**SCHOOL OF FINANCE AND APPLIED ECONOMICS**
Bachelor of Business Science – Actuarial Science, Finance & Financial Economics

**MARCH 2018 SPECIAL EXAMINATION**
**BSA 2206 FINANCIAL MATHEMATICS TWO**

DATE: 13TH MARCH 2018                                                       Time: 2 Hours

**Instructions**
1.      This examination consists of **FIVE** questions.
2.      Answer **Question ONE (COMPULSORY)** and any other **TWO** questions
**Section A**
**Question One [30 Marks]**

i.      Distinguish between each of the following:

  a.  Interest rate swap and a currency swap                                          [2]

  b.  Buying a put option and selling a call option (Use payoff diagrams)            [2]


ii.

  a.  State what is meant by a "forward contract". Your answer should include reference to the
      terms "short forward position" and "long forward position".                   [3]

  b.  Six months ago, an investor entered into a one-year forward contract to purchase a non-
      dividend paying stock. The risk-free force of interest was 4% per annum. The value of the
      stock is now 98% of its original value. Calculate the minimum value for the risk-free
      force of interest at which the original forward contract still has a positive value to the
      investor.                                                                      [4]


iii.    An investor is considering investing in a capital project. The project requires an outlay of
       £500,000 at outset and further payments at the end of each of the first 5 years, the first
       payment being £100,000 and each successive payment increasing by £10,000.  The
       project is expected to provide a continuous income at a rate of £80,000 in the first year,
       £83,200 in the second year and so on, with income increasing each year by 4% per annum
       compound. The income is received for 25 years.  It is assumed that, at the end of 15
       years, a further investment of £300,000 will be required and that the project can be sold to
       another investor for £700,000 at the end of 25 years. Calculate the net present value of the
       project at a rate of interest of 11% per annum effective.                     [9]

iv.     A one-year forward contract on a share was agreed on 1 September 2015 when the share price was £8.70 and the risk-free force of interest was 7% per annum. The stock was expected to pay a dividend of £1.10 eight months after the date of issue. The price of the share was £9.90 on 1 February 2016 and the risk-free force of interest was 6.5% per annum. The dividend expectation was unchanged. Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 February 2016.

[6]

v.      Define each of the following:
                      a.  Commercial paper                                    [1]
                      b.  Certificate of deposit                              [1]
                      c.  Eurobond                                            [1]
                      d.  Foreign Bond                                        [1]


## SECTION B
## Question Two [20 Marks]

a. Define each of the following terms and state the disadvantage of each:          [3]
   i.      Discounted payback
   ii.     Time-weighted rate of return
   iii.    Money- weighted rate of return

An investment project gives rise to the following cash flows. At the beginning of each of the first three years £180,000 will be invested in the project. From the beginning of the first year until the end of the twenty-fifth year, net revenue will be received continuously. The initial rate of payment of net revenue will begin at £25,000 per annum. The rate of payment is assumed to grow continuously at a rate of 6% per annum effective.

   i.      Calculate the net present value of the project at an effective rate of interest of 7% per annum.                                                                    [6]
   ii.     Calculate the discounted payback period of the project at an effective rate of interest of 7% per annum.                                                           [5]
   iii.    Calculate the annual effective rate of growth of net revenue which would be required if the project is to have a zero net present value at an effective rate of interest of 7% per annum.                                                           [6]

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**Question Three [20 Marks]**

a. Give the characteristics of index-linked government bonds. [3]

b. On 15 March 1996 the government of a country issued an index-linked bond of term 6 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is 3%. Interest and capital payments are indexed by reference to the value of an inflation index with a time lag of 8 months. A tax-exempt investor purchased the stock at £111 per £100 nominal on 16 September 1999, just after the coupon payment had been made. You are given the following values of the inflation index:

| Date | Inflation index |
|---|---|
| July 1995 | 110.5 |
| March 1996 | 112.1 |
| July 1999 | 126.7 |
| September 1999 | 127.4 |

   i. Calculate the amount of the coupon payment per £100 nominal stock on 15 March 2000. [3]

   ii. Calculate the effective real annual yield to the investor on 16 September 1999.

     You should assume that the inflation index will increase continuously from its value in September 1999 at the rate of 4% per annum effective. [11]

   iii. Without doing any further calculations, explain how your answer to (ii) would alter, if at all, if the inflation index for July 1995 had been more than 110.5. [3]

**Question Four [20 Marks]**

A company is adopting a particular investment strategy such that the expected annual effective rate of return from investments is 7% and the standard deviation of annual returns is 9%. Annual returns are independent and $(1 + i_t)$ is lognormally distributed

where $i_t$ is the return in year t. The company has received a premium of £1,000 and will pay the policyholder £1,400 after 10 years.

a. Calculate the expected value and standard deviation of an investment of £1,000 over 10 years, deriving all formulae that you use. [9]

b. Calculate the probability that the accumulation of the investment will be less than 50% of its expected value in ten years' time. [8]

c. The company has invested £1,200 to meet its liability in 10 years' time. Calculate the probability that it will have insufficient funds to meets its liability. [3]


**Question Five [20 Marks]**

a. In a particular bond market, the two-year par yield at time $t = 0$ is 4.15% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is £105.40 per £100 nominal. Calculate:

i. the one-year spot rate [4]

ii. the two-year spot rate [4]

b. An insurance company has a portfolio of annuity contracts under which it expects to pay £1 million at the end of each of the next 20 years, followed by £0.5 million at the end of each of the following 20 years. The government bond with the longest duration in which it can invest its funds pays a coupon of 10% per annum in arrears and is redeemed at par in 15 years' time. The yield to maturity of the government bond is 6% per annum effective and a coupon payment has just been made.

i. Calculate the duration of the insurance company's liabilities at a rate of interest of 6% per annum effective. [4]

ii. Calculate the duration of the insurance company's assets at a rate of interest of 6% per annum effective, if all the insurance company's funds are invested in the government bond with the longest duration. [4]

iii. Explain why the insurance company cannot immunize its liabilities by purchasing government bonds. [2]

iv. Without any further calculations, state the circumstances under which the insurance company would make a loss if there were a uniform change in interest rates. Explain why a loss would be made. [2]