Pricing of Weather Index Insurance Premium using Black Scholes Framework for Smallholder Farmers in Burundi

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

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This Research Proposal has been submitted for examination with my approval as the Supervisor.

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Abstract

Agriculture plays a critical role in Burundi's economy, with smallholder farmers constituting the majority of the agricultural workforce. However, reliance on rain-fed farming exposes these farmers to climate-related risks, such as erratic rainfall and prolonged droughts, leading to significant crop yield losses. Traditional crop insurance models are often inaccessible due to high costs and administrative complexities. This study explores an innovative approach by applying the Black-Scholes model to price weather index insurance premiums, offering a cost-effective and efficient alternative. The research hypothesizes that the Black-Scholes framework can be adapted to accurately determine insurance premiums based on historical weather and crop yield data. A quantitative research design is employed, utilizing secondary data on rainfall patterns and maize yields in Burundi spanning 32 years (1991–2022). Correlation analysis confirms a strong relationship between rainfall variability and maize production, justifying the use of rainfall as an index for insurance pricing. Lognormality tests validate the suitability of the Black-Scholes model for this application. The results indicate that insurance premiums fluctuate based on the chosen rainfall thresholds, with higher thresholds reducing payout probabilities and lowering premium costs. The study highlights the model's potential for improving financial resilience among smallholder farmers while addressing affordability concerns through policy recommendations such as government subsidies and public-private partnerships. Future research should explore region-specific data and extreme climate event simulations to enhance precision. The findings provide a robust foundation for expanding weather index insurance in Burundi, ultimately contributing to agricultural sustainability and economic stability.

Chapter 1. Introduction

1.1 Background Study

Agriculture is the backbone of Burundi's economy, with most of the population relying on smallholder farming for their livelihoods. The agricultural sector contributes, on average, 39.7 percent of GDP and employs 84 percent of the labor force (IMF,2022). However, like many other African countries, Burundi's agricultural sector is mainly dependent on rain-fed farming, making it highly vulnerable to climate change due to its significant reliance on rainfall.

Over the past few decades, Burundi has experienced increasing variability in weather patterns, prolonged droughts, and unpredictable rainfall, which severely impact crop yields (IFAD, 2013; FAO, 2018). The severity and frequency of droughts in Burundi have increased over time, mirroring the broader trend in Africa, where climate change has exacerbated the adverse effects on agriculture (Wagacha, 2014). Over 2.9 billion people worldwide live in rural regions, with 86% relying on agriculture for their livelihoods (Mahul & Stutley, 2010). Smallholder farmers comprise most of the farming population and are particularly vulnerable to these climatic changes. These small-scale farmers, who largely rely on rainfall for their agricultural needs, are incredibly susceptible to the negative impacts of climate change. They face specific challenges from events like droughts and heavy downpours.

In Burundi, smallholder farmers are generally poor, with many living on less than $2 per day, particularly in rural areas (Juvent et al., 2012). These farmers are particularly susceptible to the financial impacts of weather-related disasters. About 75% of the rural population relies on agriculture or agricultural labor as their main source of income (International Fund for Agricultural Development).

For example, maize farming is vital for food security in Burundi, being the leading cereal crop in both production and consumption. In 2020, maize production in Burundi exceeded 260,000 tons, ranking as the sixth most significant crop after cassava, bananas, sweet potatoes, beans, and potatoes (FAO, 2022).

However, maize in Burundi is predominantly cultivated by smallholder farmers (approximately 0.5 ha/farm) for household consumption (Keya & Rubaihayo, 2013; Ludgate & Tata, 2015). The recurring droughts and unpredictable rainfall present substantial threats to their crop yields and food security. Although these smallholder farmers possess extensive experience managing climate variability, the extraordinary variability linked to long-term climate change falls beyond conventional coping mechanisms (Pettengell, 2010).

To mitigate these risks, there is a pressing need for innovative solutions. Agricultural innovation is essential for navigating the evolving market environment and addressing the challenges presented by the current economic conditions. It serves as a key catalyst for social development and economic growth, representing the transformative power required for sustainable advancement (Elahe et al. N., 2024). Despite the potential benefits of agricultural insurance, its adoption in Burundi still needs to be improved. Traditional crop insurance schemes are often inaccessible to smallholder farmers due to high premiums, complex claim processes, and the need for individual loss assessments (World Bank 2011). The increasing frequency of extreme weather events due to climate change has heightened the risk exposure of farmers, making it imperative to explore innovative insurance solutions that are both affordable and effective.

Weather index insurance allows farmers in developing countries like Burundi to manage climate risks effectively. This type of insurance provides timely payouts based on predetermined weather indices, such as rainfall levels, rather than individual loss assessments, making it a cost-effective and efficient tool for smallholder farmers (Barnett & Mahul, 2007).

This study seeks to address this gap by investigating the application of the Black-Scholes model to determine premiums for weather index insurance in Burundi.

1.2 Definition and Brief Elaboration of Key Concepts

Agricultural Insurance

Agricultural insurance is a risk management tool designed to protect farmers from financial losses due to adverse climatic events, pest infestations, or other natural disasters affecting crop yields. It compensates farmers when predefined risks occur, ensuring they can recover from losses without severe financial hardship. Traditional agricultural insurance models typically require field assessments to determine actual damages, which can be costly and time-consuming. As a result, many smallholder farmers in developing countries, including Burundi, struggle to access such insurance due to high premiums and administrative complexities. By offering financial protection, agricultural insurance helps stabilize farm incomes, promote investment in improved farming practices, and enhance food security.

