2016

Relative performance of the single index versus mean variance optimization in equity portfolio construction in Kenya

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Relative Performance of the Single Index versus Mean Variance Optimization in Equity Portfolio Construction in Kenya

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Submitted in partial fulfilment of the requirements for the Degree of Master of Business Administration (MBA) at Strathmore University

Strathmore Business School

Strathmore University

Nairobi, Kenya

June, 2016

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

Approval

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Abstract

This study focuses on comparing the performance of the single index model and mean variance optimization model in portfolio construction in Kenya using the Nairobi Securities Exchange-20 Share Index from 2002-2015. The comparison is done by constructing portfolios using both the single index model and mean variance optimization model. The Sharpe ratio is used to determine which model is better, as indicated by the higher Sharpe ratio. The study establishes that the mean variance optimization model is better when considering investments with long time horizon, whereas the single index model is better when considering investments with short time horizon. The study also concludes that the mean variance optimization model is better if the investors are risk averse, while the single index model is better when considering investors who are risk lovers.

Key words: ***single index model, mean variance optimization model, Sharpe ratio, portfolio construction.***

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List of Definitions

Name	Definition
Diversification	a technique that reduces risk by allocating investments among various financial instruments, industries and other categories.
Discount rate	a rate that measures the growth relative to the year-ended accumulted amount
Effective rate of interest	the amount of money that one unit invested at the beginning of the period will earn during the period, when the amount earned is paid at the end of the period.
Nominal interest rate	an interest rate is called nominal if the frequency that is compunded (monthly, quarterly etc) is not equal to the basic time unit (year)
Portfolio	A grouping of financial assets such as stocks, bonds and cash equivalents, as well as their mutual, exchange traded and closed fund counterparts.
Risk	uncertainty of future outcomes
R-squared	measures variation of the explained variable against variation in the explanatory variable.
Securities	a financial instrument that represets an ownership position in a publicly traded corporation (stock), a

	creditor relationship with the government body or a corporation (bond), or rights of ownership as represented by an option.
Sharpe ratio	a risk-adjusted measure of return that is often used to measure the performance of a portfolio.
Systematic risk	the risk inherent to the entire market or an entire market segment. Also known as undiversifiable risk.
Unsystematic risk	company or industry-specific risk that is inherent in each investment. Also known as diversifiable risk.

Table 1.1: List of definitions

List of Abbreviations

ASEA	-	African Securities Exchanges Association
BSE	-	Bombay Stock Exchange
CVaR	-	Conditional Value at Risk
DSE	-	Dhaka Stock Exchange
MPT	-	Modern Portfolio Theory
MVO	-	Mean Variance Optimization
NASI	-	Nairobi Securities Exchange All Share Index
NSE	-	Nairobi Securities Exchange
NSE India	-	National Stock Exchange
VaR	-	Value at Risk
VBA	-	Visual Basic for Applications

Acknowledgment

I would first like to thank my Thesis Supervisor Dr. John Olukuru and Co-Supervisors Mr. Ferdinand Okoth respectively. The School of Finance & Applied Economics was always open for me to engage and discuss the progress of my research. Thank you for the guidance .

I would also like to thank Evelyne Mocha Nyokangi & Michelle Nduati for the numerous discussions and debates that we had a s part of this process. To Francis Sesi & Asaph Ngecha thank you for the short course that aided me to improve my VBA coding .

Finally I must express my sincere gratitude to my wife, Kerubo Rabera Nyamwaya and my 2 children, Omweri & Osike for their patience, support and encouragement during the period of the program and also wirting of this Thesis. To my parents and siblings, this would not have been possible without your support as well.

Chapter 1: Introduction to the Study

1.1. Background to the study

One of the ways investors allocate their wealth is through investing money in the stock market so as to gain returns. However, due to the often unpredictable nature in which the market moves, investors prefer to invest in a manner that maximizes their returns, whilst minimizing the risks. Modern Portfolio Theory (MPT) recommends that investors reduce their risk exposure by holding a well-diversified portfolio. Portfolio selection can be referred to as choosing the best investment mix of securities which include: equities, bonds, offshore deposits and alternative investments. Portfolio diversification and optimization requires deployment of tools such as the Sharpe single index model of Sharpe (1963) and the Mean Variance Optimization framework of Markowitz (1952).

The mean variance optimization framework of Markowitz (1952) uses a model in which risk averse investors diversify their portfolios by investing in more than one security, so as to reduce unsystematic risk. Based on the Markowitz model, a rational investor constructs an optimal portfolio that either maximises returns for a given amount of risk, or minimises risk for a given amount of return through the selection of securities whose expected returns are not perfectly positively correlated (see for instance Markowitz, 1952; Bhalla, 2010).

Sharpe (1963) contributes to the field of modern portfolio theory by offering a more simplified approach to constructing the optimal portfolio (Elton, Gruber & Padberg, 1976; Mandal, 2013; Sarker, 2013; Singh & Gautam, 2014). The single-index model solves the portfolio optimization problem by expressing the risk-return characteristics of a security linearly, which is simple to understand. The development of the simplified single index model was significant to the field of MPT where practitioners found the Markowitz model difficult to operationalize which resulted in a limited number of empirical studies utilising it despite the model being theoretically rigorous (Edwin, Gruber & Padberg, 1976; Mandal, 2013; Sarker, 2013; Singh & Gautam, 2014).

This study compares the performance of the single-index model and MVO in optimal portfolio construction for equity securities listed on the NSE-20 based on their risk-return profiles. The performance is measured using the Sharpe ratio, which was introduced by Sharpe in 1966. The Sharpe ratio of a portfolio is its excess return per unit of total portfolio risk, higher Sharpe ratios indicate better performance. There are other forms of performance measures, for example: Treynor measure, which is the excess return per unit of systematic risk, the disadvantage is that it does not put into consideration unsystematic risk unlike the Sharpe ratio (Treynor, 1965); and Jensen's alpha, which is the percentage above that of a portfolio with the same beta as the portfolio that lies on the securities market line (SML) (Jensen, 1968). The Sharpe ratio is used as it is the most recognized performance measure and puts into consideration both the systematic and unsystematic risk.

To the best of my knowledge there has not been rigorous research into the veracity of Sharpe's single-index model, as compared to the MVO framework, for optimal portfolio construction in Kenya. A study was carried out by Nduati (2015) comparing the single index model and the multifactor model but did not put into consideration the MVO model, which is considered in this study. In addition, most investment advice in Kenya is based on high level evaluation of ratios and multiples due to the computational difficulty involved in mean variance optimization and related models. The study also tests the consistency of the models between periods to see if the conclusion will be the same across time.

1.2. Problem statement

Different models have been developed to aid fund managers construct optimal portfolios. The three main models used by fund managers in Kenya include: the mean variance optimization model; the multifactor model; and multiples, in particular the price to earnings ratio (P/E). The MVO model has been challenged by scholars like Bowen (1984) stating that the model has high computational requirements making it complex and time consuming.

Index models were then developed following the MVO shortcomings; these include multifactor models developed by Fama & French (1992) which are models that capture the nonmarket influences that cause the co-movement of securities. The main disadvantage raised was that it was difficult to determine the number of factors to include as different scholars had different factors, for example, Roll & Ross (1980) propose three factors whereas Gibbons (1982) proposes six to seven factors. The P/E ratio shows the relationship between stock prices and earnings per share, it shows the amount of money an investor is willing to pay per earning. This strategy involves constructing a portfolio of stocks with low P/E ratios (believed to be undervalued) in the belief of obtaining abnormal returns (Pettersen, 2011). In the case of growth investors, they are willing to pay a high premium as the market expects significant future earnings growth, which is indicated by high P/E ratios (Gottwald, 2012). The main disadvantage is that the earnings per share are based on an accounting measure that can often be deceitful and manipulated.

This study seeks to compare the MVO model and the single index model to see which one performs better in selecting a portfolio based on the NSE 20. Since the model is based on a single common factor it reduces the number of estimates required and therefore improves computations tractability. In addition the single index model uses a benchmark as its basis which provides a better basis for measuring performance and lastly it is less reliant on historical data.

1.3. Research objectives

The objectives of this paper are:

1. To compare the performance of the mean variance model and the single index model in portfolio construction in Kenya using the Sharpe ratio.
2. To determine whether the performance of the single index model and the mean variance model varies over time using the Sharpe ratio.

1.4. Research questions

1. Is the single index model better than the mean variance model?
2. Are the models' performances consistent across time?

1.5. Scope of the study

This study uses monthly data from 2002 to 2015 to construct an optimal portfolio of securities in Kenya from the Nairobi Securities Exchange-20 Share Index using the Sharpe Single Index Model and MVO model.

1.6. Significance of the study

In addition to comparing the performance of the single index model to that of MVO in portfolio construction, the current study tracks the performance across time in a frontier market. This study therefore contributes significantly to the debate on portfolio selection models in a setting that has often been ignored by researchers. This study also aids informed decision making by the fund managers, retail investors[1], pension advisors, and insurance companies among other investors in optimum portfolio selection enabling investors to maximize return and minimise risk based on a simpler framework.

[1] The retail investors invest their money in mutual funds, thus the mutual fund managers use the models to determine the optimum portfolio.

