0^u \frac{1}{C+u} du = \ln(C+u) - \ln(C) = \ln(1 + u/C)$. Denote $\psi = \alpha^2$ and by the change of variable $y \longmapsto \ln(z) = 2\delta^2 z - \delta^2 \psi$ we get

$$\begin{aligned}
\ln \xi(u) &= -\delta^2 \int_{\psi/2}^\infty \int_0^u \frac{dv}{z+v} g_\lambda\Big(2\delta^2\big(z - \frac{\psi}{2}\big)\Big) dz - \lambda \ln\Big(1 + \frac{2u}{\psi}\Big) \\
&= -\delta^2 \int_{\psi/2}^\infty \ln(1 + \frac{u}{z}) g_\lambda\Big(2\delta^2\big(z - \frac{\psi}{2}\big)\Big) dz - \lambda \ln\Big(1 + \frac{2u}{\psi}\Big).
\end{aligned}$$

Hence, with Lemma 1.37 we obtain

$$\begin{aligned}
\ln \varphi(u) &= \ln \xi\big(\frac{u^2}{2}\big) \\
&= -\delta^2 \int_{\psi/2}^\infty g_\lambda(2\delta^2(z - \frac{\psi}{2})) \ln\big(1 + \frac{u^2}{2z}\big) dz - \lambda \ln\big(1 + \frac{u^2}{\psi}\big).
\end{aligned}$$

Applying the integral representations of Lemma 1.40 and the change of variable $z \longmapsto \lambda(y) = y + \frac{\psi}{2}$, and interchanging integrals, we obtain

$$\begin{aligned}
\ln \varphi(u) &= \delta^2 \int_{\psi/2}^\infty g_\lambda(2\delta^2(z - \frac{\psi}{2})) \int \frac{e^{iux} - 1 - iux}{|x|} e^{-\sqrt{2z}|x|} dx\, dz \\
&\quad + \lambda \int \frac{e^{iux} - 1 - iux}{|x|} e^{-\alpha|x|} dx \\
&= \int \frac{e^{iux} - 1 - iux}{|x|} \left(\int_0^\infty \delta^2 g_\lambda(2\delta^2 y) e^{-\sqrt{2y+\alpha^2}|x|} dy + \lambda e^{-\alpha|x|} \right) dx.
\end{aligned}$$

Therefore, the Lévy Khintchine representation of the characteristic function in the case of symmetric centered GH distributions is given by

$$\begin{aligned}
\ln \varphi(u) &= \int (e^{iux} - 1 - iux) g(x) dx \\
g(x) &= \frac{1}{|x|} \left(\int_0^\infty \frac{\exp(-\sqrt{2y + \alpha^2}|x|)}{\pi^2 y (J_\lambda^2(\delta\sqrt{2y}) + Y_\lambda^2(\delta\sqrt{2y}))} dy + \lambda e^{-\alpha|x|} \right),
\end{aligned}$$

where $\lambda \geq 0$. With Lemma 1.51, we obtain the Lévy measure of the skewed GH distribution. Introducing also μ proves the desired result. □