Weather Index Insurance (WII)

Weather Index Insurance (WII) is an innovative form of agricultural insurance where payouts are based on predefined weather indices, such as rainfall levels, temperature variations, or drought conditions, rather than direct damage assessments. This eliminates the need for individual loss verification, reduces administrative costs, and enables faster claims processing. When a weather parameter (e.g., rainfall) falls below or exceeds a predetermined threshold, farmers receive an automatic payout to compensate for potential crop losses. This approach is particularly beneficial in rural and developing regions where traditional insurance models are impractical due to the high cost of assessments. However, WII also presents challenges, such as basis risk, where farmers may experience losses even if the index does not trigger a payout. Despite this, WII is widely recognized as an effective and scalable solution for improving financial resilience among smallholder farmers.

Black-Scholes Model

The Black-Scholes model is a widely used mathematical framework for pricing financial options, originally developed for stock markets. It is based on the principle that, under certain assumptions, an option's fair price can be determined using a differential equation that accounts for factors such as volatility, time to expiration, and the risk-free interest rate. In the context of weather index insurance, the Black-Scholes model is adapted to calculate insurance premiums by treating rainfall as the underlying asset. The model helps determine the probability that rainfall will fall below a critical threshold, which in turn influences the premium farmers must pay to obtain coverage. By leveraging the Black-Scholes framework, insurers can offer more objective, data-driven pricing, ensuring that premiums are neither too high to discourage participation nor too low to be financially unsustainable.

1.3 Problem Statement

Small-scale farmers in Burundi are highly susceptible to the effects of climate change, including erratic weather patterns, prolonged droughts, and unpredictable rainfall, which severely impact crop yields. Traditional crop insurance compensates farmers for actual losses and typically requires individual field assessments to verify claims. This process is labor-intensive, time-consuming, and costly, making it impractical for widespread use, particularly among smallholder farmers in developing countries like Burundi. High premiums and the complexity of filing claims discourage farmers from opting for traditional insurance.

In contrast, weather index insurance (WII) offers a more efficient and affordable solution. Instead of assessing individual losses, WII bases payouts on predetermined weather indices, such as rainfall levels or temperature, closely linked to crop yields. This approach reduces administrative costs and simplifies the claims process, with payouts triggered automatically when the index reaches a specified threshold. The limited adoption of agricultural insurance in Burundi underscores the urgent need for innovative solutions, such as index insurance, to make insurance more accessible and practical for smallholder farmers.

Despite the potential of weather index insurance in various contexts, its implementation in Burundi still needs to be explored. This study aims to address this gap by examining the application of the Black-Scholes model for pricing weather index insurance premiums. The objective is to provide a cost-effective and efficient alternative to traditional crop insurance tailored to the specific needs of Burundian smallholder farmers.

1.4 Research Questions

1. How can the Black-Scholes model be adapted to price agricultural insurance premiums for smallholder farmers in Burundi?

2. What minimum premiums are required from smallholder farmers for weather index insurance products?

1.5 Research Objectives

1. To adapt the Black-Scholes model for pricing weather index insurance premiums tailored to the needs of smallholder farmers in Burundi.

2. To determine the minimum premiums required from smallholder farmers for weather index insurance products.

1.6 Justification of the research

Low-income individuals, including small-scale farmers, frequently lack the financial means to purchase formal insurance to safeguard against extreme weather events such as drought. Given that agriculture plays a crucial role in Burundi's economy, the demand for insuring smallholder farmers to reduce these risks is growing. Insurers must understand how to price premiums for weather index insurance products to ensure they are affordable for smallholder farmers.

This research is crucial in enhancing the resilience of Burundi's agricultural sector against the adverse impacts of climate change. By focusing on smallholder farmers, who are the most vulnerable, this study aims to make agricultural insurance more accessible and affordable, thereby contributing to the region's overall economic stability and food security.

The findings will offer valuable insights for policymakers and stakeholders, informing the design and implementation of robust agricultural insurance programs. By enhancing the resilience of smallholder farmers, the broader agricultural sector and Burundi's economy will benefit through more stable agricultural productivity and reduced vulnerability to climate-induced disruptions.

Chapter 2. Literature review

2.1 Introduction

Climate change has emerged as a significant threat to agricultural productivity globally, and Burundi is no exception. The country, heavily dependent on agriculture, faces increasing variability in weather patterns, prolonged droughts, and unpredictable rainfall, severely impacting crop yields. As mentioned above, studies by the International Fund for Agricultural Development (IFAD) and the Food and Agriculture Organization (FAO) highlight that smallholder farmers in Burundi are particularly vulnerable due to their limited adaptive capacity and reliance on rain-fed agriculture (IFAD, 2013; FAO, 2018). These climatic challenges worsen food insecurity and poverty, highlighting the necessity for effective risk management strategies like crop insurance. This chapter examines relevant research on crop insurance in Burundi, calculating premiums using the Black-Scholes model and exploring the advantages of index insurance. It also identifies gaps in the literature and discusses subsequent studies that have tried to address these deficiencies, noting their limitations.

2.2 Actuarial Science for Crop Insurance

Actuarial science plays a fundamental role in crop insurance by applying statistical and mathematical methods to assess risks and determine fair insurance premiums. This field helps insurers estimate potential agricultural losses caused by climatic events, pests, and diseases, ensuring that insurance products are both financially viable and accessible to farmers.

One of the key aspects of actuarial science in crop insurance is risk assessment, which involves identifying potential threats that could impact crop yields. Mahul and Stutley (2010) developed a comprehensive framework for risk assessment, incorporating various variables such as historical climate patterns and loss probabilities. However, their approach lacked considerations for dynamic climate change effects and the emergence of new pest species. Subsequent research by Brown and Murphy (2015) attempted to enhance risk assessment by incorporating predictive analytics and climate models. Despite these advancements, accurately modeling long-term climate variability remains a challenge, limiting the reliability of risk predictions.