Chapter 2: Literature Review

2.1 The Foundation of Portfolio Analysis

Modern portfolio theory originates from Markowitz (1952) seminal work on portfolio optimization. In the Markowitz model, investors are assumed to be risk averse when making decisions in which there is uncertainty about future outcomes. As a result of investors being risk averse, investors require higher levels of returns if they are to take on higher levels of risks. Hence, the objective of rational investors would be to either construct portfolios that maximize expected returns subject to a given level of risk or minimize risk subject to a given level of expected return. The Markowitz's model is designed to assist investors achieve a trade-off between risk and return portfolio construction because it allows them to forecast a portfolio's expected rate of return as well as the standard deviation of the portfolios. The model uses standard deviation of returns as a proxy for risk. Portfolios constructed this way were assumed to be efficient because the investor cannot derive a higher expected rate of return (at the given level of risk) from the other portfolios available to them.

Markowitz's model for portfolio construction was the first to encourage investors to focus on the risk-return profiles of portfolios, rather than that of individual securities. This focus allows investors to benefit from the diversification of risk that comes with holding many securities. Furthermore, the Markowitz model can be utilized by investors seeking to make long term investments since it conforms to the geometric mean optimization approach (Briec & Kerstens, 2009).

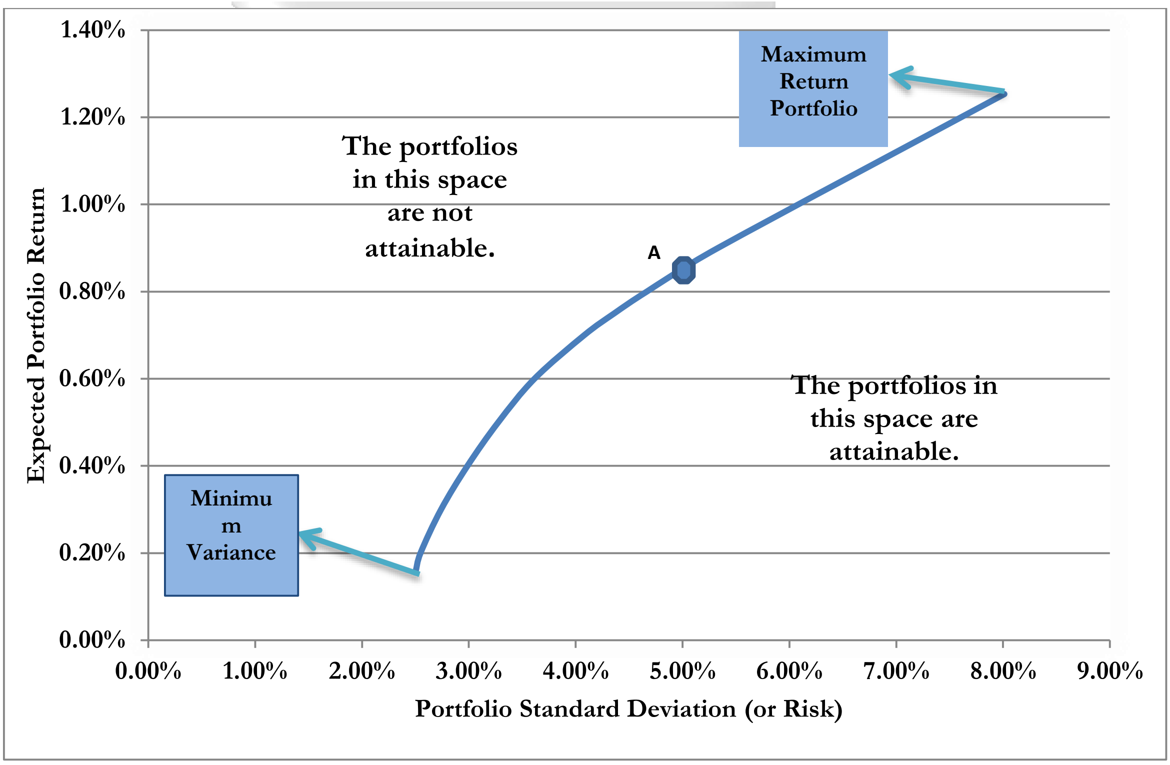
A review of the literature however shows that many portfolio managers do not use the Markowitz's model in portfolio construction despite the model being theoretically well-defined. This is attributed to a number of factors. First, the model assumes that a security's expected return is determined by its historical returns (Bilbao-Terol & Antomil-Lbias, 2006). Second, the model is complex and time consuming to construct and implement since it requires the following parameters be estimated: the expected returns of each security; the variances of each security; and the covariance between every conceivable pair of securities within the portfolio. This means that in the case of n securities, the model requires n estimates of expected returns and variances and a further $(n^2 - n)/2$ estimates of covariances to be calculated. The Markowitz model's high computational requirement makes it more complex and time consuming which undermines the ability of investors to systematically employ it on a regular basis, especially as the number of securities n increases (Bowen, 1984). Third, the accuracy of the covariance estimates decline significantly as the number of securities included in the portfolio rises, which has a negative impact on the reliability of the outputs from the model (Bowen, 1984). Fourth, the Markowitz model does not provide investors with a method of forecasting the risk premiums of individual securities. This presents a significant challenge to investors seeking to construct the efficient frontier. Finally, the theory underpinning the Markowitz model is difficult for a typical investor to grasp (Michaud, 1989).

The weaknesses of the Markowitz model led to the development of alternative approaches to constructing optimal portfolios. Markowitz (2010) suggests other alternatives for portfolio selection if the MVO is not applicable, these include: using other measures of risk, for example VaR and CVaR, or return in a risk-return analysis; determine the investor's utility function explicitly and maximize its expected value; using constraints or guidelines instead of optimizing; and using intuition to select a portfolio. The Sharpe's single index model is another alternative which has the advantage of being computationally simpler, as well as having lower data requirements (Elton, Gruber & Padberg, 1976). For instance, the covariance data requirement decreases from $(n\hat{}2 - n)/2$ to n when utilising the Sharpe model; whilst the informational requirement decreases from $(n(n+3))/2$ to $3n + 2$ (Sarker, 2013). The Sharpe model therefore outperforms the Markowitz model when the investor only has limited historical data.

2.1.1 The Markowitz Model

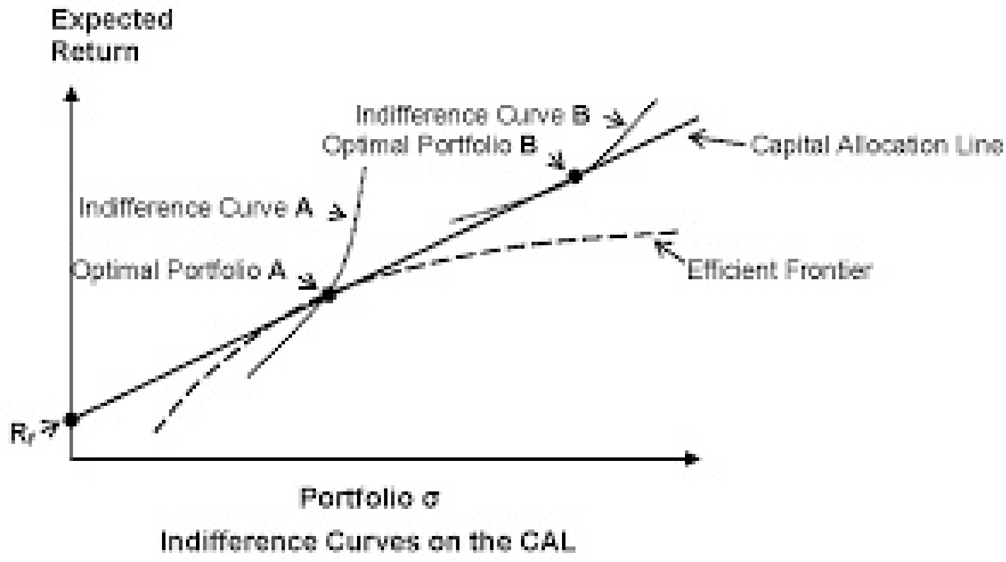
The Markowitz model is based on the following assumptions: First, that each potential investment is characterized by its probability distribution of expected returns over a specified time period; Second, that it is a single-period model and that the investor constructs a portfolio at the beginning of the period; Third, that the investor has utility curves that exhibit diminishing marginal utility of wealth, and that they will seek to maximize the expected utility of their investments over the single periods; Fourth, that the utility curves are a function of expected return and the variability of expected return, so that the investor's portfolio decision is based exclusively on these two indicators; Fifth, that the standard deviation of the portfolio's expected return is used as a proxy for risk; and sixth, that the investor is risk averse. It follows from these assumptions that the rational investor will select portfolios that lie on the efficient frontier.

The curve depicted in Figure 2.1 is known as the efficient frontier. It shows how the level of expected return varies with levels of risk for different portfolios. The efficient frontier alludes to the fact that an investor is able to construct an infinite number of portfolios by using various combinations of different securities. The aim of the model is to construct a portfolio that optimizes return for a given level of risk.



