



**THE CONSISTENCY OF CREDIT RISK PRICING IN EMERGING
MARKET SOVEREIGN DEBT: THE RELATIONSHIP BETWEEN
BOND PRICES AND CREDIT DEFAULT SWAP SPREADS**

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Table of Contents

Abstract.....	5
1 Introduction.....	6
1.1 Background of the Study.....	6
1.2 Problem Statement	10
1.3 Research Objectives	10
1.4 Research Questions	11
1.5 Research Justification.....	11
2 Literature Review	12
2.1 Determinants of Sovereign Credit Risk	12
2.2 Pricing of Credit Risk.....	13
2.3 The Link between Bond Prices and CDS Spreads: Empirical Studies	15
2.4 Research Gap	17
2.5 Conceptual Framework	18
3 Data and Methodology.....	19
3.1 Research Design.....	19
3.1.1 Population and Sampling.....	19
3.1.2 Data Collection	20
3.2 Data Analysis	20
3.2.1 The Reduced Form Pricing of Credit Risk	20
3.2.2 Pre-estimation Tests: The ADF Test for Stationarity	24
3.2.3 Testing the Relationship: The Johansen Test	25
3.2.4 Testing the Existence of a Relationship.....	27
4 Results and Analysis	28
4.1 Data Description.....	28
4.1.1 Data Trend	28
4.1.2 Descriptive Statistics.....	29

4.2	Volatility Structure.....	30
4.2.1	Testing for ARCH Effects: The White (1980) Test.....	30
4.2.2	Variation in Volatility Across Time	32
4.3	Empirical Results	33
4.3.1	Pre-estimation Tests.....	33
4.3.2	Main Results	35
5	Discussion and Implications	36
5.1	Summary of Findings	36
5.1.1	Answers to Research Questions.....	36
5.2	Discussion	37
5.2.1	The Increase in the Debt Spread and the CDS Premium.....	37
5.2.2	The Relationship Between the Debt Spread and the CDS Premium	40
5.3	Implications for Participants in Sovereign Debt Markets	43
5.3.1	Policy Implications for Sovereign Debt Issuers	43
5.3.2	Trading Implications for Investors	44
6	Conclusion	45
7	Bibliography	47
8	Appendices.....	51

Abstract

Theoretical foundations indicate that, under no-arbitrage conditions, a relationship between bond prices and Credit Default Swap prices should be observed. This study examines whether this relationship is evident in emerging market sovereign debt. By performing Johansen cointegration analysis on the debt spread and the CDS premium, mixed evidence is found: a relationship exists in some time periods but not in others. This implies inconsistent pricing of credit risk in some periods and is in line with previous empirical studies. Therefore, there exist opportunities for credit risk arbitrage although profits from these opportunities may be restricted by the limits to arbitrage. In addition, the effect of bond market volatility on the relationship between the debt spread and the CDS premium is analysed. It is found that volatility due to idiosyncratic factors does not affect the existence of the relationship. Finally, a qualitative analysis of the behaviour of credit risk in light of economic and political conditions provides some insights for macroeconomic policy makers in emerging market economies. Potential extensions to this study include empirical analysis on a different sample of sovereign emerging market securities and extension of existing credit risk pricing models.

1 Introduction

1.1 Background of the Study

1.1.1 Fixed Income Securities and Their Risks

A fixed income security allows a party to borrow from another. This entails the borrower (the issuer of the fixed income security) receiving some payment today and promising to make some future payments to the lender (the holder of the fixed income security). Entities seeking to receive finance in the form of fixed income securities are typically governments and corporate entities. Government debt is often referred to as sovereign debt.

Fixed income securities carry some risks with them. According to Fabozzi & Mann (1983), these include, among others: liquidity risk, market risk and credit risk. Liquidity risk is the risk that the investor will be unable to sell a bond or have to sell a bond below its true value where the true value is indicated by a recent transaction. Market risk is the possibility of an increase in interest rates causing a capital loss. Credit risk is the risk that either the issuer will default on its obligation or the issuer's credit worthiness will reduce resulting in a decrease in the value of the security (Fabozzi & Mann, 1983). Compensation for these risks is provided in the pricing of the fixed income security (Berndt & Obreja, 2007).

Broadly, the risks affecting securities issued by governments and corporate issuers fall into these categories. However, due to the nature of the issuing entities, the fundamental determinants of credit risk are different in corporate and governmental issuers. For corporate issuers these factors include earnings, debt-to-cash-flow and others (Demerjian, 2007). For sovereign issuers, the factors include real GDP growth, fiscal balance and government stability (Csonto & Ivaschenko, 2013). This study focusses on credit risk.

Fear of the risks inherent in fixed income securities has led financial market participants to innovate ways of mitigating them. Interest rate hedging has been practiced to reduce market risk. Credit default swaps have been used to mitigate credit risk.

1.1.2 Mitigation of Credit Risk – Credit Default Swaps

To mitigate credit risk, credit default swap contracts (CDS contracts) have been introduced. These are insurance contracts in which one party pays a price so that they will be compensated if and when the issuer of a particular security defaults. The party seeking protection (the CDS buyer) pays the party offering protection (the CDS seller) a fee of P_{CDS} basis points a year (the CDS price or spread) per dollar of notional principal protected against. The CDS seller is then contractually bound to compensate the CDS buyer in case the issuer of the underlying security (referred to as the reference entity) defaults on its obligation (Hull J. C., 1997). This compensation is often actualised by the CDS seller buying the security that is in default from the CDS buyer at a predetermined price. Naturally, the credit risk of the issuer should be priced into the price of protection (Hull & White, 2000). Figure 1 below summarises the structure of a CDS contract (figure originally by Hull J. C. (1997)).

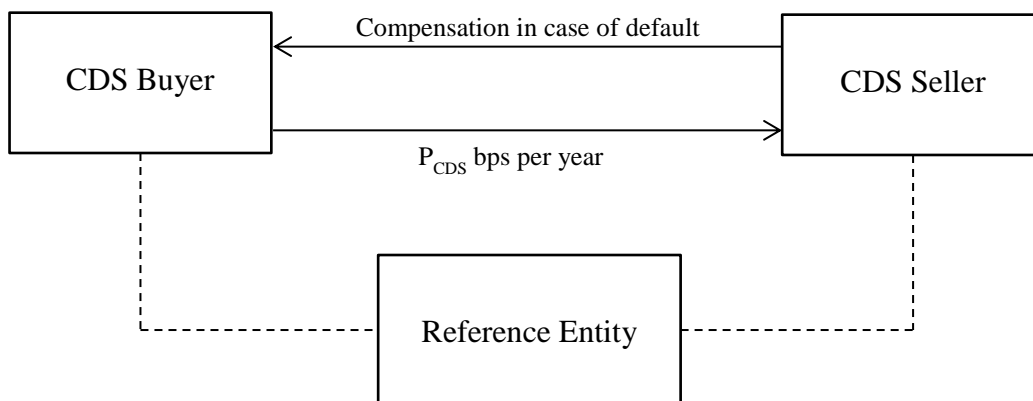


Figure 1: Credit Default Swap Contract Structure

The credit derivatives market has experienced rapid growth and, among credit derivatives, the credit default swap (CDS) has become a widely traded instrument. According to the Bank for International Settlements, by the end of 2014, the total notional amount of outstanding CDS contracts was greater than \$16 trillion (Bank for International Settlements, 2015).

1.1.3 Trends in the Sovereign Fixed Income Market

The market for fixed income securities has been growing rapidly for the past few decades. According to Greenwood & Scharfstein (2013), between 1980 and 2007, the American market for fixed income assets grew from 57% to 182% of GDP. This expansion has been observed elsewhere in the world with governments increasingly seeking money from financial markets to finance spending. In emerging market

economies, domestic sovereign debt markets have grown markedly since the mid-1990s and currently represent one of the governments' main sources of financing (Jaramillo & Weber, 2012).

However, some countries seeking to raise capital face challenges in raising the money from within the country. A primary challenge has been the presence of small and undeveloped financial markets incapable of supporting significant borrowing (Christensen, 2005). This challenge and others have seen a new trend emerge: African countries have been seeking substantial amounts of debt from the global financial markets. Vellos (2015) noted that between 2011 and 2014, twelve African nations (Kenya included) went to the international markets for debt finance. Sovereign debt issues are usually denominated in 'hard currency'¹ and this means that debt from emerging and frontier economies is now being considered (along with developed market debt) by investors in hard currency fixed income securities.

1.1.4 The Risk of Sovereign Default in Emerging and Frontier Markets

Due to elevated default risk, emerging and frontier market debt has offered higher returns than developed market debt (Jaramillo & Weber, 2012). These higher have enticed investors with several sovereign debt issues being met with oversubscription.

However, anxiety over the vulnerability of emerging market issuers to default still poses a challenge to investors seeking to take advantage of the higher returns. In light of the current global economic conditions² that pose economic challenges to some emerging market issuers, credit risk is a key concern for investors. Mozambique is as a prime example of an issuer experiencing sovereign debt challenges and that has defaulted on some of its obligations³. Ivory Coast's 2011 sovereign default is still fresh in investors' memory⁴. Even issuers with a better economic history have been victims. South Africa had its debt put on downgrade watch by Moody's in March 2016⁵. Issuers outside Africa have also faced increased credit risk. Brazil is also a notable example⁶.

¹ Defined by Goldfajn & Rigobon (2000) as a currency that is frequently used as an international means of payment and reserve value.

² Including low commodity prices and a slowdown in the Chinese economy

³ Bloomberg News - Mozambique Bond Yield Jumps to Record on Debt-Payment Default (18th May 2016)

⁴ Bloomberg News - Ivory Coast Defaults on Eurobonds, Pledges to Pay (1st Feb 2011)

⁵ https://www.moody.com/research/Moodys-places-South-Africas-Baa2-ratings-on-review-for-downgrade-- PR_344855

⁶ Bloomberg News - Brazil Credit Ratings Cut to Junk by Moody's (25th Feb 2016)

Investors may choose to shield themselves from the elevated risk of default by purchasing credit default swaps.

Therefore, the investor has a choice to make. On one hand, an investor can choose to take on credit risk and receive compensation in the form of a higher expected return. On the other hand, they can choose to protect themselves from this risk by purchasing a credit default swap and thereby sacrifice some of their return.

1.1.5 Efficiency and Arbitrage in the Market for Sovereign Credit Risk

So, the credit risk of an issuer of a fixed income security is priced in two different markets: the market for the security (where investors are rewarded in the form of higher yields for taking on credit risk) and the market for credit default swaps (where investors pay for protection against credit risk).