Another critical component is premium calculation, where insurers use historical crop yield and loss data to determine appropriate premium rates. Mahul and Stutley's (2010) pioneering work established foundational methodologies for pricing agricultural insurance premiums based on past trends. However, their approach assumed data stationarity and did not fully account for socioeconomic factors influencing crop yields, such as market dynamics and farming practices. To address these gaps, Jensen and Barrett (2016) refined premium calculations by incorporating real-time data and economic variables. While this approach improved pricing accuracy, challenges still exist in accounting for rapid market fluctuations and incomplete datasets, particularly in developing regions.

Loss assessment is another crucial aspect of crop insurance, ensuring that farmers receive timely and fair payouts in the event of adverse weather conditions. Smith and Watts (2009) developed standardized methodologies for loss assessment, primarily focusing on field inspections and direct yield measurement. While effective, these traditional approaches have limitations due to the subjectivity of manual inspections and the potential for human error. To overcome these challenges, Hill and Kotze (2018) explored the use of remote sensing technology and machine learning algorithms to improve loss assessment accuracy and objectivity. These innovations have shown promise in automating damage evaluations, but their high implementation costs and the need for specialized training present barriers to widespread adoption.

2.3 Weather Index Insurance vs Traditional Insurance in Agriculture

Traditional Insurance

Traditional crop insurance compensates farmers for their losses, necessitating individual field assessments. Hazell (1992) points out that this process is often expensive and time-consuming, making it less suitable for widespread application in regions like Burundi. The need for on-site loss verification and the involvement of trained personnel for accurate assessment contribute to the high costs and delays associated with traditional insurance. Additionally, the complexity of claim processes often discourages smallholder farmers from participating in these schemes.

Weather Index Insurance

Weather Index Insurance (WII) provides a more efficient alternative to traditional crop insurance by linking payouts to measurable weather indices, such as rainfall levels or temperature, rather than assessing individual farm losses. This approach offers several advantages, making it particularly suitable for smallholder farmers in developing regions.

One key benefit of WII is the reduction in administrative costs. Unlike traditional insurance, which requires expensive and time-consuming on-site loss verification, WII operates based on predefined weather thresholds. This eliminates the need for field inspections, significantly lowering administrative expenses for insurers while making policies more affordable for farmers.

Another advantage is the speed of payouts. Since claims are processed based on objective weather data, compensation can be distributed quickly when an index threshold is met. This ensures that farmers receive financial support without delays, helping them recover from weather-related losses and maintain their livelihoods.

Additionally, WII helps mitigate moral hazard, a common issue in conventional insurance where farmers might deliberately manipulate circumstances to claim payouts. Because WII is based on independent weather indices that cannot be influenced by individual actions, the risk of intentional losses is eliminated (Barnett & Mahul, 2007).

However, weather index insurance also has its shortcomings, particularly the issue of basis risk, which occurs when the index does not perfectly correlate with the actual losses experienced by farmers. This can result in farmers not receiving adequate compensation despite experiencing significant losses.

Subsequent studies have sought to address the gaps identified in earlier research. For instance, Greatrex et al. (2015) examined satellite technology's role in improving weather indices' accuracy. Their study demonstrated that satellite data could enhance the reliability of indices and reduce basis risk. However, the high cost of satellite data and the technical expertise required for its analysis remain significant barriers to implementation.

Further research by Clarke et al. (2012) explored the implementation of weather index insurance in several developing nations, including Burundi. They identified several barriers to widespread adoption, such as a need for more awareness among farmers, limited financial literacy, and the initial cost of premium payments. Despite the promising aspects of WII, their study underscored the need for better educational initiatives and financial support mechanisms to enhance adoption rates.

In another study, Jensen et al. (2016) focused on the economic impact of weather index insurance on smallholder farmers. They found that WII could increase farmers' resilience to weather shocks by providing timely financial support and encouraging investment in improved agricultural practices. Despite these benefits, the study highlighted that the high cost of premiums and the lack of tailored insurance products for different types of crops and farming practices still pose challenges.

Further research by Mude et al. (2019) explored the integration of community-based approaches to improve the adoption of weather index insurance. They proposed leveraging local cooperatives and farmer organizations to disseminate information and facilitate enrolment. Their findings suggested that community involvement could enhance trust and understanding of WII among farmers. Nonetheless, these community organizations' coordination and capacity-building challenges were identified.

In another effort, Carter et al. (2021) investigated the potential of combining weather index insurance with other financial products, such as savings and credit. Their study indicated that bundling WII with other financial services could make insurance more attractive and affordable for farmers. However, the complexity of managing multiple financial products and ensuring effective coordination among service providers were noted as significant obstacles.

Weather index insurance offers a promising alternative to conventional crop insurance by offering reduced administrative costs, timely payouts, and mitigation of moral hazard. While substantial progress has been made in addressing some critical barriers, significant gaps still need to be addressed, particularly concerning basis risk, data accuracy, and affordability. Continued research and targeted interventions are essential to overcoming these challenges and ensuring that weather index insurance becomes a viable and sustainable solution for enhancing the resilience and financial stability of smallholder farmers in Burundi.

2.4 Existing crop insurance in Burundi

Crop insurance in Burundi is underdeveloped, with most schemes being pilot projects or donor-funded initiatives. The World Bank (2011) highlighted the inaccessibility of traditional crop insurance for smallholder farmers due to high premiums and complex claims processes without delving into socioeconomic barriers like income levels and financial literacy.

Index-based insurance products offer a more viable alternative, reducing administrative costs and simplifying claims. The World Bank (2017) acknowledged their potential but noted challenges in data accuracy, infrastructure, and farmer trust.