Source: Authors own.

Figure 2.1 : Markowitz Model for Portfolio Selection: The Efficient Frontier



Source: Financial Planning Body of Knowledge (2016)

Figure 2.2 : Portfolio Optimization

Figure 2.2 illustrates how the investor's utility varies with different portfolio constructions. The indifference curves (ICs) depict the expected utility that investors derive from portfolios with certain risk-return profiles. Rational choice theory states that investors will select portfolios that yield the best combination of return and risk i.e. falls on the efficient frontier, and the highest level of utility. This optimal portfolio has a risk-return combination that characterizes the point (A) at which the efficient frontier is tangential to the indifference curve of the investor. Markowitz (1959) argues that under certain conditions a carefully chosen portfolio from a mean-variance efficient frontier will approximately maximize the investor's expected utility.

2.2 Applications of Sharpe's Single Index Model

A simplified variant of the Markowitz model was developed by Sharpe (1963) that reduced its data and computational requirements, for example the process of inputting data, data tabulation, and data analysis, substantially. Sharpe's single-index model solves the portfolio optimization problem by expressing the risk-return characteristics of a security linearly,

$$\boldsymbol{R_i} = \propto_i + \boldsymbol{\beta_i R_m} + \boldsymbol{e_i} \quad (1)$$

Where, R_i is the expected rate of return on security i, R_m is the expected market index rate of return, $\propto_i$ is security i's return independent of the market's performance and β_i is the change in the expected return of security i from a unit change in the expected return of the market index, or a measure of the correlation between security i and the market, m. The residual term e_i represents the unsystematic risk of security i; and is said to be an independent and identically distributed (i.i.d) random variable with a zero conditional mean $E(e|R_m) = 0$ and constant variance $Var(e|R_m) = \sigma^2$. These assumptions are essential if one is to derive unbiased estimates of $\widehat{\propto}$ and $\hat{\beta}$ using Ordinary Least Squares (OLS) method. Equation (1) is used to find the single index value that measures the desirability of including that particular security in the optimum portfolio.

In equation (1), the beta (β_i) of the security represents the market's influence on each security's expected return. Securities with an estimated negative beta coefficient are rejected as investment options for the rational investor because it indicates that the security is not linear to the market. Furthermore, securities that are expected to offer rates of return that are lower than the risk-free rate of return are also rejected as investment options because they violate the risk-averse assumption. This assumption states that investors expect higher returns for higher levels of risk, so that risk-free assets are preferred over risky securities that offer a lower expected rate of return.

According to Sharpe (1963), the desirability of a security is linked to its excess return-to-beta ratio given as,

$$\frac{(R_i - R_f)}{\beta_i} \tag{2}$$

with securities being included in the optimal portfolio if their excess return-to-beta ratio is greater than the calculated cut-off point, ($\frac{(R_i - R_f)}{\beta_i} > C^*$). Thus, more desirable securities have a higher likelihood of being included in the optimal portfolio.

MPT was advanced by Elton, Gruber & Padberg (1976) by developing a rigorous criterion for constructing optimal portfolios using a variety of models: single-index model; multi-index model; and constant correlation model. The single-index and multi-index models are simpler and can process a greater number of securities than the constant correlation model (Haugen, 1993). Elton, Gruber & Padberg (1977) extended these models further by constructing a portfolio that places upper limits on the amount that can be invested on any single security within the portfolio. This was a departure from Sharpe's single-index model which did not place limits on the amount allocated to the individual securities. Lintner (1965) also extends the model by assuming that there are no restrictions on short selling. Furthermore, a study by Briec & Kerstens (2009) found that the Markowitz model outperforms the Sharpe model in constructing optimal portfolios that are to be held over multiple periods i.e. long term investments.

Sharpe's single-index model is taken a step further by Kamal (2012) by applying it to the DSE under two stock price scenarios: ex-ante bubble and bubble burst.[2] Kamal (2012) found that the optimal portfolio was primarily comprised of bank and financial institution securities in the ex-ante scenario. This was in stark contrast to the bubble burst scenario, where no securities met the unique cut off rate of return criteria (i.e. excess return over beta must be greater than the risk free rate) for inclusion due to their being just as risky as the market index.

Referring to Saravanan & Natarajan (2012) an optimal portfolio is constructed using the single index model using four securities from NSE-India. This study found that this portfolio outperforms the NSE NIFTY, which they used as the benchmark index. Sarker (2013) used the single-index model to construct a portfolio from the 164 companies listed in DSE. The authors calculated the cut-off rate of return using five years (2007 to 2012) of data on the securities monthly closing prices and DSE all share price index. The optimal portfolio comprised of 33 securities and had an expected return of 6.17 percent. Finally, Gautam & Singh (2014) applied the single index model to nine companies listed in the NSE-Mumbai and found that the optimal portfolio only consisted of securities from two of those companies.

[2] A speculative bubble exists when the price of a commodity does not equal its equilibrium market price for reasons other than random shocks to the market. This often leads to inflated commodity prices if it persists for an extended period of time.

2.3 Kenyan stock market

In Kenya, most companies use multiples, including price to earnings ratio, price to book ratio, enterprise value to EBITDA ratio among others, to determine which stocks can be included in a portfolio. So as to determine the optimal portfolio and weights, the MVO is used. However, multiples have disadvantages such as they can be easily manipulated as well as the MVO which is complicated. Thus the study introduces the use of the single index model in Kenya that can be used to determine which stocks can be included in a portfolio, the optimal portfolio and weights.

2.4 Literature Review Summary

Markowitz (1956) pioneered the use of the MVO model to determine the optimal portfolio. The MVO model has been proven in literature to be well defined, however, the MVO model is criticized as it requires too many parameters, and it is too complicated. In the Kenyan context, some of the problems encountered when using the MVO model are that: first, the MVO model depends on historical returns which, are difficult to obtain over a long period, there may be cases of share splits in the past and also the constituents of the NSE 20 change frequently; second, the higher the number of stocks, the less accurate the covariances are, thus not all the shares in the NSE are put into consideration as they are many, and would reduce the reliability of the outputs from the model, therefore the NSE 20 is used as proxy for all tradable shares in Kenya; third, the MVO model is difficult for a typical investor to understand, thus it is difficult to simplify the model to a shareholder who does not have prior MPT and statistics knowledge.

Index models were then introduced to cater for the MVO shortcomings. Hence, this study particularly looks at the single index model which was introduced by Sharpe in 1963. The main advantages are that a few parameters are required when using the single index model and it is easy to understand, however, the single index model only considers the market as the only explanatory variable for the stock returns thus there may be a problem of omission as there are other factors that determine the stock return, for example, economic growth, corporate governance, among others.

This study uses the NSE 20 Share Index to determine the portfolio construction using the single index model and MVO model. For the single index model, the NSE 20 share index is used as the market index. Given the optimal portfolio, the Sharpe ratio is used as a performance measure, the model that has the highest Sharpe ratio is concluded as better.

Chapter 3: Methodology

This study compares the single index model and the mean variance optimization model for the Kenyan market between January 2002 and December 2015.

3.1 The Research Design

This study has taken an exploratory research design as it is concerned with calculating the return and risk of portfolios so as to draw inferences from the results.

3.1.1 Population and Sampling

There are five commonly used asset classes in Kenya including: domestic common equity, domestic fixed income, non-domestic common equity, non-domestic fixed income and alternative investment. Despite the numerous assets in Kenya, this study confines itself to domestic equities as the data is easily accessible.

The securities selected are based on the following criteria: First, stocks included in the NSE 20, this is because these stocks are liquid and 60% of the market activity in the 20 stocks is by foreign participants and 25% by fund managers; and second, companies whose stocks were trading during the period between 2002 and 2015.

3.1.2 Data and data sources

Two major stock indices are used to characterize Kenyan equities including the Nairobi All Share Index (NASI); and the NSE 20 Share Index[3]. The NSE-20 is used in this study because it is the oldest and the most widely used of the NSE's performance barometers, it is a geometrically-weighted average of the largest 20 listed companies, measured by market capitalization. The index is constructed from stock prices data (excluding dividends), adjusted for corporate actions, such as stock splits and changes in firms' market capitalization over time. The NSE-20 is a good proxy for the whole market because its 20 companies represent over 80% of the market capitalization of the entire exchange

[3] Other indices include: NSE 25 Share Index; FTSE NSE Kenya 15 Index; FTSE NSE Kenya 25 Index; FTSE NSE Kenya Govt. Bond Index; and FTSE ASEA Pan African Index.

This analysis utilizes secondary data on the monthly closing price of securities trading on the NSE; and the monthly closing value of the NSE-20 for the period ranging from 2002 to 2015. The monthly closing price is used to make the model robust because the daily prices cannot be used as they are more volatile and also the investment horizon of investors is longer to use daily prices to find returns. Historical data is also collected on the interest earned on 91-day Treasury Bills issued by the Central Bank of Kenya. The interest rate is used as proxy for risk-free rate primarily because they are backed by the Kenyan government, and are therefore considered to be risk-free.

Data are obtained from Nairobi Stock Exchange and Central Bank of Kenya.