The fact that the same factor is being priced in two different locations introduces the possibility of mispricing. This exposes the possibility of riskless profit for an investor who buys in the cheap location and sells in the expensive location. This would violate the no-arbitrage condition and hence the principle of market efficiency.

For riskless profits (arbitrage) to be non-existent, the Law of One Price must hold (Lamont & Thaler, 2003). The Law of One Price states that identical goods must have identical prices (Lamont & Thaler, 2003). While the law was originally formulated in the context of trade in the real economy, it has been extended to financial markets through the no arbitrage principle. Sharpe & Alexander (1990) define arbitrage as the simultaneous purchase and sale of the same, or essentially similar, security in two different markets for advantageously different prices. Theoretically speaking, arbitrage requires no capital and entails no risk (Shleifer & Vishny, 1997).

If mispricing exists, arbitrage will be possible and this would violate the implication of the Law of one price to financial markets: identical securities (that is, securities with identical state-contingent payoffs) must have identical prices (Lamont & Thaler, 2003).

1.2 Problem Statement

Consistent pricing is no doubt important to modern financial theory: it implies the no-arbitrage condition and forms the basis of a lot of modern financial theory (Lamont & Thaler, 2003). When it comes to credit risk, there is the same state-contingent payoff (a negative one) being traded in two different markets simultaneously: the bonds market and the CDS market. This creates the possibility of inconsistent pricing and thus violation of the no-arbitrage condition. For pricing to be consistent there must be an observed link between prices of the underlying security and CDS spreads.

Past studies such as those by Blanco, Brennan, & Marsh (2003) and Ammer & Cai (2007) have been in agreement that while pricing deviates considerably in the short run (particularly in crises and periods of volatility), there is a stable long-term equilibrium relationship. Hence, arbitrage opportunities may exist in the short run. However, previous studies either deal with corporate issuers or cover a period that is far in the past. There is therefore lack of clarity on the current state of the markets for the credit risk of emerging sovereign debt issuers.

Given this lack of clarity and the importance of pricing consistency to modern financial theory, there is need to investigate whether the pricing of emerging market sovereign credit risk is consistent across the bond market and the CDS market. Consistency implies that there is a relationship between bond prices and CDS spreads.

1.3 Research Objectives

This research aims:

- To test whether there is an observable relationship between bond prices and CDS spreads.
- To test whether volatility in the bond market affects the relationship between bond prices and CDS spreads.

1.4 Research Questions

In light of the research problem, this study compares the pricing of credit risk in the market for the sovereign fixed income security and the market for a credit default swap on the security. The study answers the following questions:

- 1) Is there an observable relationship between bond prices and CDS spreads?
- 2) Does bond market volatility affect the relationship between bond prices and CDS spreads?

1.5 Research Justification

This research is relevant to investors in sovereign debt markets as it provides a better understanding of the extent to which and consistency with which credit risk is priced by the market. This could potentially expose arbitrage or mispriced hedging opportunities that could be taken advantage of in the future. Academia also stands to gain from this research as the pricing of credit risk in security prices is explored and as the larger discussion on market efficiency and arbitrage is contributed to.

2 Literature Review

There is extensive literature on the topic of credit risk pricing in general and the consistency of its pricing in particular. The foundations of the topic of this study are clear. Empirical studies have shown that the factors affecting credit risk for sovereign issuers are the same. These factors include idiosyncratic and systematic risk factors. In totality, the effects of risk from both sources have been observed in both the CDS and the debt market. With regard to the consistent pricing of these risks, theory shows that under no-arbitrage conditions there is equivalence between the prices of credit risk in both markets (with the prices of credit risk being regarded as the debt spread⁷ and the CDS spread). Empirical tests have shown that the no-arbitrage relationship holds in the long run but short run deviations have been observed.

This section is organised as follows: first the determinants of credit risk for sovereign issuers are discussed, the theoretical foundations of credit risk pricing are then reviewed, and finally literature concerning the consistency in credit risk pricing is critiqued.

2.1 Determinants of Sovereign Credit Risk

Literature on the determinants of sovereign default risk is extensive. Generally, the determinants are both local and international variables. The effects of these variables are evident in both the debt market and the CDS market. The work by Longstaff, Pan, Pedersen, Singleton, & Kenneth (2011) and by Csonto & Ivaschenko (2013) provides a good illustration of the effects of various variables on credit risk in a sovereign context.

Longstaff, Pan, Pedersen, Singleton, & Kenneth (2011) studied the pricing of credit risk in sovereign bonds between 2000 and 2010. They considered the credit default swap spreads of 26 developed and emerging market sovereign issuers and, using a correlation approach, found substantial co-movement between them. Using regression analysis, they found that the sources of this co-movement are attributable to both local factors and international financial variables with local factors contributing an average of 43% of CDS spread variation. Default risk and macroeconomic factors are the main contributing local factors. Global financial variables include US stock market returns, VIX volatility and market risk premia. Using an affine sovereign credit valuation

⁷ The difference between the yield on a risky bond and the yield on a similar risk free bond

model, they also find that on average 33% of the CDS spread is caused by a risk premium due to the default intensity (probability) of an issuer.

In a similar study, Csonto & Ivaschenko (2013) considered the effect of global and country specific factors on the debt spreads of 18 emerging market issuers between January 2001 and March 2013. For country specific variables, they used risk indicators of the International Country Risk Guide (ICRG) database⁸ (this includes economic, financial and political risk). For global factors, they used proxies for risk aversion and liquidity). Using a fixed effects model, they find that both sets of factors play a role over the long term. However, short term volatility is mainly a factor of global factors. In addition, they find that countries with stronger economic fundamentals tend to be less sensitive to global factors (particularly risk attitudes). They are therefore less volatile in the short term. Thus, the risk of default is a key factor in determining debt spreads. When accurate information on country-specific factors is lacking, global factors tend to drive changes in spreads (this sometimes leads to mispricing). In line with their findings, they show that more than half of the tightening of credit spreads during the pre-crisis period was driven by an improvement in global and country-specific factors which led to decreased probability of default.

The findings of Longstaff, Pan, Pedersen, Singleton, & Kenneth (2011) and Csonto & Ivaschenko (2013) separately show the determinants of debt spreads and CDS spreads respectively. These determinants are both local and international factors with the economic reality of the issuer (and thus the risk of default) playing a key role in both prices. These studies lay the foundation for the foundation for the economic equivalence of credit risk prices in the bond and CDS markets. This is through their common determination by similar real-world economic factors. These factors are largely those which affect the issuer's probability of default.

2.2 Pricing of Credit Risk

While this study is not concerned with finding the economically justifiable price of credit risk, an overview of literature the subject will help contextualise and motivate this research. Generally, there are two approaches to pricing credit risk: structural form models and reduced form models⁹.

⁸ <https://www.prsgroup.com/about-us/our-two-methodologies/icrg>

⁹ The definitions are discussed in detail by Kao (2000)

In structural form models, the debt of an issuer is treated as a contingent claim to the issuer's assets. This entails pricing credit risk based on the financial fundamentals of the issuer. This approach is founded on the work by Black & Scholes (1973) and Merton (1974).

The second approach (reduced form models) bypasses the issuer's financial fundamentals and links the price of credit risk to bond spreads. It is based on no-arbitrage and risk neutral conditions. Reduced form models are discussed by Duffie (1999) and Hull & White (2000). Given the relative nature of this study, reduced form models are of primary concern.

2.2.1 Reduced Form Models

Duffie (1999) discussed a reduced form model based on the no-arbitrage approach. He assumed that there are no restrictions on short selling, no taxes and no transaction costs. Considering reference obligation maturing at par (and a CDS protecting against its issuer's default), an investor can take a short position of \$100 in the obligation at time $t=0$ and invest this amount in a risk free bond. Consequently, the investor receives r_f from the risky bond and has the obligation to pay:

$$r_f + S$$

Where: S is the spread of the risky reference obligation over the risk free rate).

Duffie (1999) also assumed no transaction costs and perfect CDS contract specification. In case of no credit event, the bonds both mature at par and there is no cash flow requirement from the investor. In case a credit event occurs, the investor sells the risk free bond at the coupon date immediately following the event for \$100 (it can be shown that this is its price at this time) and buys the reference obligation for its market value V . Their net cash flow will therefore be:

$$D = 100 - V$$

Based on this Duffie (1999) illustrates that under risk neutral conditions in a perfect market the price of a CDS should be equal to the debt spread. This is a formal statement of the theoretical consistency of credit risk pricing. It has formed the foundation of numerous studies including this one and those mentioned in Section 2.3.

Extending the work by Duffie (1999), Hull & White (2000) derived a reduced form model for CDS valuation. The initial assumption was that the possibility of default is the only reason why a bond trades above the risk-free rate. Accordingly, the price

difference is equal to the present value of the expected loss from default. Based on this, the reduced form model price of a credit default swap is found to be a function of the probability of a credit event occurring during the life of the obligation. In their discussion on default probabilities, Hull, Predescu, & White (2005) showed that the debt spread is a function of probabilities¹⁰. These two papers strengthen the theoretical equivalence stated by Duffie (1999).

2.3 The Link between Bond Prices and CDS Spreads: Empirical Studies

The equivalence between the debt spread the CDS spread that Duffie (1999) demonstrated has been investigated by a number of researchers in various contexts. Empirical findings have largely been in agreement: there is long term equivalence between the two variables but short term deviations from the no-arbitrage position have been observed.

2.3.1 Corporate Issuers

Blanco, Brennan, & Marsh (2003) empirically studied this relationship and in a sample of 33 US and European investment firms between January 2011 and June 2012. They found evidence of the theoretical equivalence of debt spreads and CDS spread. However, the relationship does not hold true over the entire period studied. When differences in pricing occur, it is shown that the credit spread is the lower bound of the price of credit risk while the CDS spread is the upper bound. Furthermore, it is shown that credit price discovery occurs in the CDS market (as it better adjusts to changes in short term fundamentals) with adjustment occurring later in the bond market. This is attributed to the fact that price discovery will occur in the market where informed traders trade most. Despite their short-term differences, both variables are shown to reflect fundamentals in the long run.

In a similar study, Zhu (2004) studied 24 investment grade corporate issuers between January 1999 and December 2002. He found that there is evidence of long term co-movement with short term deviation. The short-term deviations are attributed to differences in the response to credit conditions. As in the study by Blanco, Brennan, & Marsh (2003), the CDS market was found to lead the bond market.