Subsequent studies by Diro et al. (2018) examined the implementation of index-based insurance in Burundi, highlighting the importance of accurate and timely data collection. They emphasized that while index-based insurance reduces administrative costs, it relies heavily on the availability of reliable data, which can be a significant hurdle in regions with limited technological infrastructure. This study also pointed out that the need for more awareness and understanding of index-based insurance among farmers poses a challenge to widespread adoption.

Another study by Nguyen and Le (2019) focused on the socioeconomic impacts of index-based insurance in Burundi. Their research indicated that while index-based insurance products have the potential to increase financial inclusion and resilience among smallholder farmers, the high cost of premiums remains a barrier. They suggested that further subsidies or donor support might be necessary to make these products more affordable for farmers. However, this study also faced limitations such as a narrow sample size and a short study period, which might not capture the long-term impacts of index-based insurance.

Further research by Williams and Nkurunziza (2020) explored innovative solutions to the identified gaps, such as leveraging mobile technology for data collection and dissemination. Their study highlighted the potential of mobile platforms to increase accessibility and trust in index-based insurance among farmers. However, challenges related to mobile technology adoption, such as limited network coverage and digital literacy, were identified as ongoing obstacles.

In another effort, Johnson and Mugisha (2021) investigated community-based approaches to enhance the effectiveness of crop insurance schemes. They proposed involving local cooperatives in disseminating and managing index-based insurance products to increase farmer participation and trust. Despite these promising strategies, their study acknowledged the need for significant capacity building and financial support to implement such community-based approaches effectively.

Nyandwi and Kagiraneza (2020) examined government policies' role in promoting Burundi agricultural insurance. They found that while government support is crucial for the success of agricultural insurance schemes, inconsistent policy implementation and lack of financial resources have hindered progress. Their study recommended more robust government intervention and the establishment of public-private partnerships to enhance the effectiveness and sustainability of agricultural insurance programs.

Ndayishimiye and Dusabe (2022) conducted a comparative analysis of different agricultural insurance models in Burundi. Their study compared traditional crop insurance, index-based insurance, and hybrid models, finding that hybrid models combining elements of traditional and index-based insurance were the most effective in addressing farmers' diverse needs. Nevertheless, the study pointed out that implementing hybrid models requires substantial initial investment and robust infrastructure, often needing improvement in Burundi.

Crop insurance in Burundi is evolving with a shift from traditional models to more innovative index-based insurance products. While recent efforts have addressed some of the critical barriers, significant gaps still need to be addressed, particularly regarding data reliability, affordability, and farmer awareness. Continued research and targeted interventions are essential to overcoming these challenges and ensuring that crop insurance becomes a viable tool for enhancing the resilience and financial stability of smallholder farmers in Burundi.

2.5 Black-Scholes Framework

In 1973, the Chicago Board of Options Exchange began facilitating the trading of options on exchanges, although financial institutions had previously traded options over the counter. That year, Black, Scholes, and Merton published groundbreaking papers on option pricing theory. Their contributions were so significant that Scholes and Merton received the Nobel Prize in Economics in 1997, although Black had already passed away and thus could not share the honor (Okine A.N., 2014).

The core idea of the Black-Scholes model is that by continuously adjusting the mix of stocks and options in a portfolio, an investor can eliminate all market risks, creating a risk-free hedge portfolio. This relies on the assumptions of continuous trading and the continuous movement of asset prices. Any zero-risk portfolio must have an expected return equal to the risk-free interest rate in an efficient market without riskless arbitrage opportunities. This concept leads to the differential equation known as the "heat equation" in physics, whose solution is the Black-Scholes formula for pricing European put and call options on non-dividend-paying stocks.

A European call option grants the holder the right, but not the obligation, to buy an asset at a specified strike price only at the option's expiration date. This differs from American options, which allow early exercise at any time before expiration. Conversely, a European put option gives the holder the right to sell an asset at the agreed strike price only at maturity. These options are widely used in financial markets as tools for hedging or speculation, as they provide a structured way to manage price risk without requiring immediate asset transactions.

Critical variables in the Black-Scholes model include:

- C (S, t): the price of a European call option

- P (S, t): the price of a European put option

- S: the price of the stock, which can be a random variable or a constant

- K: the strike price of the option

- r: the annualized risk-free interest rate, continuously compounded

- μ: the drift rate of (S), annualized

- σ: the standard deviation of the stock's returns

- t: time in years; typically, now=0 and expiry=T

-δ: dividend rate, continuously compounded

- N(x): the standard normal cumulative distribution function

The Black-Scholes formulas for call and put options are:

$$\mathbf{C(S,t)} = \boldsymbol{Se^{-\delta t}}\mathrm{N}(\boldsymbol{d_1}) - \mathbf{K}\boldsymbol{e^{-rt}}\mathbf{N}(\boldsymbol{d_2})$$

$$\mathbf{P(S,t)} = \mathbf{K}\boldsymbol{e^{-rt}}\mathrm{N}(-\boldsymbol{d_2}) - \boldsymbol{Se^{-\delta t}}\mathrm{N}(-\boldsymbol{d_1})$$

Where: $\boldsymbol{d_1} = \dfrac{\left(\boldsymbol{ln}\left(\frac{S}{K}\right)\right)+(\boldsymbol{r}+\mathbf{0.5}\boldsymbol{\sigma^2})\boldsymbol{t}}{\boldsymbol{\sigma\sqrt{t}}}$, $\boldsymbol{d_2} = \boldsymbol{d_1} - \boldsymbol{\sigma\sqrt{t}}$

2.6. Conceptual Framework

Cash-or-nothing put options provide a fixed payout if the stock price falls below a certain level at expiration. Let S(t) be the stock price at time t. A cash-or-nothing put option pays c at expiry time T if S(T) < K, where c denotes the payoff. The Black-Scholes model can be applied to price these options using risk-neutral probabilities and assumes ST/S0 is lognormally distributed (Weishaus, 2009).