Variables to be used in the study

Variable	Description
Equity returns	The NSE-20 is used as the market index. The return is a monthly average return
Short-term Treasury Bill	A short term government security, yielding no interest but issued at a discount on its redemption price. 91-day Treasury Bill is used in the study (Burmeister & Wall, 1986)

Table 3.1: Description and Measurement of the Variables

3.2 Data analysis

3.2.1 The Single Index Model

The Sharpe single-index model approach to portfolio construction is based on a risk-return regression equation that estimates the expected rate of return on a security (R_i) as a function of the expected market index rate of return (R_m) and company-specific unexpected factors (e_i). According to Sharpe (1963), the risk-return regression equation is expressed as in equation (1).

$$R_i = \propto_i + \beta_i R_m + e_i$$

$$\text{where } \boldsymbol{Cov(e_i, e_j) = 0; Var(e_i) = \sigma_\varepsilon^2}$$

Pre-estimation tests including tests for autocorrelation and heteroskedasticity will be carried out (Brooks, 2008).

The $\hat{\beta}$ parameter is interpreted as the change in the expected return of security i from a one-unit change in the expected return of the market index m.[4] A security has: more systematic risk than the market index if the value of $\hat{\beta}$ is greater than one; less systematic risk than the market index if the value of $\hat{\beta}$ is smaller than one means; and has the same systematic risk as the market index if the value of $\hat{\beta}$ is equal to one.

The monthly return on security i at time t is given by the equation below.

$$\boldsymbol{R_{it}} = \frac{\boldsymbol{P_{it}}}{\boldsymbol{P_{it-1}}} - \mathbf{1} \tag{3}$$

Equation (3) shows that monthly return on security i at time t is a function of the price of security i at time t and $t-1$. Combining equation (1) and equation (3) will allow estimation of α, β and the excess return-to-beta ratio $\left((R_i - R_f)/\beta_i\right)$ (Elton, *et al.*, 1976).

The excess return to beta ratio, equation (2), simply describes the relationship between the potential reward and risk of investing in security i. The risk-free asset (R_f) is the risk free rate of return. This is the daily discount rate on treasury bills offered by governments through the Central Banks. Just as with the risk-return model, the excess return-to-beta ratio is calculated for each security. According to Sharpe (1963), this ratio denotes the additional positive return on security i (in excess of the return from risk free assets) that an investor is expected to gain per unit increase in their systematic risk. The ratio is then used to rank the individual securities in ascending order, with the rank capturing the relative desirability of each security. Rational investors prefer securities with higher excess return-to-beta ratios. A security

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[4] Systematic risk, is defined as variations in the expected return of security i that is related to variations in the market index m. This can be expressed in notational form as $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$, where σ_{im} is covariance between security i and the market index and σ_m^2 is the variance of the market index.

tends to have a lower rank if it has a negative beta coefficient, or if it provides a rate of return that is lower than the risk-free rate of return. These two conditions are grounds for a security's exclusion from the optimal portfolio because they indicate that the security is not linear to the market and that its expected return does not compensate for the investment risk.

According to Elton, Gruber & Padberg (1976) the exact securities that are selected to be included in the investor's portfolio will depend on the value of the cut-off point (C^*), which is computed from the characteristics of all of the securities being considered for inclusion in the optimal portfolio. Therefore, for a portfolio of i securities, C^* is calculated as

$$\boldsymbol{C}^* = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i - R_f)\beta_i}{\sigma_{e_i}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{e_i}^2}} \tag{4}$$

The cut-off point formula contains two variance parameters that are related to the risk of security. There is the variance that is associated with variations in the market index (i.e. systematic risk), σ_m^2; and the variance that is not associated with variations in the market index (i.e. unsystematic), $\sigma_{e_i}^2$. Securities whose excess return-to-beta ratio fall below the cut-off point, $(R_i - R_f)/\beta_i < C^*$ are excluded from the optimal portfolio since short selling is not allowed; whilst the securities whose calculated ratio fall above the cut-off point $(R_i - R_f)/\beta_i > C^*$ are included in the optimal portfolio.

The share of the portfolio (X_i) to be allocated across the total number of selected securities (n) is determined by equation (5), under the following conditions: short sales are prohibited; investors can borrow at the risk free rate; and lending is unlimited. These assumptions were also made by Lintner (1965).

$$\boldsymbol{X_i} = \frac{Z_i}{\sum_{i=1}^{n} Z_i} \tag{5}$$

where,

$$\boldsymbol{Z_i} = \frac{\beta_i}{\sigma_{e_i}^2}\left[\frac{R_i - R_f}{\beta_i} - \boldsymbol{C}^*\right] \tag{6}$$

and each $\boldsymbol{Z_i} > \boldsymbol{0}$. Furthermore, the weighted average of each security in the optimal portfolio is equal to one (full investment)

$$\sum_{i=1}^{n} X_i = 1 \quad (7)$$

The risk (σ_p) and return (R_p) of the optimal portfolio can be calculated using the formulae below.

$$R_p = \alpha_p + \beta_p R_m \quad (8)$$

where, $\beta_p = \sum_{i=1}^{n} X_i \beta_i$ and $\alpha_p = \sum_{i=1}^{n} X_i \alpha_i$.

$$\sigma_p = \sqrt{\beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} X_i^2 \sigma_{\sigma_i}^2} \quad (9)$$

The β_p and α_p of the portfolio are expressed as the weighted average of each of the selected securities respective β_i and α_i parameters. α_p will tend towards zero as the number of securities n in the portfolio increase, especially if some securities have positive alphas and others have negative alphas. Similarly, the unsystematic risk of a portfolio, equation (11), diminishes toward zero with greater diversification.

The Sharpe single-index model presented in this section can assist investors with which securities should be included in the optimal portfolio, and the share of the overall portfolio that is allocated to each of the selected securities

3.2.2 *The Mean Variance Optimization Approach*

The first step in carrying out an MVO test is estimating the mean of the portfolio which is given by:

$$E(R) = (w_1 E(R_1) + w_2 E(R_2) \ldots + w_N E(R_N)) \quad (10)$$

Since the asset classes compose a portfolio, then the summation of the individual asset class' weight should add up to one. Mathematically speaking;

$$\sum w_i = 1 \quad (11)$$

After estimating the return of the portfolio, the variation of each asset class is estimated. According to Kaplan (1998), this is given by:

$$V(R_i) = E\{[\sum w_i R_i - E(\sum w_i R_i)^2 \quad (12)$$

Since the Markowitz MVO technique accounts for the covariances between asset classes so as to reduce the risk of an entire portfolio (Markowitz, 1952), the variation of the portfolio with n asset classes is given by:

$$\boldsymbol{\delta_p = \sqrt{\sum w_i^2 \delta_i^2 + 2\sum\sum w_i w_j \delta_{i,j}}} \tag{13}$$

and the covariance matrix for an n asset class portfolio case is given by;

$$\boldsymbol{\delta_{i\ldots n} = \begin{bmatrix} \delta_1^2 & \delta_{n-1,1} & \delta_{n,1} \\ \delta_{1,n-1} & \delta_{n-1}^2 & \delta_{n,n-1} \\ \delta_{1,n} & \delta_{n-1,n} & \delta_n^2 \end{bmatrix}} \tag{14}$$

Where;

$$\boldsymbol{\delta_{i,j} = E\left[(R_i - \mu_i)(R_j - \mu_j)\right]} \tag{15}$$

$\boldsymbol{\delta_i^2}$ the variance of asset class *I* as descrived in equation (11) and,

The main diagonal of the matrix (14) contains the variances.

However, since in reality invetsors hold portfolios with n assets so as to exploit diversification benefits, MVO is extended to a scenario with several asset classes. According to Markowitz (1952), every investor is tasked with the problem of creating portfolios that would minimize the portfolio's risk given a certain return. This risk is constrained by the availablity of investable assets. This is mathematically represented as;

Maximize,

$$\boldsymbol{z = \frac{1}{2} X' V X} \tag{16}$$

Subject to

$$\boldsymbol{X \in \mathbb{R}/X',\ \mu = \mu_p,\ X'e = 1} \tag{17}$$

Where, $X = [X_1, X_2 \ldots X_n]'$ is a column vector of portfolio weights for each security, V is a covariance matrix for the returns and μ_p is the desired portfolio return.

In order to solve the problem above given equation (16) and (17), Taha (2007), suggests the use of a Lagrangian Function given by;

$$\boldsymbol{L(X,\lambda) = \frac{1}{2} X'VX - \lambda_1 X'e - 1 - \lambda_2 X'\mu - \mu_p} \tag{18}$$

Assuming that the first and second moments of the random variables (X) exist, the vectors are linearly independent and the covariance matrix is strictly positive definite, the solution to equation (18) gives the optimal portfolio (X*).

