Strathmore UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBERSECURITY
END OF SEMESTER EXAMINATION
CNS 2206 PROBABILITY AND STATISTICS

Date: 02nd December, 2024 Time allowed: 2 Hours

INSTRUCTIONS

1. This examination consists of **FIVE** questions.
2. Answer ALL the questions in section I and any other TWO questions in section II.
3. Show all your workings clearly in the answer sheet.

SECTION I

QUESTION ONE - COMPULSORY

(a) Let X be a continuous random variable with probability density function $f(x)$. Give two axioms of $f(x)$. [2]

(b) The number of infections which arose in a sample of families of a certain village gave the following frequency distribution for the number of infections per family in the last year:

Table 1:

Number of infections (x)	0	1	2	3	4
Number of families (f)	15	20	10	5	0

Calculate the fourth order moment about the origin for these data. [4]

(c) Suppose that the finishing times of those men who complete a city 10 km run are normally distributed with mean 61 minutes and variance 81 minutes. Let X be the time of a randomly selected finisher. Find the probability

(i) that his time will be greater than 75 minutes. [2]

(ii) that his time will be between 50 and 75 minutes. [3]

[5 marks]

(d) Electricity Company introduced a new payment system and they visited a number of houses in the city to educate them on the use of the new device. The table below shows the number of houses visited and the number of times they were visited.

Table 2:

Number of houses	**Frequency of visits**
4	4
5	1
6	0
7	6
8	3
9	2

Calculate the following

(i) Mean [2]

(ii) Median [2]

(iii) Mode [2]

[6 marks]

(e) Dominic throws two fair dice and notes the numbers obtained. A is the event 'The product of the two numbers is 12'. B is the event 'One of the numbers is odd and one of the numbers is even'. By finding appropriate probabilities, determine whether events A and B are independent.

[5 marks]

(f) An exam is graded on terms of percentage from 0 to 100, with 60% needed to pass. Students' scores are modeled by the following density.

$$f(x) = \begin{cases} 4x, & 0 \leq x \leq 0.5 \\ 4 - 4x, & 0.5 \leq x \leq 1 \\ 0, & \text{otherwsise} \end{cases}$$

Determine the following:

(i) What is the probability that a random student passes the exam? [2]

(ii) What is the expected score of this distribution? [3]

[5 marks]

(g) Consider a geometric random variable X with $Var(X) = 3E(X)$.

(i) Find the probability of success p. [1]

(ii) Find the probability that $x = 5$. [2]

[3 marks]

[**Total marks: 30 marks**]

QUESTION TWO - OPTIONAL QUESTION

(a) The following back-to-back stem-and-leaf diagram shows the completion times in seconds in an hackathon competition involving two teams, A and B.

Table 3:

A		**B**
5 4 4 2 0 0	**20**	5 6 7
9 8 5 2 2 2 0	**21**	1 2 2 3 7 7
9 8 7 5 3 2 2 2	**22**	1 3 5 6 6 8 9
8 7 6 5 2 1	**23**	4 5 7 8 8 9 9 9
8 6 4 3	**24**	2 4 5 6 7 8 8
0	**25**	0 2 3 7 8

Key: 5|22|6 means a completion time of 0.225 seconds for A and 0.226 seconds for B.

(i) Find the median and the interquartile range for the two teams. [6]

(ii) Sketch a box-and-whisker plots for teams A and B and compare the distribution of the two groups [4]

[10 marks]

(b) The heights of the 11 members of the SU team are denoted by x cm. It is given that $\sum x = 1923$ and $\sum x^2 = 337221$. The SU team are joined by 3 new members whose heights are 166 cm, 172 cm and 182 cm.