In the context of index insurance, similar principles apply. Index-based insurance responds to an objective parameter, such as rainfall or temperature, measured at a defined location over a specified period. The policyholder receives a payout if the index measurement falls below a trigger value calculated based on historical data.

The Black-Scholes model can be used to price index insurance as follows:

- The trigger measurement in index insurance is R_T

- The payout structure is a lump sum, say Q

- The index follows a lognormal distribution.

The premium of the index insurance is calculated as:

$$\boldsymbol{Premium = Qe^{-rt}N(-d_2)}$$

Where N $(-d_2)$ is the probability that the index parameter(rainfall) is less than the threshold for rainfall and r is the continuously compounded interest rate over duration t.

Chapter 3. Methodology

3.1 Research Design

This study uses a quantitative research design, which is well-suited for analyzing data to find the relationship between rainfall and maize production and calculating insurance premiums using the Black-Scholes model. By focusing on numbers and statistical methods, this approach helps to accurately determine premiums based on historical data, adapting the Black-Scholes model, initially used in finance, to the context of weather index insurance.

3.2 Population and Sampling

The research will focus on smallholder farmers in Burundi, particularly those engaged in maize farming. Maize is selected as the crop of interest since it is a staple food crop predominantly rain-fed in Burundi. The regions for data collection will include key maize-producing areas such as Gitega, Ngozi, and Kirundo. Maize grows well when the rainfall is between 600 and 900 mm, and the optimum temperature for good maize yields ranges between 18°C and 30°C. These study areas are chosen because they represent different climatic conditions, ensuring a diversified risk assessment for the index-based insurance product.

3.3 Data Collection

This study relies on secondary data, specifically historical rainfall and maize yield data. The data covers at least 32 years, from 1991 to 2022, to ensure sufficient variability in weather patterns and crop yields for robust analysis. Rainfall data will be sourced from the World Bank data set, while maize yield data will be obtained from Burundi's Ministry of Agriculture and Livestock. These sources provide reliable and comprehensive datasets necessary for accurate premium calculation.

3.4 Data Analysis

The data collected will be analyzed using inferential statistics to determine the relationship between rainfall variability and maize yield levels and to calculate the premium for the index-based insurance product.

3.4.1 Correlation Analysis

A Pearson correlation coefficient will be calculated to determine the relationship between rainfall and maize yield. This statistical measure will help establish the strength and direction of the relationship between the two variables, thereby confirming that maize production in Burundi is significantly influenced by rainfall. This analysis is crucial, as a strong correlation between these variables is necessary for the effectiveness of weather index insurance. The higher the correlation, the more accurate the index insurance will be in reflecting the actual losses experienced by farmers.

3.4.2 Distribution Analysis of Rainfall

The Black-Scholes model assumes that the underlying asset follows a lognormal distribution. In this study, rainfall is treated as the underlying asset. To verify this assumption, the rainfall data will be transformed using logarithmic functions, and normality tests such as the Shapiro-Wilk test will be performed to confirm that the transformed data follows a lognormal distribution. This step is essential to ensure the validity of the pricing model used in this study.

3.5 Pricing of Weather Index Insurance Products using the Black-Scholes Model

As mentioned, the pricing of weather index insurance products in this study will follow the methodology used for pricing cash-or-nothing put options under the Black-Scholes model. A put option is said to be in the money when the underlying asset's price is below the strike price. A cash or nothing put option is a binary option that provides a predetermined payout if the underlying asset's price is below the strike price at the option's expiration date or no payoff when it is above the strike price.

The price of the cash or nothing put option is:

$$\boldsymbol{P_t = C * e^{-rt} * N(-d_2)}$$

Where: C is the predetermined payout when the price of the underlying asset is less than the strike price at time **T**

r is the risk-free interest rate, assumed to be constant over the insurance period.

t is the time to maturity of the insurance contract.

and N ($\boldsymbol{d_2}$) is the probability that the underlying asset's price is lower than the strike price.

For pricing weather index product, the following assumptions will be made:

The farmer is paid a lump sum (**Q**) when claiming.

The index parameter, rainfall, denoted by **R** should follow a lognormal distribution.

The predefined threshold in weather index insurance is $\boldsymbol{R_T}$

The essential feature of index-based insurance is that the insurance contract responds to an objective parameter (e.g., measurement of rainfall or temperature) at a defined weather station during an agreed-upon period (Okine A.N., 2014). An index insurance policy purchaser receives a payoff if the rainfall measurements fall below the trigger measurement, which is calculated based on historical rainfall data.

Therefore, the premium equation will be as follow:

$$\boldsymbol{Premium = Qe^{-rt}N(-d_2)}$$

And $\boldsymbol{d_1 = \dfrac{\left(ln\left(\frac{R}{R_T}\right)\right)+(r+0.5\sigma^2)(T-t)}{\sigma\sqrt{t}}}$, $\boldsymbol{d_2 = d_1 - \sigma\sqrt{t}}$

Where N $(-d_2)$ is the probability that the index parameter(rainfall) is less than the threshold for rainfall and r is the continuously compounded interest rate over duration t.