The optimal portfolio is;

$$\boldsymbol{X^* = V^{-1}(\lambda_{1e} + \lambda_{2\mu})} \tag{19}$$

Where the parameters λ_1 and λ_2 are given by;

$$\boldsymbol{\lambda_1 = \frac{(c - b\mu_p)}{(ac - b^2)}} \tag{20}$$

$$\boldsymbol{\lambda_2 = \frac{(a\mu_p - b)}{(ac - b^2)}} \tag{21}$$

And,

$$a = e^T V^{-1}$$

$$b = e^T V^{-1} \mu$$

$$\boldsymbol{c = \mu^T V^{-1} \mu} \tag{22}$$

3.2.3 *Comparing the Single Index and MVO Model*

To compare the performance of the single index model and the MVO model, the portfolio used for the single index model is determined using the cut off rate in equation (4) given the NSE 20 index whereas an optimization code using visual basic for applications (VBA) in excel, shown in Appendix 4, is used to determine the portfolio for the MVO model given the NSE 20 index. The optimal weights are determined using equation (5) and the code, shown in Appendix 4, for the single index and MVO respectively. Given the optimal weights, the return of the portfolios is computed using equation (8) for the single index model and equation (10) for the MVO model. The standard deviation is also computed using equation (9) and (13) for the single index model and MVO model respectively. Finally, the Sharpe ratio for the portfolio given the weights of the single index model and MVO model is determined as shown in equation (23) and (24) respectively;

$$\frac{R_p - R_f}{\sigma_p} \tag{23}$$

$$\frac{E(R_p) - R_f}{\delta_p} \tag{24}$$

Therefore, the model with the highest Sharpe ratio will be concluded as the best model.

To determine whether the performance of the models is time invariant, the period between January 2002 and December 2015 will be divided into three sub periods: 2002 to 2006; 2007 to 2011; and 2012 to 2015. The same process is used as in the first objective to see if the results will be consistent with the results obtained in the first objective.

Chapter 4: Presentation of Research Findings

The analysis is divided into four parts: (1) the whole period from 2002-2015, which responds to the research question on which model is better; and (2) research question on the consistency of the results, obtained from the first research question, will be answered given the periods, 2002-2006, 2007-2011 and 2012-2015.

4.1 Variable selection and transformation

The portfolios used in both the single index model and mean variance optimization model are based on the NSE 20 Index. For each period, the composition of the NSE 20 is based on the last year for that period, for example, for the period 2002-2006, NSE 20 will be composed of the NSE 20 stocks as at 2006. This is to show that despite the NSE 20 components, the results obtained should be consistent.

Given the stocks in each period, the closing price is used in calculating the returns as this is the last trading price of a stock. In cases where the stocks had a share split, the stock prices were adjusted by dividing the prior prices by the share split ratio before calculating the return. The mean return of each stock was then calculated.

The risk free rate, 91-day T-bill rate, is given as a nominal discount rate, thus, for the rate to be used in the analysis, the rate is converted to a monthly effective interest rate using equation (25) and (26),

$$[\mathbf{1} - \boldsymbol{d}] = \left[\mathbf{1} - \frac{\boldsymbol{d}^{(\boldsymbol{m})}}{\boldsymbol{m}}\right]^{\boldsymbol{m}} \tag{25}$$

Where, d is the effective interest rate, $d^{(m)}$ is the nominal discount rate and m is the period. Equation (25) is used to convert the nominal discount rate to an effective discount rate. Equation (26), shown below is then used to convert the effective discount rate to an effective interest rate (i), which is then divided by 12 to convert it into a monthly effective interest rate.

$$\frac{\mathbf{1}}{(\mathbf{1}-\boldsymbol{d})} = (\mathbf{1} + \boldsymbol{i}) \tag{26}$$

The descriptive statistics for the companies included in the portfolio given the single index model and the MVO model in each period are shown in part II under Appendices.

4.2 Determining the best model

In order to determine which model is better, the single index model and MVO model are compared using the Sharpe ratio, the higher the Sharpe ratio the better. The period that is used is from 2002-2015. The analysis is shown in part III under Appendices, for the single index model and part IV under Appendices for the MVO model.

4.2.1 *Full period (2002-2015)*

The single index model portfolio is based on the cut off rate as shown in part III under Appendices, whereas the MVO model portfolio is based on the solver function as shown by the VBA code in part V under Appendices, which calculates the weights by minimizing standard deviation. The use of the cut off rate and solver function result in different portfolio compositions which are used to calculate the portfolio return, portfolio standard deviation and the Sharpe ratio as shown in Table 4.1 below:

Portfolio composition for the period 2002-2015

Portfolio composition for the period 2002-2015								
	Single index model				Mean variance optimization model			
i	w_i	R_p	σ_i	SR	w_i	R_p	σ_i	SR
ARM	28.45%	2.07%	6.06%	0.2285	0.00%	1.47%	1.06%	0.7339
BAT	27.49%				0.00%			
BAMB	14.74%				63.53%			
EABL	24.38%				0.00%			
KENO	4.94%				0.00%			
BRIT	0.00%				36.47%			

Table 4.1: Comparison of the single index model and MVO model portfolios for the period 2002-2015; composition, return, standard deviation and Sharpe ratio

Given, i is the security; w_i is weight; R_p is the portfolio return; $\sigma_i$is the portfolio risk; and SR is the Sharpe ratio

The portfolio composition of the single index model is comprised of three sectors from the NSE, that is; construction and allied, ARM and BAT; energy and petroleum, KENO; and manufacturing and allied, BAT and EABL. The portfolio composition of the MVO model includes two sectors, that is: construction and allied, BAMB; and insurance, BRIT. Therefore, the single index model gives a more diversified portfolio in the period 2002-2015 compared to the MVO model as it includes more sectors as well as more stocks.

The single index model portfolio has a higher return (2.1%) than the MVO model portfolio (1.5%) but at the cost of higher risk as shown in Table 4.1.

The Sharpe ratio, which is used to decide which model is better, shows that the MVO model is better as it has a higher Sharpe ratio at 0.7 compared to that of the single index model at 0.2.

4.3 Measuring the consistency of the models

A similar process, as in the full period in section 4.2, is used in analyzing the models in the sub periods to measure the consistency of the models.

4.3.1 Period one (2002-2006)

The resulting portfolio composition, return, standard deviation and Sharpe ratio is as shown in Table 4.2:

Portfolio composition for the period 2002-2006

Portfolio composition for the period 2002-2006								
	Single index model				Mean variance optimization model			
i	w_i	R_p	σ_i	SR	w_i	R_p	σ_i	SR
BAMB	22.92%	3.90%	5.36%	0.6377	0.00%	2.84%	5.05%	0.4668
KQ	14.51%				0.29%			
EABL	27.85%				48.37%			
BOC	17.02%				0.00%			
NMG	7.13%				0.00%			
TPSE	3.46%				1.25%			
SASN	1.82%				12.15%			
BAT	5.29%				0.00%			
DTK	0.00%				6.97%			
UTKL	0.00%				30.97%			

Table 4.2: Comparison of the single index model and MVO model portfolios for the period 2002-2006; composition, return, standard deviation and Sharpe ratio

The single index model portfolio is composed of more stocks compared to the MVO model portfolio. The single index model includes four sectors in the portfolio, they include; agricultural, SASN; commercial and services, KQ, NMG and TPSE;

construction and allied, BAMB; and manufacturing and allied, BOC, BAT and EABL. The MVO model also entails four sectors in the portfolio, they include; agricultural, SASN and UTKL; banking, DTK; commercial and services, KQ and TPSE; and manufacturing and allied, EABL.

The return of the single index model portfolio (3.9%) is higher than the return of the MVO model portfolio (2.8%) but at a cost of a higher risk at 5.4% compared to 5.1%.

The single index model in the 2002-2006 period is better than the MVO model since it has a higher Sharpe ratio at 0.6 compared to that of the MVO model at 0.5.

4.3.2 *Period two 2007-2011*

The resulting portfolio composition, return, standard deviation and Sharpe ratio is as shown in Table 4.3:

Portfolio composition for the period 2007-2011

Portfolio composition for the period 2007-2011								
	Single index model				Mean variance optimization model			
i	w_i	R_p	σ_i	SR	w_i	R_p	σ_i	SR
ARM	100%	1.31%	5.20%	0.1287	0.00%	-1.68%	3.59%	-0.6437
BAMB	0.00%				34.21%			
UCHM	0.00%				65.79%			

Table 4.3: Comparison of the single index model and MVO model portfolios for the period 2007-2011; composition, return, standard deviation and Sharpe ratio

The single index model contains one sector, that is, the construction and allied, ARM whereas the MVO model includes two sectors, that is; commercial and services, UCHM; and construction and allied, BAMB. This shows that the MVO model has a more diversified portfolio compared to the single index model portfolio.

However, during the 2007-2011 period, the market performance was poor, given that the return in both models are low at 1.3% for the single index model and -1.7% for the MVO model. A negative return is not attractive to an investor even though the MVO model portfolio is more diversified than the single index model portfolio. The single index model portfolio has a higher risk at 5.2% whereas the MVO model portfolio has a 3.6% risk.

The single index model is deemed to be better since it has a higher Sharpe ratio than the MVO model. Investors are not attracted to portfolios with a negative Sharpe ratio, which is the case in the MVO model.