(i) Calculate the mean of the heights before and after the new members joined. [2]

(ii) Calculate the standard deviation of the heights before and after the new members joined. [3]

(iii) If the median and modal height remain equal and unchanged with the addition of new members, compute Karl Pearson's coefficient of skewness for the two groups. [4]

(iv) Does the addition of new members affect the shape of the data? [1]

[10 marks]

[Total marks: 20 marks]

QUESTION THREE - OPTIONAL QUESTION

(a) Strathmore chess team play a match every weekend. Each match can be won, drawn or lost. At the beginning of the season the probability that Strathmore chess team win their first match is $\frac{3}{5}$, with equal probabilities of losing or drawing. If they win the first match, the probability that they win the second match is $\frac{7}{10}$ and the probability that they lose the second match is $\frac{1}{10}$. If they draw the first match they are equally likely to win, draw or lose the second match. If they lose the first match, the probability that they win the second match is $\frac{3}{10}$ and the probability that they draw the second match is $\frac{1}{20}$.

(i) Draw a fully labelled tree diagram to represent the first two matches played by Strathmore chess team in the season. [2]

(ii) Given that Strathmore chess team win the second match, find the probability that they lose the first match. [3]

[5 marks]

(b) The amount of data mined by investigating company during 2020 US elections are modelled by a continuous random variable, X, with probability density function

$$f(x) = \begin{cases} r\exp^{-0.01x} & , x > 0 \\ 0 & , otherwise \end{cases}$$

(i) Calculate the value of r. [2]

(ii) Find the probability that the amount of data mined is more than 100. [2]

(iii) Show that for 200, the average amount of data mined is 100 and the standard deviation of the is also 100. [3]

[7 marks]

(c) The discrete random variable X has the following probability distribution:

Table 4:

X	1	2	3	6
P(X = x)	0.15	p	0.4	q

(i) Write down equations satisfied by p and q. [2]

(ii) Given that $E(X) = 3.5$, find p and q. [3]

(iii) Calculate the variance of Y, where $Y = 2X + 10$. [3]

[8 marks]

[Total marks: 20 marks]

QUESTION FOUR - OPTIONAL QUESTION

(a) Prove that the mean and variance of a geometric distributed random variable i are, respectively, $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$.

$$f(i) = p(1-p)^{i-1}, \quad \text{for } i = 1, 2, \ldots, n$$

[6 marks]

(b) Darrel attempts the chess puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75, independently of all other days.

(i) If Darrel attempts the puzzle over n days,find the probability that he completes the puzzle exactly x times. [1]

(ii) Find the probability that he will complete the puzzle at least three times over a period of five days. [3]

(iii) What will the expected completion rate be if he competes over a period of 100 days? [2]

[6 marks]

(c) The number of messages sent per hour over a computer network has the following distribution:

Table 5:

X = number of messages	10	11	12	13	14	15
F(X)	0.08	0.23	0.53	0.73	0.93	1

(i) Obtain the pmf of X. [1]

(ii) Obtain the mgf of X [2]

(iii) Using the mgf obtained above, find $E(X)$ and $var(X)$ [5]

[8 marks]

[Total marks: 20 marks]

QUESTION FIVE - OPTIONAL QUESTION

(a) The amount of twits on twitter during elections are modelled by a distribution with moment generating function $M(t)$ given by

$$M(t) = (1 - 10t)^{-2}$$

Show that $E(X^3) = 24,000$

[3 marks]

(b) A show is scheduled to start at 9 : 00 A.M., 9 : 30 A.M., and 10 : 00 A.M. Once the show starts, the gate will be closed. A visitor will arrive at the gate at a time uniformly distributed between 8 : 30 A.M. and 10 : 00 A.M. Determine

(i) The cumulative distribution function of the time (in minutes) between arrival and $10 : 00 A.M.$ [2]

(ii) The probability that a visitor waits less than 10 minutes for a show. [2]

(iii) The standard deviation of the distribution in part (a) above. [3]

[7 marks]

(c) A random variable X has density function given by;

$$f(x) = \begin{cases} 4x(9 - x^2)/81, & 0 \leq x < 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the first four moments about the origin [8]

(ii) Compute the moment coefficient of Kurtosis β_1 and interprete [2]

[10 marks]

[Total marks: 20 marks]

End of Paper