Chapter 4. Data Analysis

4.1 Data Sources and Limitations

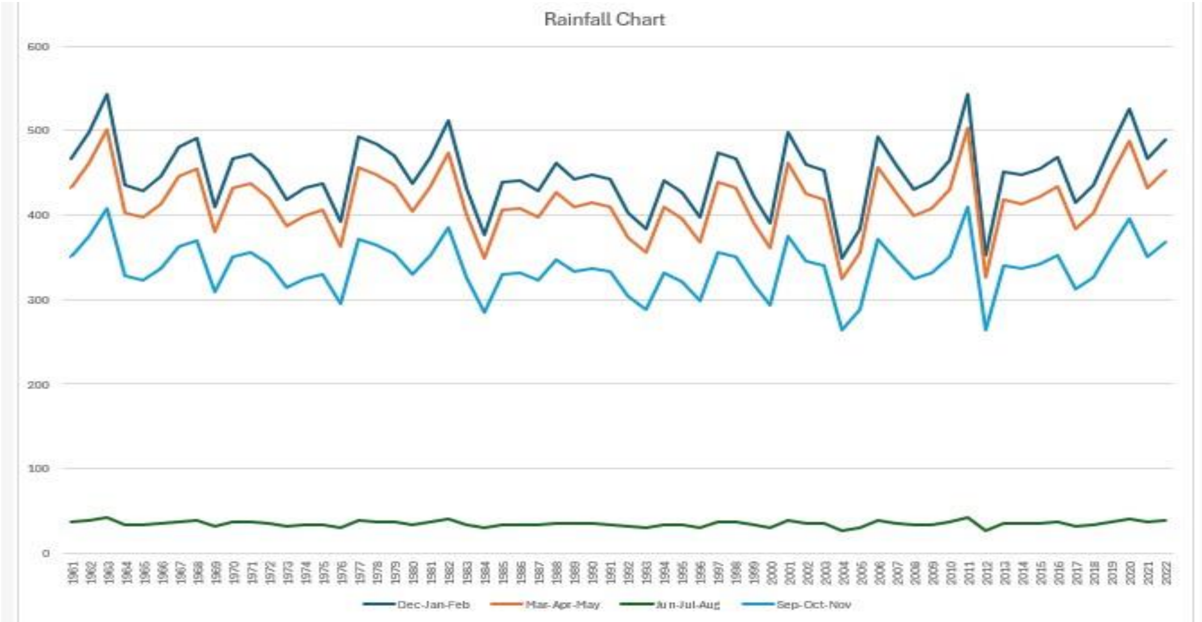
This study utilizes historical data spanning 32 years (1991–2022) to examine the relationship between rainfall and maize yields in Burundi. As stated, rainfall data was obtained from the World Bank Climate Database, while maize yield statistics were sourced from the Ministry of Agriculture and Livestock through FAO. Although these datasets provide valuable insights, several limitations must be acknowledged.

First, the analysis used nationwide rainfall and maize yield data rather than region-specific datasets, as regional-level data was unavailable. This limitation introduces potential inaccuracies in capturing localized climatic effects on maize yields, which may vary significantly across different regions in Burundi. Furthermore, occasional gaps in the historical data required interpolation, and variability in data collection methodologies over the years could influence trend consistency. Lastly, while the 32-year dataset is sufficient for meaningful analysis, extending the data further would enhance the robustness of findings, as suggested by studies recommending at least 40 years of historical data for similar analyses (Okine, 2014).

Despite these limitations, the data utilized remains a strong foundation for exploring the correlation between rainfall and maize yields and for applying the Black-Scholes framework for weather index insurance pricing.

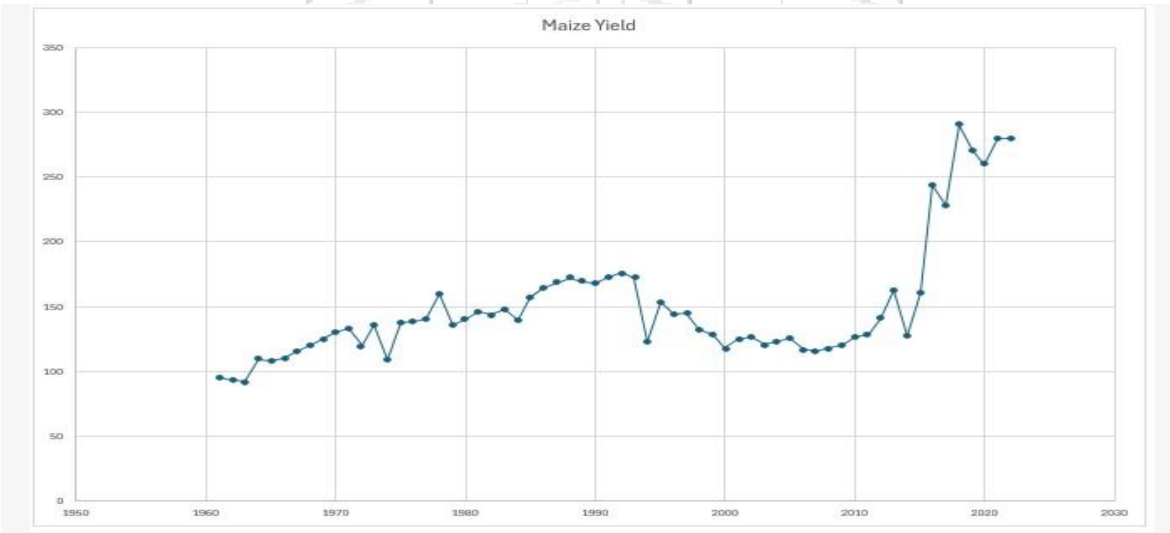
4.2 Analysis of Crop Yield and Rainfall Levels (Descriptive Statistics)

The descriptive analysis of rainfall and maize yield data reveals important patterns and seasonal trends. Quarterly rainfall levels exhibit notable variability, with averages ranging from 337.7 mm during September–November (SON) to 448.8 mm during December–February (DJF), which corresponds to the primary planting season. Rainfall variability is highest in DJF, with a standard deviation of 40.3 mm, and lowest in June–August (JJA), where variability is minimal, as indicated by a standard deviation of 3.1 mm. These seasonal trends reflect Burundi's reliance on rainfall for agricultural cycles, with DJF serving as a critical period for ensuring adequate soil moisture for planting.