4.3.3 Period three (2012-2015)

The resulting portfolio composition, return, standard deviation and Sharpe ratio is as shown in Table 4.4:

Portfolio composition for the period 2012-2015

Portfolio composition for the period 2007-2011								
Single index model					Mean variance optimization model			
i	w_i	R_p	σ_i	SR	w_i	R_p	σ_i	SR
BAT	22.43%	3.42%	6.19%	0.4064	39.76%	0.026	0.027	0.6332
SCOM	77.57%				41.93%			
SASN	0.00%				18.31%			

Table 4.4: Comparison of the single index model and MVO model portfolios for the period 2012-2015; composition, return, standard deviation and Sharpe ratio

The single index model portfolio contains two stocks: one from the manufacturing and allied, BAT; and telecommunication and technology, SCOM. On the other hand, MVO model has 3 stocks from three sectors, which are; agricultural, SASN; manufacturing and allied, BAT; and telecommunication and technology, SCOM. The models have common stocks, which assures the investor that these stocks are a good investment. In this case, BAT and SCOM are good investments.

The single index model portfolio return is 3.4% at 6.2% risk, whereas the MVO model portfolio has a lower return and risk at 2.6% and 2.7% respectively.

In the period 2012-2015, the MVO model is considered better as it has a higher Sharpe ratio than the single index model at 0.6 compared to that of the single index model which is at 0.4.

Chapter 5: Discussion

The practitioners of the single index model, such as Elton, Gruber & Padberg (1976); Sarker (2013) among others, show that the mean variance optimization model is difficult to operationalize despite the MVO being a theoretically rigorous model. In this study 20 stocks are used, thus 20 estimates each are needed for the return and variance, and 190 estimates of covariances ($(n^2 - n)/2$). In the case where more stocks are put into consideration, more estimates will be needed which is time consuming and also the covariance accuracy decreases. In addition, the VBA code cannot be employed regularly unless one is using the same number of stock, if the number of stocks changes, then the code has to change as well. The single index model on the other hand requires less information as shown by Sarker (2013). The only information needed are 20 expected return, variance, alpha and beta each. The single index model can be used regularly and one uses less time to construct the optimum portfolio.

The results show that holding a well-diversified portfolio is better than holding an individual asset as was proposed in the Modern Portfolio Theory despite the model being used. This can be illustrated in Table 4.5, where the individual risks are higher than the portfolio risk when using both the single index model and MVO model.

Portfolio risk versus individual stock risk

Single index model		
Stocks	Standard deviation	Portfolio standard deviation
BAMB	10.65%	5.36%
KQ	13.14%	
EABL	8.43%	
BOC	7.70%	
NMG	12.68%	
TPSE	14.60%	
BAT	10.41%	
Mean variance optimization model		
Stocks	Standard deviation	Portfolio standard deviation
DTK	16.54%	5.05%
EABL	8.43%	
KQ	13.14%	
SASN	22.37%	
TPSE	14.60%	
UTKL	9.20%	

Table 5.1: Comparison of individual stock risk versus portfolio risk given the single index model and the mean variance optimization model for the period 2002-2006

Therefore, as shown in Table 4.5, it is better to hold a portfolio of stocks rather than individual stocks as it reduces the risk that an investor bears. The MVO model also works best if the investor is risk averse, as was assumed by Markowitz (1952). This has been shown in Table 4.1, 4.2, 4.3 and 4.4 where the single index model portfolio risk is higher than the MVO model portfolio risk. It is therefore advisable for a risk averse investor to use the MVO model and a risk lover investor to use the single index model.

It is also shown by Markowitz (1952) that the MVO model can be utilized by investors seeking to make long term investments since it conforms to the geometric mean optimization approach. This study agrees with Markowitz's findings as the MVO model performs better than the single index model in the period 2002-2015 which is a long time horizon as shown in Table 4.1. This was also shown by Briec & Kerstens (2009) who found that the MVO model outperforms the single index model in constructing optimal portfolios that are to be held over long periods.

Kamal (2012) found that, by using single index model, most of the stocks in the portfolio comprised of the bank and financial institutions. However, in this study, the portfolio mainly comprised of the construction and manufacturing companies.

Chapter 6: Conclusion and Recommendations

This study focused on comparing the relative performance of the single index model and the mean variance optimization model in equity portfolio construction in Kenya based on the NSE 20 Index. The conclusion, on which model is better, is based on the Sharpe ratio, as rational investors want to invest in portfolios with high Sharpe ratios. In the period 2002-2015, the study concludes that the MVO model is superior to the single index model as it has a higher Sharpe ratio. When looking at the sub periods, the single index model outperforms the MVO in 2 of the sub periods, that is, 2002-2006 and 2007-2011, where the single index model has higher Sharpe ratios than the MVO model.

From the results obtained, there is no clear cut on which model generally outperforms the other as there are instances where the single index model outperforms the MVO model and vice versa. However, we can conclude that the MVO model is superior when considering investments with a long time horizon, say, above 10 years, whereas the single index model is superior when considering investments with a relatively short time horizon, say, less than 7 years.

The study also shows that the single index model has higher portfolio returns than the MVO model but at the price of a higher risk. Thus, if an investor is a risk lover, then they can use the single index model to determine which stocks they can invest in, whereas a risk averse investor can use the MVO model as it has lower portfolio risk.

6.1 Limitations of the study

The conclusion of the study is based on the Sharpe ratio, which despite its advantages such as conceptual simplicity and a good comparison investment tool, has disadvantages. The main disadvantage is that it is based on normal distribution, thus in cases where there are non-normal returns making the distribution skewed, then the Sharpe ratio obtained will be inflated which may lead to false conclusion. In this study, the stocks are skewed, thus the Sharpe ratios may not be accurate.

The single index model also has a limitation as the market has a low R-squared, meaning there are explanatory variables which have been omitted that better explain the stock returns.

6.2 Further research

There is still room for further research in determining the best models to be used in determining the optimum portfolio. This is due to the following assumptions: First, the study only considers NSE 20 Index stocks, which are considered blue chip companies with big market capitalization, whereas there can be high returns in other stocks in either the NASI or NSE 25 Index. Second, only equities have been put into consideration whereas in reality there are other asset classes such as fixed income, real estate among others. Third, there are other models that can be put into consideration, for example multiples such P/E ratio and constant correlation among others.

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Appendices

I. Market indices in Kenya

NASI, which is a market cap weighted index consisting of all securities on the NSE; NSE 20 Share Index, which is a price weight index and measures the performance of 20 blue-chip companies with strong fundamentals and which have consistently returned positive financial results; NSE 25 Share Index, which was developed to provide the exchange with opportunities to develop structured products in the equities and the upcoming derivative market; FTSE NSE Kenya 15 Index, which represents the performance of the largest 15 stocks trading on the NSE, ranked by full market capitalization ; FTSE NSE Kenya 25 Index, which represents the performance of the 25 most liquid stocks trading on the NSE; FTSE NSE Kenya Govt. Bond Index, which measures the performance of Kenyan government bonds with the constituent screened on the basis of size and liquidity and include bullet bonds, unstripped strippable bonds and zero-coupon bonds denominated in KES; and FTSE ASEA Pan African Index, which represents the performance of eligible securities listed on ASEA member exchanges.

II. Descriptive statistics of the stocks contained in the portfolios in each period

STOCKS	MEAN	STD DEV	MEDIAN	MAXIMUM	MINIMUM	SKEWNESS	KURTOSIS	WHITE TEST	DURBIN-WATSON STAT	
SINGLE INDEX MODEL 2002-2015 DESCRIPTIVE STATISTICS										
ARM	0.0240	0.1137	0.0098	0.5603	-0.2103	1.2751	3.2656	homoskedastic	1.7674	no autocorrelation
BAT	0.0166	0.0829	0.0072	0.4310	-0.2613	1.1255	4.8314	homoskedastic after adjustments	1.7986	no autocorrelation
BAMB	0.0144	0.0833	0.0000	0.3913	-0.2000	1.2504	3.6373	homoskedastic	1.8618	no autocorrelation
EABL	0.0174	0.0828	0.0193	0.2605	-0.2695	-0.0604	1.4493	homoskedastic after adjustments	2.1755	no autocorrelation
KENO	0.0146	0.1106	0.0055	0.5462	-0.3277	0.9968	3.9076	homoskedastic	2.2551	no autocorrelation
MVO MODEL 2002-2015 DESCRIPTIVE STATISTICS										
BAMB	0.0144	0.0833	0.0000	0.3913	-0.2000	1.2617	3.6373	homoskedastic	1.8618	no autocorrelation
BRIT	0.0151	0.1346	0.4490	0.4490	-0.2535	0.8837	1.4727	homoskedastic	1.9777	no autocorrelation
SINGLE INDEX MODEL 2002-2006 DESCRIPTIVE STATISTICS										
BAMB	0.0462	0.1065	0.0226	0.3913	-0.1351	1.0515	1.1801	homoskedastic	1.9342	no autocorrelation
KQ	0.0464	0.1314	0.0123	0.4860	-0.1758	1.3295	2.3675	homoskedastic	1.6789	no autocorrelation
EABL	0.0394	0.0843	0.0364	0.2549	-0.1379	0.2769	-0.1776	homoskedastic	2.0616	no autocorrelation
BOC	0.0288	0.0770	0.0000	0.2792	-0.0815	1.4869	1.7614	homoskedastic after adjustment	1.4081	no autocorrelation
NMG	0.0363	0.1268	0.0276	0.5096	-0.3443	0.8425	4.0605	homoskedastic after adjustment	2.0551	no autocorrelation
TPSE	0.0280	0.1460	0.0063	0.6111	-0.2832	1.2285	3.2829	homoskedastic	2.5177	no autocorrelation
SASN	0.0388	0.2237	0.0000	1.2857	-0.2200	3.3314	15.5577	homoskedastic	1.7202	no autocorrelation
BAT	0.0269	0.1041	0.0091	0.4310	-0.2613	1.1038	4.1836	homoskedastic	1.5894	no autocorrelation
MVO MODEL 2002-2006 DESCRIPTIVE STATISTICS										
DTK	0.0379	0.1654	0.0294	0.7321	-0.2586	1.1851	3.7589	homoskedastic after adjustment	2.1599	no autocorrelation
EABL	0.0394	0.0843	0.0364	0.2549	-0.1379	0.2842	-0.1776	homoskedastic	2.0616	no autocorrelation
KQ	0.0464	0.1314	0.0123	0.4860	-0.1758	1.3644	2.3675	homoskedastic	1.6789	no autocorrelation