¹⁰ The discussions by Hull & White (2000) and Hull, Predescu, & White (2005) will be revisited in more detail in Section 3.2.1.

Celic (2012) studied 32 corporate issuers between 2010 and 2011. With regard to the relationship between debt spreads and CDS spreads, his findings were consistent with prior work: the no-arbitrage relationship holds reasonably well over the long and medium term but deviates significantly in the short term. The CDS market is also shown to lead the bonds market. This relationship is weaker for lower graded entities and reverses during times of crisis. He suggests two explanations for this: “first, trading in investment-grade bonds might increase during times of crisis such that bond prices provide more information, second, increased counterparty risk inherent in CDS might disturb CDS spreads such that the information value of CDS prices is decreased”.

To test the relationship in times of economic distress, Bai & Collin-Dufresne (2011) investigated variation in the difference between CDS spreads and debt spreads for a sample of 484 firms during the financial crisis. The entire period of the study is from January 2006 to September 2009. This was a period in which there was high bond market volatility. They found that during the crisis the arbitrage relationship was violated. While the findings are unable to point at a single cause of this, several factors are shown to be contributors. These include funding risk, counterparty risk, and collateral quality. They also investigate the lead-lag relationship between debt spreads and CDS spreads and find that the share of price-discovery occurring in the CDS market falls significantly during the crisis. This change was more pronounced for high yield issuers some of which showed the debt spread leading the CDS spread in the crisis period. This is attributed to the effect of deleveraging during and after the crisis of the debt spread. Deleveraging causes price discovery in the bond market through the pricing anew of debt obligations (this is related to the first reason stated by Celic (2012)).

2.3.2 Sovereign Issuers

When it comes to sovereign issuers, the findings are similar to those of corporate issuers. A stable long run relationship is observed with short term deviations. As with corporate issuers, the CDS market tends to lead in price discovery.

Ammer & Cai (2007) studied sovereign emerging market issuers when they examined nine emerging market sovereign borrowers between 2001 and 2005. In line with findings on corporate debt, they showed the presence of a stable long-term equilibrium relationship for most countries. However, there were considerable short term deviations. CDS premiums often moved ahead in the short-term time horizon (to a

larger extent than is implied by the earlier findings of Blanco, Brennan, & Marsh (2003)). It is also found that the liquidity of the two markets relative to each other is a key factor in determining which market moves first. While the study by Ammer & Cai (2007) was conducted on sovereign emerging market issuers, it is set in a different global macroeconomic environment. Moreover, we are now living in a world after the global financial crisis of 2007-2008 whose effects were exacerbated by CDSs (Harrington (2009)) hence the dynamics of the CDS market may have changed.

Coudert & Gex (2010) studied 18 sovereign issuers and 17 financial sector corporate issuers between January 2007 and March 2010. The sovereign issuers included both emerging and developed market issuers. As Ammer & Cai (2007), they found that there is a long-term relationship between the two. Causality tests indicate that short run interactions are bidirectional (the two variables cause each other). However, in the long run the CDS market leads the bond market in terms of price discovery. The results also highlighted that the CDS market's lead was exacerbated by financial stress in emerging markets and Southern Europe.

2.4 Research Gap

Existing literature provides a good theoretical foundation for a relationship between debt prices and CDS spreads. Numerous studies such as the one by Ammer & Cai (2007) have tested whether this holds true and findings have largely been consistent: there is a long-term relationship with short term deviations.

However, these studies have either been done on corporate issuers or cover a period that is significantly in the past (they only cover the period until 2012). Moreover, they have focussed on the relationship between the two prices of credit risk without much attention to whether the relationship holds true in cases where there is volatility in the market for the issuer's security. This study addressed these gaps by covering emerging market debt over a more recent time period and by considering whether volatility has an impact on the relationship between bond prices and CDS spreads.

2.5 Conceptual Framework

This study contributes to the existing body of work that tests the relationship between the debt spread and the CDS spread. This and past research is primarily based on testing the relationship that Duffie (1999) theoretically showed (under several assumptions which are not necessarily true in the real world). The relationship is theoretically supported by Hull & White (2000) and Hull, Predescu, & White (2005) who showed that both variables should be affected by default probabilities. Further evidence to supports Duffie’s claim comes from Longstaff, Pan, Pedersen, Singleton, & Kenneth (2011) and Csonto & Ivaschenko (2013) whose findings empirically show that the debt spread and the CDS spread have similar real-world determinants.

The place of this study within existing literature is summarised in figure 2 below.

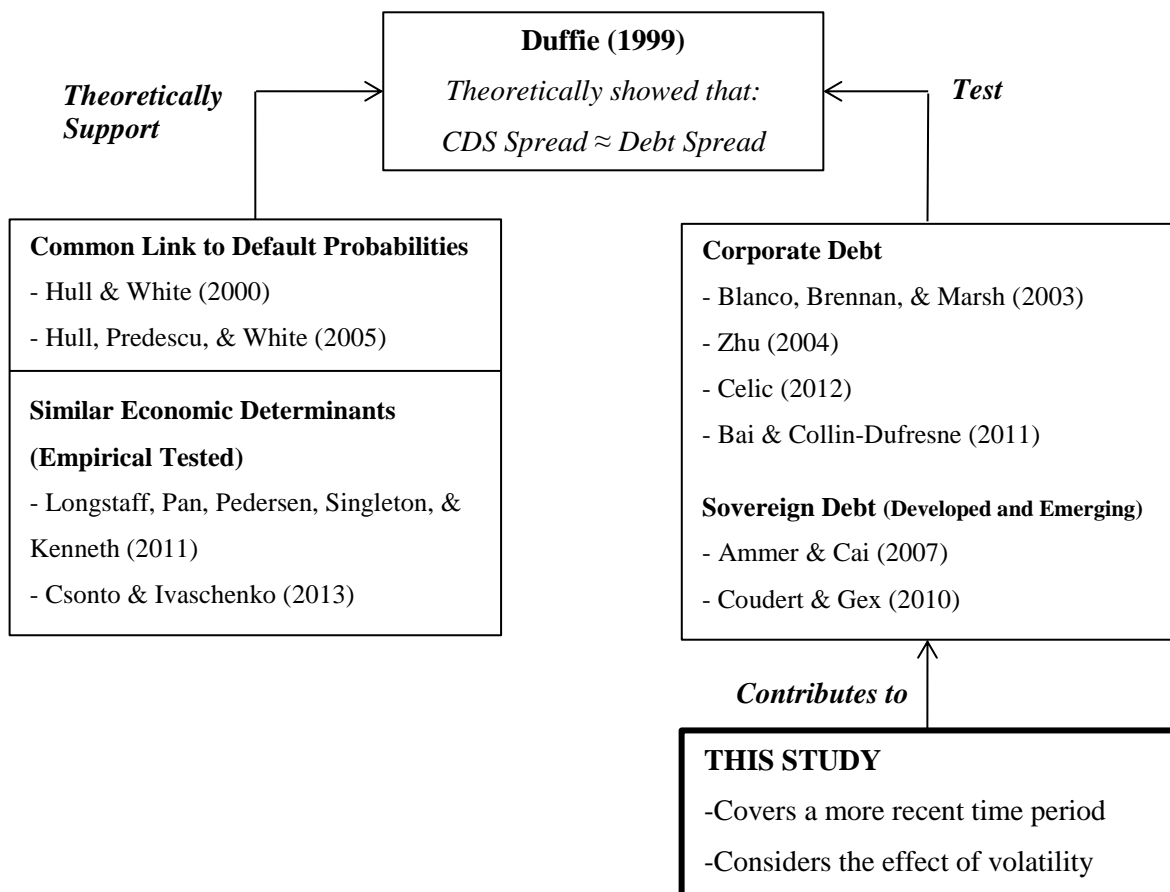


Figure 2: Conceptual Framework

3 Data and Methodology

3.1 Research Design

Due to the research gap that exists in the field, this study considered the relationship between the price of a bond (observed as the debt spread) and the price of a credit default swap that protects the buyer from the bond issuer's default. The research looked at the extent to which there has been co-movement between these two variables. This implies the use of quantitative analysis on secondary data.

3.1.1 Population and Sampling

The ideal population for a study of the pricing consistency between emerging market sovereign bonds and the CDS protecting against the issuers' default would be the all emerging market sovereign bonds with CDS contracts written on them. As of May 2013, this population included 1,075 securities issued by 50 entities (Vanguard Research, 2013). However, collecting and analysing all this data would be time consuming. Therefore, this study used a sample. The sampling technique used was convenient sampling: choosing securities which are compatible (for the sake of comparison), are well representative of the market for emerging market sovereign bonds and are influenced interesting economic realities.

For simplicity and to clearly bring out the arbitrage implications of price consistency, the study considered one bond from one emerging market sovereign issuer: Brazil. The choice of issuer is based on the fact that it has had a recent past that has been very volatile but a relatively calm long term history. This provides a sample that spans a wide range of conditions. The study used a security that covers the aforementioned periods of calmness and volatility. This enhances its representativeness and allows the study to answer both questions with one data set. The study therefore considered the Brazilian sovereign bond issued on 22 January 2010 and maturing on 22 January 2021. This security pays a coupon of $4\frac{7}{8}$ and is denominated in USD.

For the CDS contract, the study uses the credit default swap index maturing in January 2021. This matches the maturity of the bond and therefore forms a representative basis for comparison.

3.1.2 Data Collection

As both bonds and credit default swaps are traded, the values of their prices are market determined and fluctuate over time. Therefore, in comparing them the study compared two different financial time series variables over time.

The data required was the time series data of the variables being studied. Given the empirical nature of the data to be used, objective secondary data sources were used. This is in line with earlier studies discussed in Section 2.3. The data was sourced from Bloomberg and is daily between 22nd April 2010 to 15th March 2016.

3.2 Data Analysis

The analysis is based on comparing the price fluctuations of a bond and a CDS that protects from the issuer's default. Given the multiple measures of a bond's price (such as market price and yield to maturity) the question then becomes: which specific variables should be compared? This study compares the debt spread and the CDS spread in accordance with Duffie's (1999) proof.

Credit risk is accounted for in the pricing of both bonds and CDS contracts as both are functions the probability of future default. Section 3.2.1 demonstrates this. As a result, it is inferred that at a given probability of default (at any particular point time), the debt spread and the CDS spread should have be related as they are both on the same entity. As probability changes over time, they should both change in a similar manner.