Maize yields show a mean of 149.6 kilotons (kt) over the study period, with a maximum of 290.5 kt and a minimum of 91.9 kt. The standard deviation of 46.6 kt highlights moderate yield variability. Yield trends demonstrate steady growth, particularly after 2000, which could be attributed to advancements in agricultural practices or favorable climatic conditions. The correlation between rainfall and maize yield aligns with expectations, as seasonal rainfall is critical for crop growth, as similarly observed in other studies (Okine, 2014; Kadenge, 2017).

The rainfall-yield relationship exhibits significant seasonal dependency, emphasizing the importance of timely and adequate rainfall during key agricultural cycles.



4.3 Correlation Analysis and Lognormality Test

The correlation analysis reveals a strong positive relationship between rainfall and maize yields, with a Pearson correlation coefficient of **0.8249**. This indicates that rainfall plays a critical role in determining maize productivity, which aligns with findings from similar studies in Kenya and Ghana (Kadenge, 2017; Okine, 2014; Munyaka, 2021). Regression analysis further supports this relationship, with an R^2 value of **0.6805**, implying that 68% of the variability in maize yields can be explained by rainfall. The statistical significance of the regression model is confirmed by an F-statistic of **521.97** and a p-value of **1.19E-62**, which demonstrate the reliability of the results in capturing the relationship between these variables.

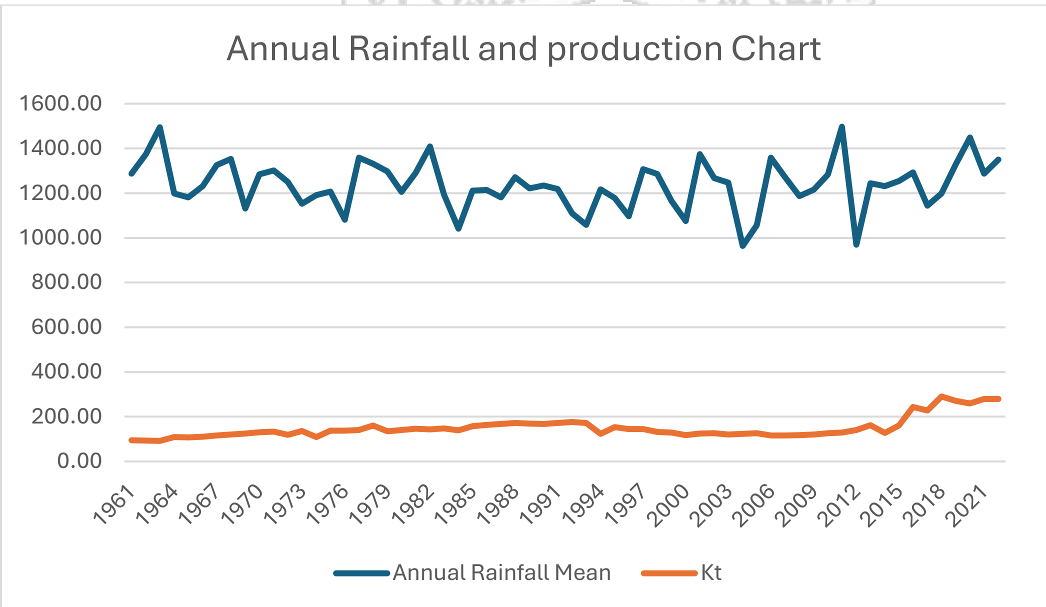
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.824961344
R Square	0.680561219
Adjusted R Square	0.679257387
Standard Error	13.48025659
Observations	247

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	94851.01015	94851.01015	521.9701199	1.19439E-62
Residual	245	44520.74286	181.7173178		
Total	246	139371.753			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.075645035	1.806992174	0.595268231	0.552213614	-2.483576465	4.634866535	-2.483576465	4.634866535
466.9766029	0.117916998	0.005161235	22.8466654	1.19439E-62	0.107750946	0.12808305	0.107750946	0.12808305



This chart illustrates the annual rainfall trends and maize production (Kt) over the period from 1961 to 2021, providing a visual representation of their correlation. The chart shows that while maize production remained relatively low and stable for several decades, it started increasing significantly after the early 2000s. Meanwhile, annual rainfall exhibits noticeable fluctuations, with occasional sharp declines. The overall trend suggests that while maize production is influenced by rainfall variability, external factors such as improved farming techniques or irrigation may have contributed to stabilizing production in later years. However, the observed correlation reinforces the importance of weather index insurance in mitigating climate-related risks for smallholder farmers in Burundi.

