Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)
BACHELOR OF SCIENCE IN STATISTICS AND DATA SCIENCE
MAT 1102 DISCRETE MATHEMATICS
END OF SEMESTER EXAMINATION

Date: 07 February 2024 **Time: 2 Hours**

Instructions

- Answer **QUESTION ONE** and any other **TWO** questions.
- Show all your workings in the booklet provided.

QUESTION ONE (30 Marks)[COMPULSORY]

Q1). (a) State if each of the following statements are **TRUE** or **FALSE**. Provide a brief justification for your answer.

(i) The complete graph K_{12} is planar. **(2 marks)**

(ii) $P(95^5, 35^4) = C(95^5, 35^4) \cdot 35^4!$ **(2 marks)**

(iii) In a group of 13 passengers in a van, at least 2 of the passengers have their birthdays in the same month. **(2 marks)**

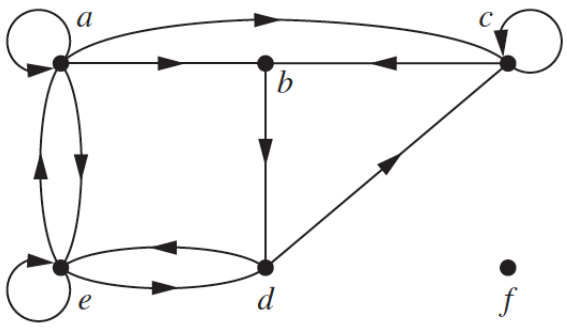
(iv) The total number of ways of flipping 5 two sided coins is 10. **(2 marks)**

(b) Simplify $\dfrac{6001! + 6000!}{6002!}$. **(3 marks)**

(c) How many different odd 4-digit numbers can be formed with the digits of the number $90,423$? **(3 marks)**

(d) Find the number of ways in which 12 people can sit around a circular table. **(3 marks)**

(e) Consider the following graph G, and let $V = \{a, b, c, d, e, f\}$.



G

Verify that

$$\sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}(v).$$

(4 marks)

(f) Using Binomial theorem, compute $(0.98)^5$ to two decimal places. **(3 marks)**

(g) Suppose that a connected simple planar graph has 30 vertices each of degree 2. Into how many regions does the representation of the graph split the plane?

(3 marks)

(h) By drawing the cycle graph C_6, show that graph C_6 is bipartite. **(3 marks)**

QUESTION TWO (20 Marks)[OPTIONAL]

Q2). (a) A committee of 12 people is to be formed from a group of 10 gentlemen and 12 ladies. In how many ways can the committee be formed if:

(i) at most 5 ladies are to be part of the committee? **(3 marks)**

(ii) two thirds gender rule is satisfied in forming the committee? (That is, there should be not more than two-thirds of either gender in the committee) **(3 marks)**

(b) Ben is accompanied by his three sisters to a dinner at Dan's home. If Dan stays with his wife and two children. In how many ways can all of them sit at a circular dining table during the dinner? **(3 marks)**

(c) The government of Kenya is planning to restart the registration of the number plates of government vehicles. If the number plate is supposed to have 6 characters with the first two characters as G and K, then followed by any three letters of the English Alphabet and the last character given by any digit.

(i) How many number plates can be produced using this method? **(3 marks)**

(ii) How much should the government budget with to produce all the number plates if the cost of producing 5 number plates is KES $15,000$. **(2 marks)**

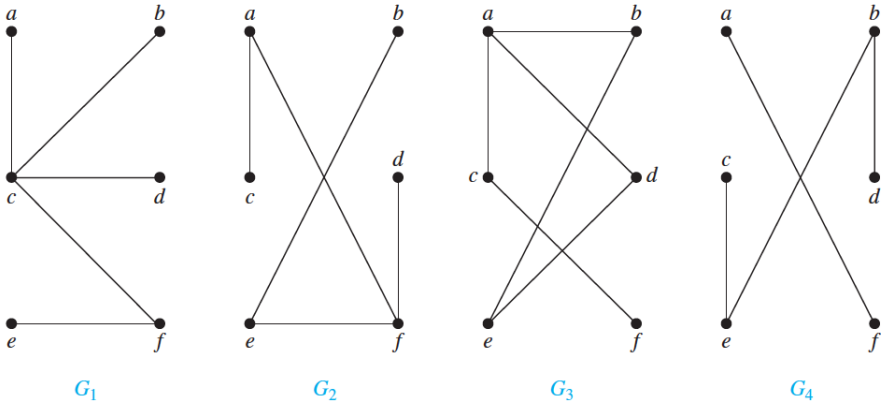
(d) Find the coefficient of x^5 in the expansion of **(6 marks)**

$$(2x^2 - \frac{1}{3x})^{10}.$$

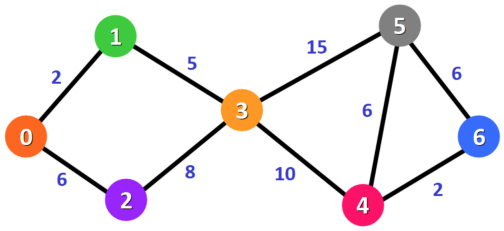
QUESTION THREE (20 Marks)[OPTIONAL]