3.2.1 The Reduced Form Pricing of Credit Risk

Credit Risk in Bond Pricing

Hull & White (2000) assumed that the possibility of default is the only factor that causes a corporate bond to sell at a price less than a similar risk free security is (this implies that all other risks are identical for both bonds). The assumption implies that equation (1) below holds.

$$G_0 - B_0 = \text{Present Value of Cost of Defaults} \dots \dots \dots (1)$$

Where:

B_0 =Price of the risky bond today

G_0 = Price of the bond today if there were no probability of default (the price of a risk free bond)

Now consider a risky bond that matures at time T . It makes payments at some times t where: $0 < t < T$ and let:

$v(t)$ = Present value (at time j) of \$1 received at time t with certainty

$C(t)$ = Claim by holders of the bond in the event of a default at time t

$R(t)$ = Recovery rate for holders of the bond in the event of a default at time t

α_t = Present value (at time j) of the loss, relative to the value the bond from a default on the bond at time t

p_t = The risk-neutral probability of default at time t

From the initial assumption above, about the reason for price differential between the bond and the risk free security, the present value of defaults is the sum of the expected losses given default. Formally, Hull & White (2000) stated this at some time j such that $0 < j < t < T$ as:

$$G_j - B_j = \sum_{t=0}^T [p_t * \alpha_j] \dots \dots \dots (2)$$

It is then possible to incorporate the recovery rate (the proportion of the bond value that bond holders will be able to recover in case of default). This is similar to what was done by Hull & White (2000). This gives:

$$G_j - B_j = \sum_{t=j}^T [p_t * X_t * (1 - R(t))v(t)] \dots \dots \dots (3)$$

Take expected recovery rate on the reference obligation in a risk-neutral world and denote it as $\tilde{\mathcal{R}}$, as Hull & White (2000) did. This is assumed to be independent of the time of the default. Over continuous time the price of a risky bond at time j is given by:

$$B_j = G_j - \int_j^T p_t * X_t * (1 - \tilde{\mathcal{R}}) v(t) dt \dots \dots \dots (4)$$

So, at a given deterministic interest rate and a reliable estimate of default the recovery rate, the price of a risky bond has a negative relationship to the probabilities of default during the life of the bond. This implies that the yield has a positive relationship to default probabilities.

The value of a bond is the value of its future payments discounted at its yield. Taking $w(t)$ to be the present value (at time j) of \$1 promised to be received from the issuer at time t and X_t to be the expected payment from the bond at time t :

$$B_j = \int_j^T X_t * w(t) dt \dots \dots \dots (5)$$

Incorporating a similar expression for the risk-free bond, we can rewrite equation 4 as:

$$\int_j^T X_t * w(t) dt = \int_j^T K_t * v(t) dt - \int_j^T p_t * X_t * (1 - \tilde{\mathcal{R}}) v(t) dt \dots \dots \dots (6)$$

Recall that the two securities are identically structured hence: $X_t = K_t$ Taking r_t as the spot rate on the risky bond the discount factor from time t to time 0 is given by:

$$w(t) = \left(\frac{1}{1 + r_t} \right)^{t-j} \dots \dots \dots (7)$$

So, taking the risk-free spot rate to be r_{fj} , equation (6) can be written as:

$$\int_j^T \frac{1}{(1 + r_t)^{t-j}} dt = \int_j^T \frac{1}{(1 + r_{fj})^{t-j}} dt - \int_j^T p_t * (1 - \tilde{\mathcal{R}}) v(t) dt \dots \dots \dots (8)$$

Taking the risk-free rate (r_{fj}) and the recovery rate $\tilde{\mathcal{R}}$ as exogenous, if the probability of default p_t increases the yield on the bond will also increase. Replacing the exogenous some exogenous parameters a_t , b and c_t gives:

$$\int_j^T \frac{1}{(1 + r_t)^{t-j}} dt + \int_j^T [p_t * b * c_t] dt = \int_j^T a_t dt \dots \dots \dots (9)$$

It can be seen that the yield of the risky bond is positively related to probabilities of future default. Hull, Predescu, & White (2005) summarised this relationship through the average default intensity per year (the average probability of default per year for the remaining life of the bond, \bar{p}_t). This is a function of the difference between the yield of the risky bond and the risk-free bond (known as the debt spread). Taking r_j and r_{fj} as the yields to maturity of the two bonds, under risk-neutrality¹¹:

$$\frac{r_j - r_{fj}}{1 - \tilde{\mathcal{R}}} \approx \bar{p}_t \dots \dots \dots (10)$$

Rearranging gives:

$$r_j - r_{fj} \approx \bar{p}_t (1 - \tilde{\mathcal{R}}) \dots \dots \dots (11)$$

¹¹ This is demonstrated to be a good approximation true under risk neutrality by (Jorion, 2011). See Appendix I

Credit Risk in CDS Pricing

According to Hull & White (2000) the value of the credit default swap to the buyer is the present value of the expected payoff minus the present value of the payments made by the buyer. Based on this, they showed that assuming risk neutrality and no counterparty risk:

$$P_{CDS,j} = \frac{\int_0^T [1 - (\tilde{\mathcal{R}} + A(t)\tilde{\mathcal{R}})] [q(t)v(t)] dt}{\int_0^T [u(t) + e(t)] q(t) dt + \pi u(T)} \dots \dots \dots (12)$$

Where:

q(t): Risk-neutral default probability density at time t

u(t): Present value of payments at the rate of \$1 per year on payment dates t= 0 and t =T

e(t): Present value of an accrual payment at time t equal to t-t where t* is the payment date immediately preceding time t*

π: The risk-neutral probability of no credit event during the life of the swap

A(t): Accrued interest on the reference obligation at time t as a percent of face value

This equation implies that the price of a CDS is primarily determined by probabilities of default (*q(t)* and *π*). This is because *v(t)*, *u(t)* & *e(t)* are discount rates and therefore exogenously determined, *A(t)* varies uniformly over the life of the bond and is not subject to unexpected change, and *tilde{mathcal{R}}* is a time-invariant constant.

The Link between the Debt Spread and the CDS Spread

The above provides the foundation of most empirical analysis in this area of study: the price of a credit default swap and the spread of the underlying bond are both determined primarily by the probabilities of future default. Therefore, there should be some observed relationship between them.

The study tested the relationship between the price of the CDS index and the spread of a risky bond over a similar risk-free bond (shown by Duffie (1999)). This implies a comparison of the market observations (the left-hand sides) of equations 11 and 12¹²:

$$X_t = r_j - rf_j \quad \text{and} \quad Y_t = P_{CDS,j} \dots \dots \dots (13)$$

For the benchmark risk-free rate, this study used the LIBOR swap rate. This is due to the fact that the LIBOR rate is a commonly used benchmark for derivative pricing (Hull J. C., 1997). It was used by Blanco, Brennan, & Marsh (2003) and others.

¹² A more rigorous comparison was shown by Celic (2012). The conclusion is similar.

3.2.2 Pre-estimation Tests: The ADF Test for Stationarity

Stationarity of time series can be considered in two senses: the strict and the weak sense. A time series M_t is said to be strictly stationary if its joint probability distribution is invariant under time shift. A time series is said to be weakly stationary if its mean, variance and auto covariance are time invariant (Brooks, 2008). This study considered weak stationarity. This is because testing strict stationarity is empirically difficult.

The stationarity of time series has implications for what methods of estimation and testing can be applied on it. A non-stationary time series will not be subject to the central limit theorem making some estimation techniques such as Ordinary Least Squares estimation inappropriate.

A non-stationary time series, which can be transformed to a stationary time series by differencing is said to be difference stationary. If it needs to be differenced k times it is said to be integrated of order k (it is $I(k)$). The Augmented Dickey-Fuller (ADF) test described by Brooks (2008) can be used to test a series' stationarity and order of integration. It is given by:

$$\Delta M_t = \psi M_{t-1} + \sum_{i=1}^p [\delta_i \Delta M_{t-i}] + e_t \dots \dots \dots (14)$$

Where:

M_t is some time-varying stochastic process

p is the number of lags in the Augmented Dickey-Fuller Test

It can be concluded that the time series M_t is non-stationary if $\psi = 0$.

The appropriate number of lags (p) may be determined by the Schwartz Bayesian Information Criterion (SBIC) which is given by:

$$SBIC = T \ln(SSR) + N \ln(T) \dots \dots \dots (15)$$

Where:

T is the number of usable residuals

SSR is the sum of squared residuals

N is the total number of observations

The number of lags chosen will be that which minimises the value of the SBIC. This ensures the use of a model that best fits the data and minimises autocorrelation.

Whether two series are stationary or non-stationary has implications for the method that can be best used the relationship between them (Johansen, 1991). As a result, this study tested the stationarity of both the variables (Y_t and X_t) before estimation.

Applying the ADF test with aid of the SBIC criterion, the hypothesis below was tested:

$H_0: \psi = 0$ Variable is non-stationary

The variable has a unit root such that is integrated of order k i.e. $I(k)$; k is an integer > 0

$H_1: \psi \neq 0$ Variable is stationary

The variable is integrated of order 0 ie $I(0)$

(We Reject H_0 when: test statistic $>$ Dickey-Fuller Critical Value)

3.2.3 Testing the Relationship: The Johansen Test

There are several ways to test whether there is a relationship between the two non-stationary time series. Two common methods are the Engle & Granger (1987) two step method and the Johansen (1991) cointegration test.

The Engle & Granger (1987) method is based on employing the Augmented Dickey-Fuller test to determine whether the residuals obtained from an OLS regression of the two variables is stationary. A stationary residual term is an indicator of cointegration. This method is not appropriate for this study because it is likely that the debt spread and the CDS spread, like many time series, are likely not be stationary or be stationary of different orders. This would make the Engle & Granger method difficult to structure. Moreover, the original method consists of assuming a linear relationship between the two variables. Although the estimators are consistent, their asymptotic distribution theory is non-normal (it is skewed). As a result, inference of structural hypotheses is difficult (Stock, 1987).

Due to the limitations of the Engle & Granger (1987) method, the study's test for cointegration was based the Johansen (1991) test.