To validate the assumptions of the Black-Scholes model, a Shapiro-Wilk test was performed to assess the normality of the rainfall data. The test returned a W-statistic of **0.98347** and a p-value of **0.5693**, indicating that the rainfall data does not significantly deviate from normality ($p > 0.05$). This result confirms that the rainfall data meets the normality assumption necessary for premium pricing using the Black-Scholes framework. Consequently, further log transformation was deemed unnecessary as the data inherently aligns with the model's statistical requirements.

```
> x<-c(1286.93,1373.52,1495.04,1198.65,1181.44,1231.27,1326.44,1352.94,1130.82,1284.65,
1301.70,1250.29,1152.90,1190.59,1206.92,1081.35,1358.83,1332.02,1296.72,1205.93,1289.7
5,1408.46,1192.96,1040.30,1211.00,1214.45,1180.97,1270.65,1221.53,1234.41,1218.05,1110.
26,1057.65,1217.11,1177.95,1096.06,1306.46,1286.16,1163.33,1075.64,1374.14,1266.62,124
6.38,963.79,1056.85,1358.33,1269.48,1186.67,1215.16,1283.04,1497.03,969.16,1244.55,123
1.89,1254.60,1292.90,1143.40,1198.40,1330.26,1449.11,1286.50,1349.76)
> shapiro.test(x)

        Shapiro-Wilk normality test

data:  x
W = 0.98347, p-value = 0.5693
```

The correlation and normality results provide a robust basis for linking rainfall variability with maize yield outcomes, enabling the application of advanced statistical models such as the Black-Scholes framework for pricing weather index insurance. These findings offer valuable insights into how rainfall influences agricultural productivity and sets the stage for premium calculation.

4.4 Premium Pricing

The Black-Scholes model was utilized to calculate weather index insurance premiums, with rainfall as the underlying index. The following parameters were used in the calculations:

- t=3 months (0.25 years)
- σ=0.4629 (rainfall volatility)
- R0=368.5 mm (most recent level of rainfall)
- r=5% (assumed risk-free interest rate)

10th Percentile:

For a rainfall threshold (R_t) of $294.2\,\mathrm{mm}$:

$$d_2 = \frac{\ln \frac{R_0}{R_t} + \left(r - \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = \frac{\ln \frac{368.5}{294.2} + \left(0.05 - \frac{0.4629^2}{2}\right) 0.25}{0.4629 \times \sqrt{0.25}}$$

$$d_2 = 0.911$$

Using the standard normal cumulative distribution function:

$$N(-d_2) = 0.8189$$

The premium is then calculated as:

$$\text{Premium} = P \times e^{-rt} \times N(-d_2)$$

Assuming $P = 100$ currency units:

$$\text{Premium} = 100 \times e^{-0.05 \times 0.25} \times 0.8189 = 80.87\,\text{currency units}$$

Similar calculations were conducted for the 25th and 50th percentiles, resulting in adjusted premiums based on corresponding rainfall thresholds.

Percentile	d2	N(-d2)	Premium
(10th)294.2	0.911	0.8189	80.87
(25th)322.5	0.514	0.69651	68.79
(50th)336.7	0.328	0.62862	62.08

4.5 Analysis of Premium Affordability and Reasonability

The results indicate that insurance premiums decrease as the threshold rainfall level increases. This trend is expected since lower rainfall thresholds correspond to a higher probability of triggering payouts, requiring insurers to charge higher premiums to compensate for the increased risk exposure. Conversely, at higher thresholds, the likelihood of payouts decreases, resulting in lower premium costs. This pattern emphasizes the need for a careful balance when selecting rainfall thresholds to ensure both affordability and adequate coverage for farmers.

A lower threshold provides better protection against extreme droughts by increasing the likelihood of payouts. However, this comes at the expense of higher premium costs, which may reduce accessibility for smallholder farmers who often operate with limited financial resources. In contrast, a higher threshold lowers premium costs, making insurance more affordable, but it may provide less comprehensive coverage, potentially leaving farmers exposed to residual risks if rainfall levels remain critically low but do not trigger a payout.

This affordability concern is particularly significant in the case of smallholder farmers, who may struggle to pay premiums as high as 80.87 for a benefit of 100—amounting to nearly 81% of the insured amount upfront. Such a high premium deviates from industry benchmarks, where agricultural insurance premiums typically range between 5% and 20% of the insured amount. If not adjusted, this pricing could limit participation, reducing the insurance scheme's effectiveness in providing climate risk protection.

To enhance affordability, several strategies should be considered. Adjusting the rainfall threshold, introducing government or donor-backed subsidies, and exploring risk-sharing mechanisms could help make premiums more accessible. Additionally, a more refined region-specific pricing model that considers localized climate variability and crop-specific risks could further optimize premium levels.

Chapter 5: Conclusion

This study aimed to analyze the relationship between rainfall and maize yields in Burundi and to apply the Black-Scholes model to price weather index insurance premiums. The findings revealed a strong positive correlation between rainfall and maize yields, emphasizing the critical role rainfall plays in agricultural productivity.

The calculated premiums, based on different rainfall thresholds, demonstrated sensitivity to variability in rainfall levels. By incorporating rainfall thresholds corresponding to the 10th, 25th, and 50th percentiles, the premiums were shown to decrease as thresholds increased, reflecting the reduced likelihood of payouts. This result highlights the importance of carefully selecting trigger thresholds when designing insurance products, as they directly influence both affordability for farmers and risk coverage for insurers.

Although the application of the Black-Scholes model offers a robust and objective framework for pricing, the study encountered several limitations. The reliance on national-level rainfall and maize yield data restricted the ability to capture localized variations in climatic conditions and their impact on yields. Additionally, affordability for smallholder farmers remains a significant concern, as premiums, though quantitatively reasonable, may still be prohibitive without subsidies or government interventions. Public-private partnerships or similar mechanisms could play a pivotal role in enhancing the accessibility and adoption of weather index insurance.

Future research could address these limitations by incorporating region-specific data to improve the precision of rainfall thresholds and premiums. Additionally, scenario analyses simulating extreme climatic events could further refine the model and ensure its applicability under various climatic conditions.

In conclusion, the results demonstrate the potential of the Black-Scholes model to provide transparent, data-driven pricing for weather index insurance. This approach presents a promising solution for mitigating agricultural risks in Burundi, offering a pathway for improving the resilience of smallholder farmers to climatic variability.

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