STOCKS	MEAN	STD DEV	MEDIAN	MAXIMUM	MINIMUM	SKEWNESS	KURTOSIS	WHITE TEST	DURBIN-WATSON STAT	
SASN	0.0388	0.2237	0.0000	1.2857	-0.2200	3.4189	15.5577	homoskedastic	1.7202	no autocorrelation
TPSE	0.0280	0.1460	0.0063	0.6111	-0.2832	1.2608	3.2829	homoskedastic after adjustment	2.5177	no autocorrelation
UTKL	0.0046	0.0920	-0.0055	0.2963	-0.1439	1.1265	1.6257	homoskedastic	1.8395	no autocorrelation
SINGLE INDEX MODEL 2007-2011 DESCRIPTIVE STATISTICS										
ARM	0.0113	0.0845	0.0000	0.2857	-0.1706	0.7628	1.2637	homoskedastic	2.5106	no autocorrelation
MVO MODEL 2007-2011 DESCRIPTIVE STATISTICS										
BAMB	-0.0098	0.0597	-0.0050	0.2083	-0.2000	0.0820	3.3106	homoskedastic after adjustment	1.4299	no autocorrelation
UCHM	-0.0107	0.0437	0.0000	0.0556	-0.1940	-2.9891	9.3624	homoskedastic	1.7006	no autocorrelation
SINGLE INDEX MODEL 2012-2015 DESCRIPTIVE STATISTICS										
BAT	0.0237	0.0778	0.0140	0.25	-0.1303	0.6061	1.2162	homoskedastic	2.2479	no autocorrelation
SCOM	0.0352	0.0627	0.0491	0.1428	-0.1246	-0.4451	-0.0112	homoskedastic	1.888	no autocorrelation
MVO MODEL 2012-2015 DESCRIPTIVE STATISTICS										
BAT	0.0237	0.0778	0.0140	0.25	-0.1303	0.6263	1.2162	homoskedastic	2.2479	no autocorrelation
SASN	0.0098	0.0764	0.0062	0.2935	-0.1055	1.2794	2.7049	homoskedastic	2.2425	no autocorrelation
SCOM	0.0352	0.0627	0.0491	0.1428	-0.1246	-0.4599	0.0627	homoskedastic	1.888	no autocorrelation

Table 0.1: Descriptive statistics of the stocks in the portfolios constructed using the single index model and MVO model in each period

Skewness is a measure of asymmetry. A data set, or distribution, is symmetric if it looks the same to the left and right of the center point. The skewness of a standard normal distribution is 0, thus if a data set has a positive value, it is skewed to the right and if it has a negative value, it is skewed to the left (Pearson, 1895). Kurtosis is a measure of whether the data are heavily-tailed or light-tailed relative to a normal distribution. The kurtosis of a standard normal distribution is 0, thus if the kurtosis is positive, the data set will have a heavy tail and if the kurtosis is negative, the data set will have a light tail (Cisar & Cisar, 2010) . The kurtosis and skewness are used to determine the distribution of the stock returns as shown by a histogram, for example, the distribution of ARM is shown in Figure 0.1 Most of the stock returns are found to have a non-normal distribution.

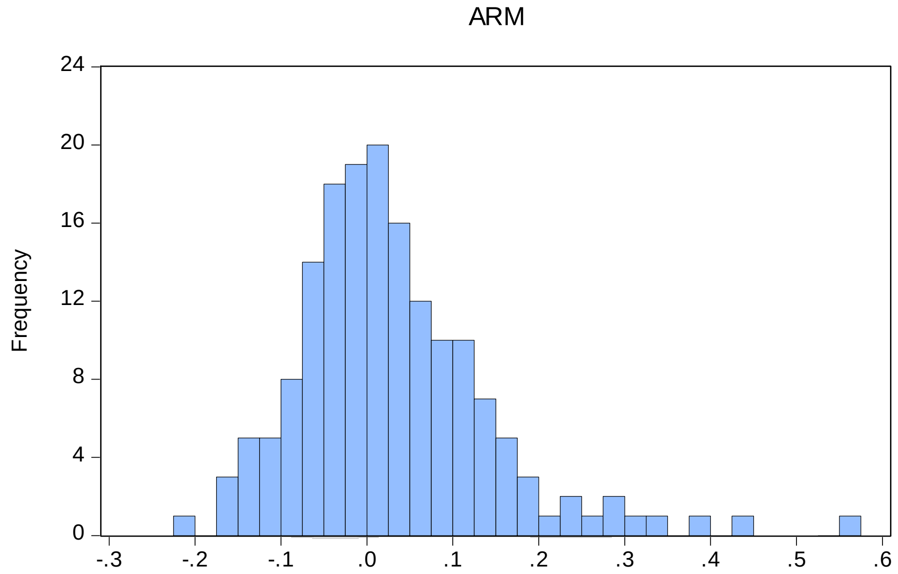

Figure 0.1: ARM stock return distribution for the period 2002-2015

Pre-estimation tests were also carried out for each stock in the portfolio. The white test was used to test for heteroskedasticity using the Eviews software. If the null hypothesis is rejected, then there is presence of heteroskedasticity. To correct heteroskedasticity, HAC Newey-West is used and thus the standard error and probability change, and the error terms will become homoskedastic after the adjustment. Most stock returns were found to be homoskedastic, for the returns that were heteroskedastic, they were corrected using the HAC Newey-West. Thus the variance of the residuals was constant throughout the periods. The Durbin-Watson statistic is used to determine whether the error terms are autocorrelated using the Eviews software. If the statistic is near 2, then there is no autocorrelation, if the statistic is near 0, then there is positive autocorrelation and if the statistic is near 4, then there is negative autocorrelation. All the stock returns exhibited no autocorrelation.

III. Single Index Model Portfolio Selection

							A_i	B_i	C_i	
STOCKS	$\bar{R}_i$	α_i	$\boldsymbol{\sigma_i^2}$	β_i	σ_e^2	$(\overline{R}_i - R_f)/\beta_i$	$[(\overline{R}_i - R_f)\beta_i]/\sigma_e^2$	β_i^2/σ_e^2	$(\sigma_m^2 * Sum\ A_i)/[1 + (\sigma_m^2 * Sum\ B_i)]$	C^*
ARM	0.0240	0.0238	0.01292	0.7055	0.0110	0.0242	1.0952	45.2085	0.0036	0.0085
BAT	0.0166	0.0160	0.00687	0.4482	0.0061	0.0218	0.7163	32.9181	0.0054	
BAMB	0.0144	0.0132	0.00694	0.5273	0.0059	0.0143	0.6789	47.3822	0.0065	
EABL	0.0174	0.0141	0.00686	0.7871	0.0045	0.0134	1.8579	138.3140	0.0083	
KENO	0.0146		0.01223	0.6938	0.0104	0.0111	0.5167	46.3900	0.0085	
KCB	0.0195	0.0146	0.01748	1.4841	0.0090	0.0085	2.0730	244.3885	0.0085	
ICDC	0.0157		0.02258	1.2128	0.0169	0.0073	0.6324	86.9107	0.0084	
NMG	0.0130		0.01315	0.8970	0.0101	0.0068	0.5461	79.9475	0.0083	
CFC	0.0133		0.02053	0.9760	0.0169	0.0065	0.3693	56.4713	0.0082	
BRIT	0.0151		0.01812	1.3041	0.0116	0.0063	0.9270	146.7512	0.0079	
SASN	0.0110		0.02861	1.0896	0.0241	0.0038	0.1875	49.3634	0.0078	
EQTY	0.0127	0.0236	0.02524	1.7219	0.0138	0.0034	0.7285	214.1740	0.0071	
KPLC	0.0110		0.02611	1.5640	0.0167	0.0026	0.3859	146.3563	0.0067	
SCOM	0.0088	0.0142	0.00825	1.0908	0.0037	0.0018	0.5757	323.6195	0.0059	
SCBK	0.0075		0.00622	0.7265	0.0042	0.0009	0.1085	125.8429	0.0056	
COOP	0.0063		0.00923	1.0372	0.0051	-0.0005	-0.1116	210.9039	0.0050	

Table 0.2: Determining the components of the portfolio (2002-2015)

The average return ($\bar{R}_i$) is obtained from the average of the monthly returns of the stocks from 2002-2015. α and β are obtained from running regressions using Eviews software. The unsystematic risk (σ_e^2) is calculated from subtracting the systematic risk ($\beta_i^2\sigma_m^2$) from the total risk (σ_i^2). The risk free rate is 0.69% and the variance of the market is 0.0038.