The Johansen Cointegration Test

This approach to testing for cointegration¹³ begins from a multivariate autoregression model. Consider an $n \times 1$ vector V_t of n potentially endogenous variables within the unrestricted vector autoregression (VAR) model:

$$V_t = \sum_{i=0}^k A_i V_{t-i} + u_t \dots \dots \dots (16)$$

Where: k is the number of lags

u_t is a $n \times 1$ vector of residuals

A_i is a $n \times n$ matrix of parameters to be estimated

Each V_t is regressed on lagged values of itself and all other variables being considered. This VAR model allows testing of cointegrating relationships without making assumptions about the structural form of the relationship or the exogeneity of variables. The VAR model can be expressed in the following vector error-correction model (VECM):

$$V_t = \Pi \Delta V_{t-1} + \sum_{i=1}^{k-1} \Pi_i \Delta V_{t-i} + u_t \dots \dots \dots (17)$$

The matrix Π comprises of parameters to be estimated and can also be expressed in terms of the vector of adjustment parameters α (an $n \times 1$ vector) and the matrix of cointegrating vectors β (an $n \times 1$ vector).

$$\Pi = \alpha \beta' \dots \dots \dots (18)$$

The n coefficients in the cointegrating vector β' multiply the variables to give the linear combinations of variables that do not have a unit root $\beta' X_{t-1}$. The n coefficients in alpha are the adjustment coefficients which multiply the cointegrating relationship $\beta' X_{t-1}$ to give the response of the variables in the two equations to deviations of the cointegrating relationship from zero.

If Π is a matrix of zeroes, the variables are not cointegrated. If the variables are cointegrated, then $\text{rank}(\Pi) \neq 0$ [$\text{rank}(\Pi)$ is in fact the number of cointegrating vectors].

¹³ This illustration shown is heavily influenced by Dwyer (2015). Johansen (1991) provides a more rigorous derivation.

There are two tests for this: the maximum eigenvalue test and the trace test. Both of these have their foundations in likelihood ratios and require a calculation of n eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$.

- i. The maximum eigenvalue tests whether the rank of the matrix Π is zero. The null hypothesis is that $\text{rank } \Pi = 0$ and the alternative hypothesis is that $\text{rank } \Pi = 1$. The test statistic is:

$$LR(r_0, r_0 + 1) = -T \ln(1 - \lambda_{r_0+1}) \dots \dots \dots (19)$$

Where: T is the sample size

- ii. The trace test tests whether the rank of the matrix Π is r_0 . The null hypothesis is that $\text{rank}(\Pi) = 0$ while the alternative hypothesis is that $r_0 < \text{rank}(\Pi) \leq n$. The test statistic is:

$$LR(r_0, n) = -T \sum_{i=r_0+1}^n \ln(1 - \lambda_i) \dots \dots \dots (20)$$

The Johansen cointegration test thus provides a clear method for testing whether there is a relationship between the debt spread and the CDS spread.

3.2.4 Testing the Existence of a Relationship

In order to facilitate the testing of the relationship under a wide range of conditions and thereby answer the research questions, the sample is split into 10 sub-periods of 150 consecutive trading days each. This is similar to the approach taken by Bai & Collin-Dufresne (2012) who split their study period into pre, during and post crisis periods.

- To test whether there is an observable relationship between the variables, the study tests for Johansen cointegration between the variables over the entire period of the study (April 2010 to March 2015) and in each of the ten sub-periods. Statistical inference using p-values discussed by Mackinnon, Haug & Michelis (1999) determines whether a relationship exists.
- To test whether the existence of a relationship is affected by volatility, the study compares the statistical significance of the cointegrating relationships across the sub-periods (which are shown to differ in terms of the volatility of the variables).

4 Results and Analysis

4.1 Data Description

The data used for this study were numerical values of the closing trading price of the debt spread (using the LIBOR of matching maturity as a benchmark) and the CDS Premium. The data was sourced from Bloomberg and covers the trading days 22nd April 2010 to 15th March 2016 and is daily. This includes 1501 observations.

4.1.1 Data Trend

From an initial glance, it seems as though the two series have a common trend. This conforms to the theory that implies the existence of a relationship between the variables. Chart 1 below¹⁴ shows the trend of the data.

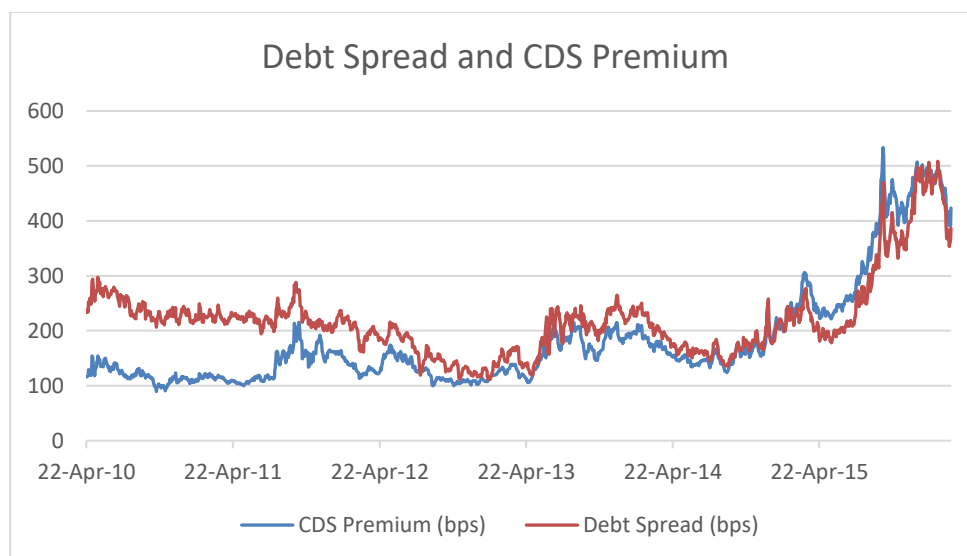


Chart 1: Debt Spread and CDS Premium (Basis Points)

A notable trend is observed in both series. The debt spread and CDS premium remained relatively low (with slight increases and decreases) for a prolonged period of time. They then experienced a rapid increase beginning in late 2014. High levels persisted until the end of the sample period.

The objective of this study is to empirically test whether a relationship between the two variables exists and whether its existence varies depending on the volatility present in the bond market. Before conducting an in-depth examination of the relationship between the debt spread and the CDS premium and the influence of volatility on this relationship, it is necessary to better understand the variation within the data.

¹⁴ Appendix II contains charts of the debt spread and the CDS premium over each sub-sample. In each of the subsamples, the charts display some degree of co-movement between the time series.

4.1.2 Descriptive Statistics

Table 1 below shows the mean and standard deviation of both time series. It is apparent that both mean and volatility vary across time. Both variables seem to have co-movement between their summary statistics (i.e. higher mean in the debt spread occurs at the same time as higher mean in the CDS premium).

150 Trading Days Ending	Debt Spread (bps)		CDS Premium (bps)	
	Mean	Standard Deviation	Mean	Standard Deviation
17-Nov-10	247.84	21.55	121.31	14.92
15-Jun-11	226.07	7.65	111.06	5.20
11-Jan-12	223.45	19.48	151.69	25.81
8-Aug-12	190.77	22.72	139.88	14.70
13-Mar-13	137.29	14.70	113.69	9.12
16-Oct-13	188.30	36.51	161.07	31.52
23-May-14	210.33	26.81	178.20	18.99
30-Dec-14	167.89	19.48	156.19	20.28
4-Aug-15	215.38	22.03	246.38	26.37
15-Mar-16	391.90	70.19	433.08	55.54
Entire Period	220.15	71.95	181.59	95.86

Table 1: Mean and Standard Deviation of the Debt Spread and CDS Premium

Both the mean and standard deviation of the two variables fell within a wide range over the sample period. This theoretically indicates that credit risk varied significantly over the period with higher risk being most present after August 2015.

The varying nature of the mean and volatility indicate that the two series are most likely non-stationary.

4.2 Volatility Structure

Of particular concern to this study is the volatility of the data. This is because one of the aims of the study is to investigate the relationship between the debt spread and the CDS premium under different volatility regimes. For this to be tested, the data must exhibit different volatility regimes. A precondition to the existence of different volatility regimes is variation in volatility across the time period.

Table 1 above indicates that volatility is not constant over the period. Debt spread volatility varies between a low of 7.65 bps and a high of 70.19. Similarly, the CDS Premium seems to have varying volatility. This indicates that volatility of both series changes over time. Tests for Autoregressive Conditional Heteroskedasticity (ARCH) effects can confirm whether this is statistically true.

4.2.1 Testing for ARCH Effects: The White (1980) Test

Consider the AR (1) Ordinary Least Squares regression equation of a variable M.

$$M_t = c_1 + \theta M_{t-1} + e_t \dots \dots \dots (21)$$

To test for ARCH effects, the test statistic is computed from the OLS regression of squared error terms from the above equation. This produced the auxiliary regression equation

$$e_t^2 = c_2 + \phi e_{t-1}^2 + u_t \dots \dots \dots (22)$$

Hypothesis Test

H_0 : There are no ARCH effects (Volatility is not time varying)

$$TS = \text{Test Statistic} = n * R^2$$

Where: n = Number of observations in the auxiliary regression,

R^2 = R – squared from the auxiliary equation

$$CS = \text{Critical Value} = \chi_{\alpha}^2(1)$$

Reject H_0 if: $TS > CV$

ARCH Test Results

Regressing Equation 22 above for both the debt spread and the CDS premium over 1500 adjusted observations results in the results in Table 2 below. The R-squared in the auxiliary regression for the debt spread is 0.037 for the debt spread and 0.043 for the CDS premium.

Included observations: 1500 after adjustments				
Debt Spread				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	55.54	5.15	10.76942	0.00
EX2(-1)	0.19	0.03	7.574296	0.00
R-squared	0.036885	F-statistic	57.37	
CDS Premium				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	28.97	3.17	9.12	0.00
EY2(-1)	0.21	0.03	8.17	0.00
R-squared	0.042685	F-statistic	66.79	

Table 2: Auxiliary Regression of the Debt Spread

Based on the results in Table 2 above, the test statistics are:

For the debt spread:

$$Test\ Statistic = n * R^2 = 1500 * 0.036885 = 55.33$$

For the CDS premium:

$$Test\ Statistic = n * R^2 = 1500 * 0.042685 = 64.03$$

Critical Value at 95 % Confidence:

$$\chi^2_{\alpha}(1) = 3.841$$

For both variables, the test statistic is greater than the critical value therefore the null that there are no ARCH effects is rejected.

The conclusion is therefore that ARCH effects are present in the data i.e. volatility in both time series is time-varying.

