



STRATHMORE BUSINESS SCHOOL
BACHELOR OF SUPPLY CHAIN MANAGEMENT
END OF SEMESTER EXAMINATION
SCM 4205: ADVANCED OPERATIONS RESEARCH

Date: Friday 4th August 2023

Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE

(TOTAL 30 MARKS)

- a) Differentiate between the following terms;
 - i. a state and a stage in dynamic programming algorithm **(2 Marks)**
 - ii. (a *source* and a *sink* of a network flow. **(2 Marks)**
 - iii. an absorbing State and a transient state of a Markov matrix **(2 Marks)**
- b) Explain any four characteristics of dynamic programming problems **(4 Marks)**
- c) Explain any two properties of Markov chains **(2 Marks)**
- d) An insurance company is introducing two product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit in mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-hours per unit		Work- hours available
	Special risk	Mortgages	
Underwriting	3	2	120
Administration	4	6	260
Profits per unit	\$5	\$2	

- i. Formulate a Linear programming problem for this problem. **(3 Marks)**
 - ii. Use graphical method to solve this problem. **(4 Marks)**
- e) The number of units of an item that are withdrawn from inventory on a day to day basis is a Markov chain process in which requirements for tomorrow depend on today's requirements. A one-day transition matrix for the number of units withdrawn from inventory is given below

$$B = \begin{matrix} & \begin{matrix} \text{tomorrow} \\ 05 & 10 & 12 \end{matrix} \\ \begin{matrix} 05 \\ 10 \\ 12 \end{matrix} & \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \end{matrix}$$

- i. Represent the matrix in a transition diagram **(3 Marks)**
 - ii. Clasify the states of the Markov Matrix **(3 Marks)**

- iii. Develop a two-day transition matrix (3 Marks)
- iv. Comment on how a two-day transition matrix might be helpful to a manager who is responsible for inventory management. (2 Marks)

QUESTION 2

(TOTAL 20 MARKS)

- a) Five projects are being evaluated over a 3 year planning horizon. The following table gives expected returns for each project and the associated yearly expenditure

Project	Expenditure (in millions kshs)			Returns (in millions Kshs)
	1	2	3	
1	5		8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available funds (in million, Kshs)	25	25	25	

Formulate the model as an integer programming model.

(6 Marks)

- b) Research students studying the level of addiction to common drugs by street children undergoing day care rehabilitation found out by their survey that the mobility of the street children, in percent in the three classifications of their addition states; Severe (S), Mild(M) and Recovered (R) states were in the following percentages.

	<i>S</i>	<i>M</i>	<i>R</i>
<i>S</i>	0.5	0.3	0.2
<i>M</i>	0.1	0.7	0.2
<i>R</i>	0.1	0.4	0.5

If the present population in states S, M and R are in proportions of 0.7, 0.2, and 0.1 respectively determine the proportions of the children in the three states

- i. After **two** years (4 Marks)
- ii. In the long run (10 Marks)

QUESTION 3

(TOTAL 20 MARKS)

- a) Explain the difference between linear programming problems and non-linear programming problems (3 Marks)
- b) Define a general non- linear programming problem (4 Marks)
- c) A research firm has come up with a mathematical data for two products X_1 and X_2 which a firm manufacture. It has been found that this is a non linear programming problem having linear constraints and the objective function is of quadratic form. The data gathered is as follows:

$$\text{Maximise } Z = 8x_1 - x_1^2 + 8x_2 - x_2^2$$

$$x_1 + x_2 \leq 12$$

Subject to the constraints $x_1 - x_2 \leq 4$

$$x_1, x_2 \geq 0$$

Determine the maximum contribution and the number of units that can be expected from these products. (13 Marks)

QUESTION 4 **(TOTAL 20 MARKS)**

- a) Explain any two assumptions in Linear programming problem (2 Marks)
- b) The following primal problem

$$\text{Maximize } Z = 5x_1 + 4x_2$$

Subject to

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_3 \leq 2$$

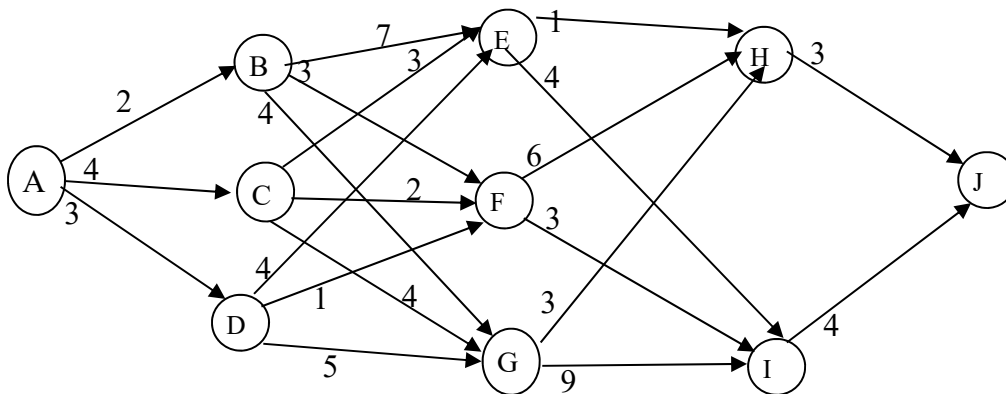
$$x_1, x_2, x_3 \geq 0$$

Required:

- i. Write the dual for the above Linear Programming Problem. (3 Marks)
- ii. Write the standard form for the dual problem (3 Marks)
- iii. Using the primal problem, solve for the optimum values of solution, Z using the **Simplex method**. (12 Marks)

QUESTION FIVE **(TOTAL 20 MARKS)**

A print media company needs to transport newspapers from their premise located at the city Centre, A to town J. The cost of transportation between different cities in hundred dollars is given in the following network.



- a) Find the optimal route connecting cities A and J using **Dynamic Programming** algorithm. (18 Marks)
- b) Hence obtain the optimal distance (2 Marks)