The same procedure is used in determining the portfolio components of the other sub periods: 2002-2006; 2007-2011; and 2012-2015.

IV: The mean variance optimization model portfolio selection

	ARM	BAMB	BAT	BBK	BRIT	CFC	COOP	EABL	EQTY	ICDC	KCB	KEGN	KENO	KPLC	KQ	NMG	SASN	SCAN	SCBK	SCOM
ARM	0.007	0.004	0.002	0.005	0.006	0.004	0.005	0.003	0.005	0.004	0.005	0.005	0.003	0.004	0.004	0.002	0.002	0.005	0.004	0.002
BAMB	0.004	0.004	0.002	0.004	-0.003	0.001	0.003	0.002	0.004	0.003	0.003	0.002	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.002
BAT	0.002	0.002	0.003	0.003	0.000	0.003	0.001	0.002	0.004	0.003	0.003	0.001	0.002	0.004	0.001	0.002	0.000	0.001	0.003	0.000
BBK	0.005	0.004	0.003	0.011	0.002	0.005	0.005	0.003	0.007	0.007	0.007	0.005	0.004	0.007	0.004	0.004	0.008	0.005	0.005	0.005
BRIT	0.006	-0.003	0.000	0.002	0.011	0.005	0.006	0.000	0.006	0.011	0.003	0.002	0.000	0.001	0.007	0.002	0.002	0.000	0.003	0.003
CFC	0.004	0.001	0.003	0.005	0.005	0.015	0.006	0.003	0.008	0.005	0.006	0.002	0.006	0.010	0.004	0.002	0.002	0.005	0.004	0.004
COOP	0.005	0.003	0.001	0.005	0.006	0.006	0.008	0.003	0.005	0.003	0.004	0.001	0.004	0.006	0.003	0.002	0.001	0.003	0.003	0.003
EABL	0.003	0.002	0.002	0.003	0.000	0.003	0.003	0.034	0.006	0.003	0.005	0.004	0.003	0.005	0.004	0.004	0.000	0.004	0.003	0.004
EQTY	0.005	0.004	0.004	0.007	0.006	0.008	0.005	0.006	0.012	0.013	0.010	0.006	0.003	0.013	0.005	0.007	0.007	0.009	0.006	0.007
ICDC	0.004	0.003	0.003	0.007	0.011	0.005	0.003	0.003	0.013	0.012	0.006	0.005	0.002	0.007	0.005	0.005	0.010	0.009	0.005	0.006
KCB	0.005	0.003	0.003	0.007	0.003	0.006	0.004	0.005	0.010	0.006	0.008	0.006	0.004	0.014	0.004	0.005	0.004	0.006	0.006	0.007
KEGN	0.005	0.002	0.001	0.005	0.002	0.002	0.001	0.004	0.006	0.005	0.006	0.018	0.003	0.008	0.004	0.004	0.003	0.003	0.003	0.005
KENO	0.003	0.002	0.002	0.004	0.000	0.006	0.004	0.003	0.003	0.002	0.004	0.003	0.024	0.005	0.006	0.002	0.002	0.002	0.002	0.004
KPLC	0.004	0.003	0.004	0.007	0.001	0.010	0.006	0.005	0.013	0.007	0.014	0.008	0.005	0.012	0.007	0.004	0.004	0.008	0.005	0.006
KQ	0.004	0.002	0.001	0.004	0.007	0.004	0.003	0.004	0.005	0.005	0.004	0.004	0.006	0.007	0.014	0.002	0.003	0.003	0.002	0.006
NMG	0.002	0.003	0.002	0.004	0.002	0.002	0.002	0.004	0.007	0.005	0.005	0.004	0.002	0.004	0.002	0.024	0.004	0.006	0.003	0.003
SASN	0.002	0.002	0.000	0.008	0.002	0.002	0.001	0.000	0.007	0.010	0.004	0.003	0.002	0.004	0.003	0.004	0.004	0.005	0.003	0.005
SCAN	0.005	0.002	0.001	0.005	0.000	0.005	0.003	0.004	0.009	0.009	0.006	0.003	0.002	0.008	0.003	0.006	0.005	0.012	0.003	0.003
SCBK	0.004	0.002	0.003	0.005	0.003	0.004	0.003	0.003	0.006	0.005	0.006	0.003	0.002	0.005	0.002	0.003	0.003	0.003	0.002	0.002
SCOM	0.002	0.002	0.000	0.005	0.003	0.004	0.003	0.004	0.007	0.006	0.007	0.005	0.004	0.006	0.006	0.003	0.005	0.003	0.002	0.007

Table 0.3: The variance-covariance matrix (2002-2015)

STOCKS	Weights	Returns by Asset Class	Portfolio return	$\sum_{i=1}^{n} w_i^2 \delta_i^2$	$2 * \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \delta_{i,j}$	Portfolio Standard Deviation	Sharpe Ratio
ARM	0.00%	2.40%		0.00	0.00		
BAMB	63.53%	1.44%		0.0014	-0.0007		
BAT	0.00%	1.66%		0.00	0.00		
BBK	0.00%	-0.28%		0.00	0.00		
BRIT	36.47%	1.51%		0.0014	-0.0007		
CFC	0.00%	1.33%		0.00	0.00		
COOP	0.00%	0.63%		0.00	0.00		
EABL	0.00%	1.74%		0.00	0.00		
EQTY	0.00%	1.27%		0.00	0.00		
ICDC	0.00%	1.57%		0.00	0.00		
KCB	0.00%	1.95%		0.00	0.00		
KEGN	0.00%	-1.41%		0.00	0.00		
KENO	0.00%	1.46%		0.00	0.00		
KPLC	0.00%	1.10%		0.00	0.00		
KQ	0.00%	-0.28%		0.00	0.00		
NMG	0.00%	1.30%		0.00	0.00		
SASN	0.00%	1.10%		0.00	0.00		
SCAN	0.00%	0.46%		0.00	0.00		
SCBK	0.00%	0.75%		0.00	0.00		
SCOM	0.00%	0.88%		0.00	0.00		
SUM	100.00%		1.47%	0.28%	-0.27%	1.06%	0.73

Calculate weights by minimising standard deviation

Table 0.4: MVO portfolio selection (2002-2015)

The same procedure is used in determining the portfolio components of the other sub periods: 2002-2006; 2007-2011; and 2012-2015.

V. Visual Basic for Applications code used to calculate the weights by minimizing the standard deviation

```
Sub OptPort2()
    Range("sdresult") = "=((V75+V76)^0.5)"
    SolverReset
    SolverOk SetCell:="$V$77", MaxMinVal:=2, ValueOf:=0, ByChange:="$B$72:$U$72", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverAdd CellRef:="$V$72", Relation:=2, FormulaText:="1"
    SolverAdd CellRef:="$B$72:$U$72", Relation:=3, FormulaText:="0"
    'SolverAdd CellRef:="V$73", Relation:=3, FormulaText:="$I$3"
    SolverSolve UserFinish:=True
    SolverFinish KeepFinal:=1
    Range("A1").Select

End Sub
```

```
Sub OptPort2()

SolverReset

   SolverReset
    SolverOk SetCell:="$V$78", MaxMinVal:=1, ValueOf:=0, ByChange:="$B$72:$U$72", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverAdd CellRef:="$V$72", Relation:=2, FormulaText:="1"
    SolverAdd CellRef:="$B$73:$U$73", Relation:=3, FormulaText:="0"
    SolverOk SetCell:="$V$78", MaxMinVal:=1, ValueOf:=0, ByChange:="$B$72:$U$72", _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    SolverOk SetCell:="$V$78", MaxMinVal:=1, ValueOf:=0, ByChange:="$B$72:$U$72", _
        Engine:=1, EngineDesc:="GRG Nonlinear"


SolverSolve UserFinish:=True
SolverFinish KeepFinal:=1
Range("A1").Select

End Sub
```


Figure 0.2: VBA code for calculating MVO weights

VI. Portfolio stock definition

ARM - Athi River Mining

BAMB - Bamburi Cement Ltd.

BAT - British-American Tobacco Kenya Ltd.

BOC - B.O.C Kenya Ltd.

BRIT - British-American Investments Company (Kenya) Ltd.

DTK - Diamond Trust Bank Kenya Ltd.

EABL - East African Breweries Ltd.

KENO - KenolKobil Ltd.

KQ - Kenya Airways Ltd.

NMG - Nation Media Group

SASN - Sasini

SCOM - Safaricom Ltd.

TPSE - TPS Eastern Africa (Serena) Ltd.

UCHM- Uchumi Supermarket Ltd.

UTKL - Unilever Tea Kenya Ltd.