This means that the data indeed spans periods of different volatility. As a result of this, it is possible to study the relationship between the debt spread and the CDS premium under different volatility environments.

4.2.2 Variation in Volatility Across Time

The foregoing ARCH tests indicate that the volatility of both series of data varies over time. Chart 2 below shows the 10-day moving standard deviation of the two series. Both clearly have time-varying volatility. This visually confirms the results obtained from the empirical test. Volatility clustering is also apparent in the chart.

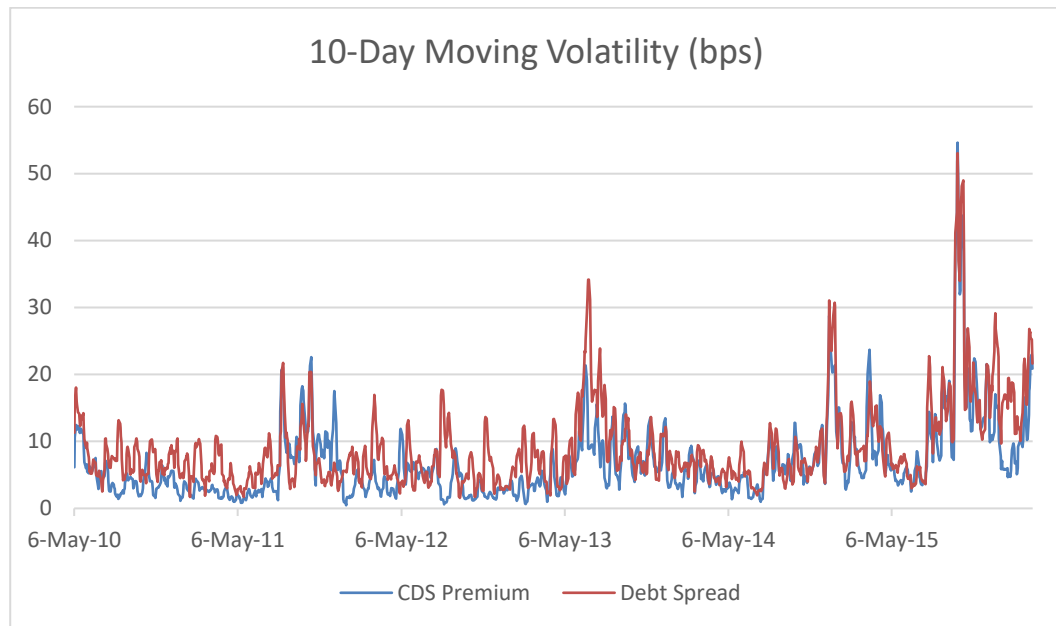


Chart 2: 10 Day Moving Standard Deviation

There are notable periods in which there is higher volatility than in other periods. These periods include the end of 2014 and mid-2015 onwards. These were the same periods in which there was a rapid increase in both the debt spread and the CDS premium. This implies that in periods of high volatility high levels of debt spread and CDS premium are also observed.

4.3 Empirical Results

4.3.1 Pre-estimation Tests

4.3.1.1 Testing for Stationarity: The ADF Test for a Unit Root

The results in Table 1 indicate that stationarity is not likely to be present in the data. The Augmented Dickey-Fuller (ADF) test empirically tests whether a time series is stationary (Dickey & Fuller, 1979).

Whether two series are stationary influences which is the most appropriate method for testing the relationship between the two. Non-stationarity limits the applicability of the Engle & Granger (1987) method of testing for cointegration (Stock, 1987).

The decision criterion for the ADF Test is given by:

H_0 : The series has a unit root (It is non – stationary)

Reject H_0 if: $|Test\ Statistic| > |Critical\ Value|$

Table 3 below contains the results obtained from ADF tests on both time series.

Augmented Dickey-Fuller Test		
Test critical values: 1% level		-3.434502
5% level		-2.863261
10% level		-2.567735
Debt Spread		
Lag Length: 1 (Automatic - based on SIC, maxlag=23)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.905888	0.3298
CDS Premium		
Lag Length: 5 (Automatic - based on SIC, maxlag=23)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.046083	0.9531
* MacKinnon (1996) one-sided p-values		

Table 3: ADF Test Results

Results for both the debt spread and the CDS premium indicate that we fail to reject the null hypothesis (at even a 10% significance level) and therefore conclude that both the series are non-stationary. This is in line with previous research such as the work by Blanco, Brennan, & Marsh (2003) and Celic (2012).

This means that the Johansen (1991) test is more appropriate for testing the cointegrating relationship between the two variables.

4.3.1.2 A Preliminary Test for Cointegration

First, a cointegration test that covers the entire sample period is run. The findings in Table 4 below indicate that the debt spread and the CDS premium are not cointegrated over the period (at a 90% confidence level).

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized		Trace	0.10	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None	0.006371	9.923935	13.42878	0.2866
At most 1	0.000238	0.356367	2.705545	0.5505
Trace test indicates no cointegration at the 0.10 level				
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypothesized		Max-Eigen	0.10	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None	0.006371	9.567568	12.29652	0.2419
At most 1	0.000238	0.356367	2.705545	0.5505
Max-eigenvalue test indicates no cointegration at the 0.10 level				
**MacKinnon-Haug-Michelis (1999) p-values				

Table 4: Entire Sample Cointegration Test Results

In order to more clearly understand the dynamics of the interaction of the two variables and thereby enhance robustness, this study splits the period into ten sub-periods as discussed in Section 3.2.4 of this research. A test for cointegration is then run on each period. Comparing the p-values in each sub-period with the significance level indicates whether a cointegrating relationship exists. Comparing p-values across different levels of volatility indicates whether the existence of a relationship is influenced by volatility.

Before delving into these results, the following issues should be paid attention to:

- Debt spread and CDS Premium volatility are highly correlated (they have a correlation coefficient of 0.94). As a result, the decision of which to use to describe market volatility is somewhat arbitrary. This study uses debt spread volatility.
- Lütkepoh & Saikkone (2001) found that while the Trace Test tends to produce more distorted sizes in small samples, its power is superior to the Maximum Eigenvalue Test. Based on this, emphasis is laid on the results that are based on the trace test.
- The decision criterion under the Johansen test for two variables is given by (Mackinnon, Haug, & Michelis, 1999):

H₀: There is no cointegrating equation (CE)

Reject H₀ if: p-value < Level of significance

This implies that the closer the p-value is to 0%, the more statistically certain the existence of a cointegrating relationship.

4.3.2 Main Results

The results over the split samples are mixed: there is an indication of cointegration in some periods but not in others. Table 5 below reports these results. At a 90% confidence level, of the ten subsamples, five subsamples display cointegration. In the table, samples in which cointegration is present are indicated in bold. P-values range between 0.01% and 51.47% indicating that there is wide variation in the extent to which a relationship exists. P-values display no obvious time-based trend.

End of Period	P-values Under the Trace Test			$\sigma_{Debt Spread}$
	No CEs	1 CE	Cointegrated?	
17-Nov-10	25.6%	18.8%	No	21.55
15-Jun-11	0.2%	6.7%	Yes	7.65
11-Jan-12	28.8%	6.0%	No	19.48
8-Aug-12	51.5%	7.1%	No	22.72
13-Mar-13	8.7%	8.5%	Yes	14.70
16-Oct-13	0.0%	15.2%	Yes	36.51
23-May-14	1.1%	31.5%	Yes	26.81
30-Dec-14	28.7%	70.4%	No	19.48
4-Aug-15	40.4%	12.4%	No	22.03
15-Mar-16	9.0%	13.5%	Yes	70.19

Table 5: Split Sample Cointegration Test Results

Moreover, there is no apparent relationship between standard deviation and the extent to which there is a relationship between the variables. Chart 3 below plots the standard deviation of the debt spread and the p-values under no CE. The plot of the two does not indicate the existence of a relationship. In fact, the correlation coefficient of the two is -0.19.

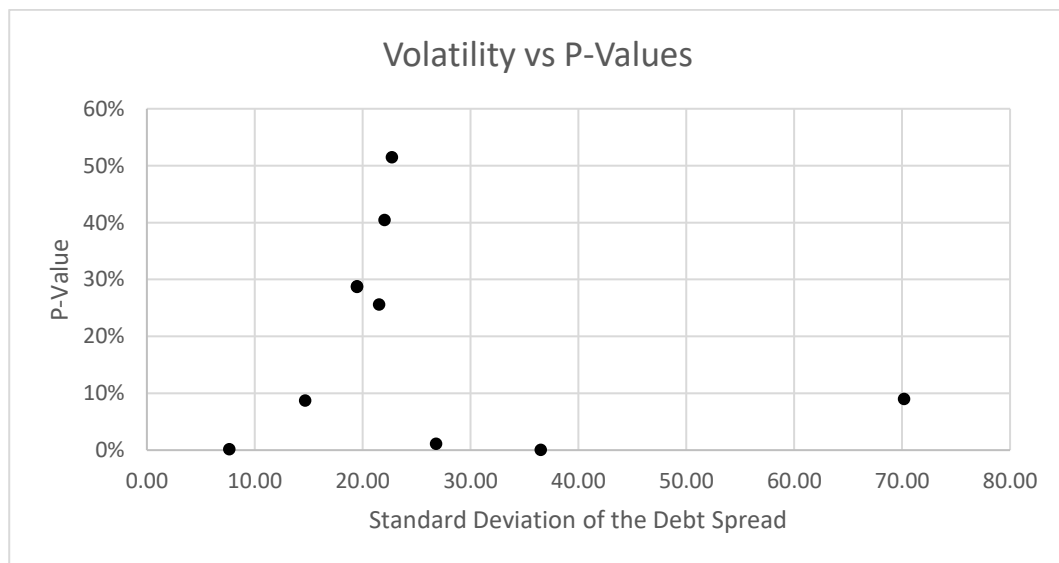


Chart 3: Cointegration p-values and debt spread volatility

5 Discussion and Implications

5.1 Summary of Findings

Sections 4.1 and 4.2 describe the data on the debt spread and the CDS premium yields. Some interesting facts are evident:

1) Both the mean and standard deviation of both time series varied significantly over the period. Theory indicates that increases in the level of the two variables is symptomatic of increased perceptions of credit risk.