Q3). (a) Give the recurrence formula for computing Sterling numbers of the second kind and hence find $S(4,2)$. Interpret your answer. **(4 marks)**

(b) Consider the following graphs G_1, G_2, G_3 and G_4. Check whether each of them is a tree or not. **(8 marks)**



(c) Consider the following graph.



By applying Dijkstra's Algorithm compute the shortest path between vertices 0 and 6. **(8 marks)**

QUESTION FOUR (20 Marks)[OPTIONAL]

Q4). (a) How many edges are there in a graph with 100 vertices each of degree 3?

(2 marks)

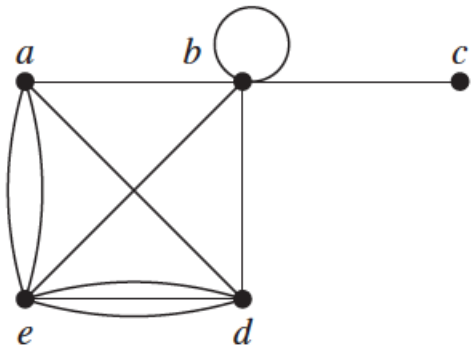
(b) Draw and name each of the following graphs, and check whether they are planar or not:

(i) K_3 **(3 marks)**

(ii) K_5 **(3 marks)**

(iii) $K_{2,2}$ **(3 marks)**

(c) Consider the following graph H.



H

(i) Find the degree of each vertex. **(3 marks)**

(ii) Using your answer in part (i) above, verify the Handshaking Theorem. **(2 marks)**

(d) Draw an undirected graph represented by the following adjacency matrix.

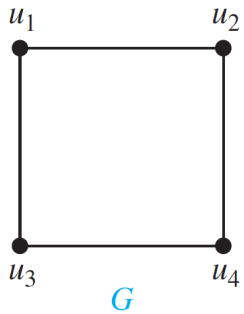
$$\begin{pmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{pmatrix}$$

(4 marks)

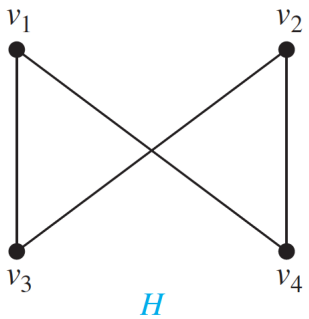
QUESTION FIVE (20 Marks)[OPTIONAL]

Q5). (a) Give an example of a graph with an Hamilton path but with no Hamilton circuit. **(2 marks)**

(b) By finding an isomorphism f of graphs, show that the following graphs, G and H, are isomorphic. **(4 marks)**



G



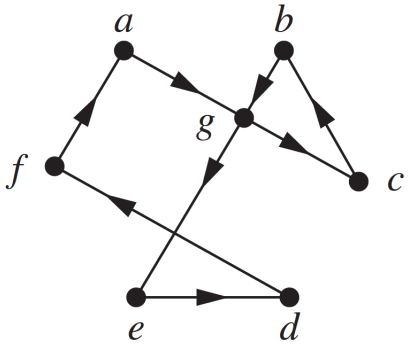
H

(c) Find the chromatic number of each of the following graphs. (Clearly draw and show the colouring in your answer booklet).

(i) K_5 **(3 marks)**

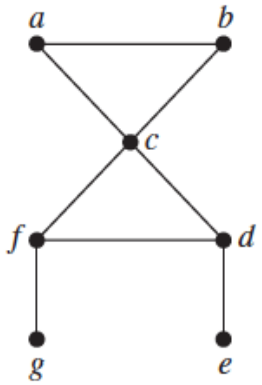
(ii) $K_{5,5}$ **(3 marks)**

(d) Check whether the following graph H_2 has an Euler circuit or not. **(3 marks)**

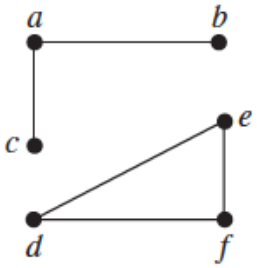


H_2

(e) Check whether each of the following graphs (G_1 and G_2 shown below) are connected or not. Justify your answers. **(5 marks)**



G_1



G_2

THE END