2) Increases in the mean of both time series was accompanied by increases in their standards deviations. An increase in standard deviation has generally been thought to indicate that perceptions of risk have increased (Duxbury & Summers, 2011). In this context, high standard deviations indicate increased uncertainty of credit risk. As a result of the similarity in causes of high mean and high standard deviation, it should not be surprising that there is some perceived similarity between the periods in which they both escalate (Compare Charts 1 and 2).

5.1.1 Answers to Research Questions

Analysis of the relationship debt spread and CDS premium (using Johansen cointegration) and the effect of volatility on this relationship yields the following conclusions which answer the research questions.

- 1) Is there an observable relationship between bond prices and CDS spreads?
→An observable relationship between the debt spread and the CDS premium is observed in some periods but not others.
- 2) Does bond market volatility affect the relationship between bond prices and CDS spreads?
→Bond market volatility does not seem to affect the relationship between bond spreads and CDS spreads.

These findings are further discussed in Section 5.2.2 and their implications for investors in debt securities are addressed in Section 5.3.2.

5.2 Discussion

5.2.1 The Increase in the Debt Spread and the CDS Premium

Variations in the debt spread and CDS premium indicate variations in credit risk. Over the sample period, credit risk varied significantly. It remained comparatively low until late 2014 when a rapid increase began. High levels persisted until the end of the sample period.

This can be explained by the fact that Brazil has been experiencing strenuous macroeconomic conditions since mid-2014 with real GDP falling by 7.2% between Q1 2014 and Q4 2015 (Barua, 2016). This has weighed down on investor perceptions of credit risk. The adverse macroeconomic conditions can primarily be blamed on a decline in commodity prices on the global market. Other factors also contributed to increased credit risk. The factors that theoretically could have led to an increase in credit risk include:

1) A decline in commodity prices

Global commodity prices experienced a significant drop beginning in the second half of 2014. This was driven by declines in petroleum products, agricultural commodities and industrial metals. Chart 4 shows the IMF Commodity Price Index from Jan 2010 to Mar 2016.

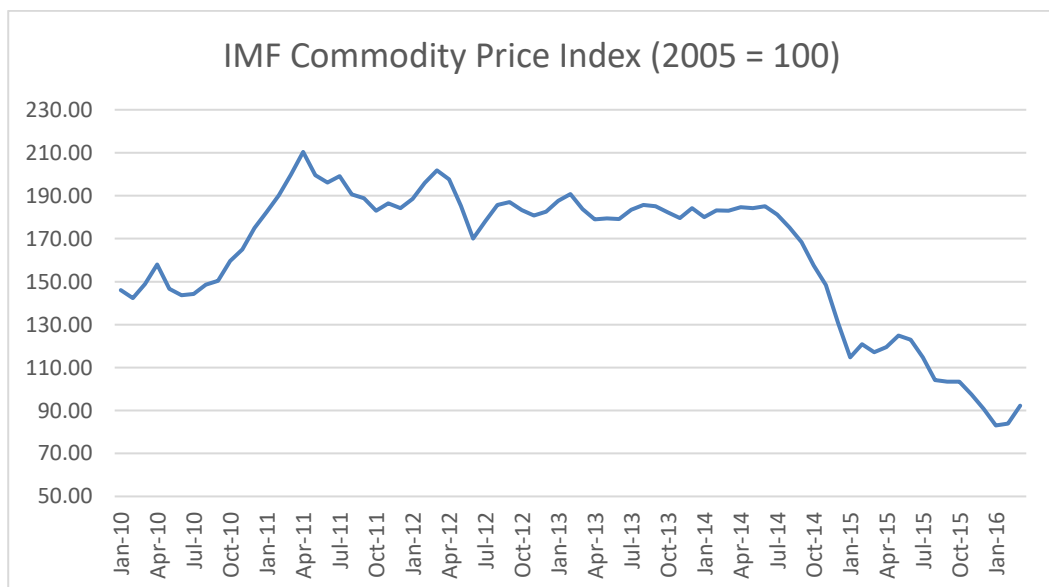


Chart 4: IMF Commodity Price Index

Brazil was hard hit as its exports are concentrated on commodities with iron ore, soy beans and crude petroleum being leading exports at 12% 10% and 7.2% respectively. Chart 5 below shows Brazil's exports in 2014¹⁵.

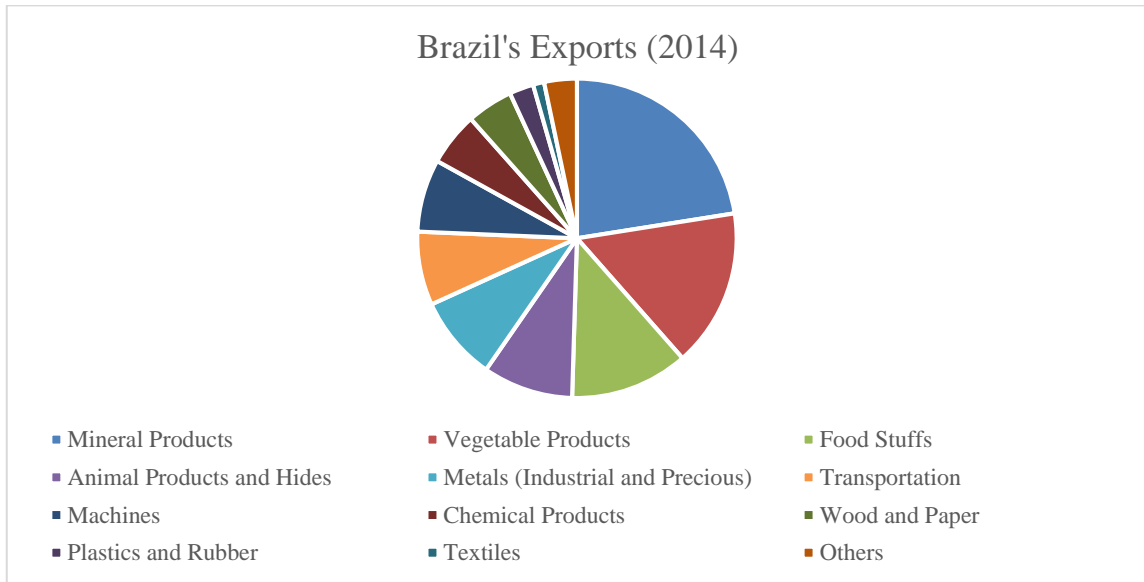


Chart 5: Brazil's Exports (2014)

Low prices of a country's exports results in lower economic activity within the country (lower output). This in turn reduces the revenue collected by the government through taxation. As a result of this, the government is in a worse position to service debts (such as sovereign bonds). Investors recognise this and attach a higher credit risk premium to the bonds resulting in a higher debt spread and CDS premium.

2) Errors in Fiscal Management

Brazil has long experienced sub-optimal fiscal policy. This has included unchecked spending, misdirection of funds to non-beneficial projects and rampant corruption (International Monetary Fund, 2015).

Fiscal policy was not in well positioned to withstand the adverse macroeconomic conditions (such as lower commodity prices). Moreover, policy was not able to respond fast enough to the changing environment. Unchecked spending continued¹⁶ and the government was already contractually bound to spend on massive projects that failed to yield the desired returns¹⁷ (such as The 2014 Football World Cup and The 2016

¹⁵ Data Source: MIT Atlas <http://atlas.media.mit.edu/en/profile/country/bra/>

¹⁶ Reuters - Brazil posts 2014 primary budget deficit, first in over a decade (30 Jan 2015)

¹⁷ Bloomberg News - Rio Olympics No Help to Brazil Economy Based on World Cup (16 Jan 2015)

Olympic Games). The result of inappropriate fiscal policy was budget deficits in both 2014 and 2015¹⁸. The budget deficit in 2014 was the first since 2001.

Fiscal mismanagement exacerbated the effects of the decline in commodity prices on the local economy by allowing for output leakages. It also limited the ability of the government to potentially intervene (perhaps through Keynesian fiscal policy) in order to stimulate the economy and thus counteract the effect of global macroeconomic conditions. More importantly, fiscal errors place the government in a cash-tight situation which impacts their ability to service debts.

3) Political Uncertainty

In early October 2015, Brazil's Federal Accounting tribunal began to call into question accounting in President Dilma Rousseff's government for the 2014 fiscal year. In December 2015, the Brazilian Chamber of Deputies (the lower house of Brazil's parliament) launched impeachment proceedings against the President¹⁹. (It is worth noting that on the dates on which both these events occurred, there was a spike in both the debt spread and CDS premium.) The process culminated in Rousseff's impeachment on 31st August 2016. During this time period, Brazil continued to experience a string of high profile corruption scandals in both the private and public sectors²⁰.

Political uncertainty has the potential to lead to regime change which in turn would lead to a change in fiscal policy. Fiscal policy could change for the worse resulting in the government finding it more difficult to meet fiscal obligations. (A worst-case scenario is the outright repudiation of the prior government's debt.) Investors recognise this possibility and attach a higher credit risk premium to the issuer. Moreover, political uncertainty due to corruption could lead to revelations of misuse of funds. This reduces the willingness of investors to invest in future debt issuances by the country. This reduced ability to refinance debt increases likelihood of default on existing obligations.

The reasons discussed above carry implications for economic policy makers in the issuing nation. These are discussed in Section 5.3.1.

¹⁸ Bloomberg News - Brazil's Budget Deficit Swells to Record as Economy Shrinks (30 Mar 2016)

¹⁹ BBC News - Brazil Crisis: A Timeline (12 May 2016)

²⁰ BBC News - Brazil's Continuing Corruption Problem (15 Sep 2015)

5.2.2 The Relationship Between the Debt Spread and the CDS Premium

At a 90% confidence level, the findings presented in Table 5 indicate that a cointegrating relationship between the debt spread and the CDS premium exists in half the sub-periods. There is a wide range in the degree to which this relationship exists. As a result, it cannot be said that a stable relationship exists across the entire period. This conforms to most empirical studies, most notably those by Blanco, Brennan, & Marsh (2003) and Celic (2012), who found mixed evidence of a relationship.

Situations in which pricing deviates from the equilibrium allow investors to perform credit risk arbitrage: selling protection in the expensive market and buying protection in the cheap market in order to make a profit without being exposed to credit risk.

Reasons why a relationship may not exist across the entire period

Previous research has provided several explanations of why the relationship between the bond price and CDS premium may not exist in some situations. These reasons include:

1) Liquidity Constraints

Nashikkar, Subrahmanyam, & Mahanti (2010) found that lack of liquidity in one of the markets for credit risk could prevent investors from taking advantage of mispricing. This is because they are unable to find sellers and buyers of credit risk to meet the other side of their trades. As a result, apparent arbitrage opportunities could persist for some period of time until traders are able to find enough liquidity to take advantage of them. Illiquidity is mainly present in the bond market.

2) Transaction Costs

According to Nashikkar, Subrahmanyam, & Mahanti (2010), transaction costs could also be a factor. High transaction costs could outweigh the profits from an arbitrage strategy making the strategy unattractive. As a result, arbitrage opportunities would appear to be present but would not actually be profitable. Hence, they are not exploited and remain.

3) Counterparty Risk

A credit risk arbitrage strategy would require an investor to take a position in a credit default swap. For the credit default swap position to pay off in case of default, the counterparty in the contract would have to honour the contractual payment. According to Celic (2012) the risk of this not happening is priced into the CDS premium and therefore might cause a relationship between the bond price and CDS premium not to hold.

4) Volatility of Positions and Temporary Losses

A credit risk arbitrage strategy would require the investor to hold positions in both the bond and a credit default swap. While this positions would eventually result in profit by the end of the bond's life, the investor is exposed to temporary losses as the market fluctuates. According to Lamont & Thaler (2003), this discourages investors from pursuing arbitrage strategies causing arbitrage opportunities to persist in financial markets.

5) Difficulty Shorting Bonds

The difficulty in shorting bonds makes it difficult for investors to sell credit risk in the bond market. As a result, mispricing of credit risk may persist in bond markets relative to the CDS market (where it is easier to short). Therefore, mispricing is likely to persist (Nashikkar, Subrahmanyam, & Mahanti, 2010).

6) Market segmentation

Zhu (2004) and Blanco, Brennan, & Marsh (2003) attribute the existence of arbitrage opportunities in the market for credit risk to market segmentation: the traders in the two markets are different and trade the securities for different reasons. "Microstructural factors ... make the CDS market the most convenient location for the trading of credit risk"²¹ while traders in bond markets trade based on a wider range of risks such as interest rate risks. As a result of this, the bond market is claimed to be slower in responding to credit risk. This results in the breakdown of the relationship between bond prices and the CDS premium.

²¹Quote from Blanco, Brennan, & Marsh (2003)

The Effect of Volatility

With regard to the effect of volatility on the existence of a relationship between the debt spread and the CDS premium, Chart 3 indicates that there is no effect. This is somewhat at odds with the findings of Bai & Collin-Dufresne (2011) who found that the relationship breaks down in periods of high volatility.

However, the study by Bai & Collin-Dufresne (2011) differs from this study in that their study covered the global financial crisis which was a period of high systematic risk in financial markets as a whole. On the other hand in this study, the risk was primarily due to idiosyncratic factors to do with the issuer of the security.

Bai & Collin-Dufresne (2011) attributed the breakdown of the relationship to the deleveraging process that took place during the crisis: "Financial institutions were forced to off-load risky assets to reduce their leverage, resulting in fire-sale prices for risky bonds. Combined with limits to arbitrage, this led to drops in bond prices as a result of the price pressure and only a slow lagged reaction of the CDS spread as arbitrageurs step in and equilibrate markets." This is essentially, an effect of systematic risk in financial markets which results in a need for the financial reorganisation in several financial institutions. As volatility in the case of this study was brought about by idiosyncratic factors, there would be no need for a deleveraging process. As a result, the distortive effects of volatility were not observed.

5.3 Implications for Participants in Sovereign Debt Markets

5.3.1 Policy Implications for Sovereign Debt Issuers

High perceptions of credit risk result in an increase in the yield on an issuer's bonds. The result is a higher cost of debt should the issuer decide to raise more debt finance. This is of concern to fiscal authorities as it increases their debt service burden and limits the amount of finance they can raise to plug any budget shortfall or fund projects. Moreover, perceptions of high credit risk result in increased uncertainty which causes high volatility in the cost of debt. This is undesired as it limits the ability to make fiscal debt plans.

For these reasons, issuers of sovereign debt securities should be concerned about managing their credit risk profile. The Brazilian case provides some areas that fiscal authorities in emerging markets should focus on:

1) Proper Fiscal Management

Fiscal policy should be positioned to be resilient to shocks. Uncontrolled spending and leakages should be limited. Moreover, fiscal policy should be responsive to shocks as they arise. This ensures that the economy remains strong under adverse conditions. This puts the government in a better position to service its debt in strenuous economic times.

2) Diversifying the Economy

The economic drivers of the issuing country should be diversified. The country should not be over dependent on one sector. As a result of diversification, the economy will be less vulnerable to shocks in one particular aspect of the global economy. In Brazil's case (and those of many emerging market countries), the economy was heavily dependent on commodity exports. Increased resiliency to shocks makes debt service easier under adverse economic conditions.

3) Transparent Accounting

Expenditure, particularly of funds raised through debt, should be accounted for. This will inspire confidence that the money is being used appropriately. Investors will therefore be more comfortable providing funding to the country. It will thus be easier for the issuer to raise new debt resulting in lower credit risk on existing obligations.

4) Political Stability

The political environment should be kept as stable as possible to increase certainty on the direction of fiscal policy and debt service. This reduces the perception of credit risk.

5.3.2 Trading Implications for Investors

For investors, the findings of this study imply that it is sometimes possible to take advantage of arbitrage opportunities in the market for credit risk. This is because in some periods the relationship between bond prices and CDS spreads deviates from the theoretically supported equilibrium resulting in arbitrage opportunities.

Consider the following simple example:

Entity X has issued a fixed-coupon bond. The bond is currently trading at a yield to maturity of 9%. The corresponding risk-free rate is 3.5%. Therefore, the debt spread is 5.5%. A credit default swap of matching maturity with X as its reference entity is currently trading at a premium of 4%.

Arbitrage opportunity: an investor could buy the risky fixed income security, short a risk-free bond and buy a CDS to protect against default. The average profit received per year of the holding period would be:

$$\begin{aligned} \text{Profit} &= (9\% - 3.5\%) - 4\% \\ &= 1.5\% \end{aligned}$$

Although the opportunities available in the market may not be as simple to identify and take advantage of as the one provided above, investors should be keen to identify and take advantage of them.

However, there are obstacles to the extent to which profit can be made from mispricing of credit risk. These obstacles were discussed in Section 5.2.2 and include

- 1) Liquidity Constraints
- 2) Transaction Costs
- 3) Counterparty Risk
- 4) Volatility of Positions and Temporary Losses
- 5) Difficulty Shorting Bonds

6 Conclusion

This research contributes to literature on the relationship between bond prices and Credit Default Swap premiums. Consistent with previous literature, mixed evidence is found: a relationship exists in some periods and not others. This is in spite of a the well documented theory that for arbitrage to be absent, there should exist a strong relationship between the two. Literature has proposed several reasons why the relationship may not always be present. These include: liquidity constraints, transaction costs, counterparty risk and other limits to arbitrage.

With regard to the effect of market volatility on the relationship, this study finds that volatility has no effect on the existence of a relationship. This is at odds with previous findings. However, the differences in results can be explained by the fact that the sources market volatility are different across different studies: in previous studies volatility was due to systematic risk while in this study it was due to idiosyncratic risk.

The implication of the findings is that investors may be able to make risk free profits by selling credit risk protection where it is expensive while purchasing credit risk protection where it is cheap. This is however subject to the limits to arbitrage. Also, periods of high volatility (due to idiosyncratic risk) do not expose more arbitrage opportunities than relatively less volatile periods.

In addition to the core findings, a survey of the behaviour of Brazil's credit risk (as measured by both the debt spread and CDS premium) in light of economic and political conditions in the country provides some insights. High credit risk perceptions adversely affect a country's cost of borrowing. Perceptions of Brazil's credit risk experienced an increase beginning in late 2014 and remained high until March 2016. This was due to economic and political conditions some of which are avoidable. These include: a decline in the price of exports, errors in fiscal management, corruption and political uncertainty. Countries, especially emerging and frontier ones, should be keen to avoid mistakes in these areas if they are to experience stability in their cost of borrowing. Specifically, they should diversify their economies, prudently plan fiscal policy, properly account for public funds and maintain political stability.

Recommendations for Future Research

This study leaves plenty of areas for future research. Future studies can extend the work of this study by:

- 1) Conducting similar analysis on a portfolio of emerging market securities as opposed to a single security to reduce noisiness in the data.
- 2) Using an interest rate benchmark that would result in more accurately isolating the price of credit risk.
- 3) Using a larger number of sub-samples. This will allow for a more granular analysis of the effect of volatility on no-arbitrage credit risk pricing. This could potentially form the foundation for the extension of current credit risk pricing models to account for diffusion and not just drift effects.

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8 Appendices

Appendix I: Debt Spreads and Default Probability

Adapted from Jorion's (2011) Financial Risk Manager Hand Book

Consider a risky zero-coupon bond of par value \$100 due in exactly one period's time. The bond is currently trading at a yield to maturity of r . At maturity, the bond may either default or not. In the first case, the bond pays $\$100 * (1 - R)$. In the second case, it pays the full amount due of \$100. The probability of default is denoted by π .

Using risk-neutral pricing, the current price must be the mathematical expectation of the values in the two states, discounting the payoffs at the risk-free rate.

$$Price = \frac{\$100}{1+r} = \left[\frac{\$100}{1+rf} * (1 - \pi) \right] + \left[\frac{\$100 * R}{1+rf} * \pi \right] \dots \dots \dots (Appendix Eq. 1)$$

Where: rf is the risk-free rate

Dropping second order terms and assuming the default probability and interest rates are small allows the equation to be simplified to:

$$r \approx rf + \pi(1 - R) \dots \dots \dots (Appendix Eq. 2)$$

Consider a bond with multiple (T) periods until maturity with compound interest rates and default rates being compounded over each period and $\bar{\pi}$ being the average annual default rate. Assuming one payment only, the present value is:

$$Price = \frac{\$100}{(1+r)^T} \dots \dots \dots (Appendix Eq. 3)$$

$$= \left[\frac{\$100}{(1+rf)^T} * (1 - \bar{\pi})^T \right] + \left[\frac{\$100 * R}{(1+rf)^T} * \{1 - (1 - \bar{\pi})^T\} \right] \dots (Appendix Eq. 4)$$

For a bond spanning several payment periods, a very rough approximation of this equation is:

$$r = rf + \bar{\pi}(1 - R) \dots \dots \dots (Appendix Eq. 5)$$

Appendix II: Debt Spread and CDS Premium Charts

