



Strathmore Institute of Mathematical Sciences (SIMS)
Special Examination for the Degree of Bachelor of Business
Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE: 31st March 2026

Time: 2 Hours

Instructions

- This examination consists of **FIVE** questions.
- Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

Question 1 (Compulsory)

(a) Consider a Bernoulli utility function given by:

$$u(w) = \frac{w^{1-\sigma}}{1-\sigma}, \quad \text{for } \sigma \geq 0, \sigma \neq 1$$

- (i) Derive the coefficient of absolute risk aversion. (2 marks)
- (ii) Derive the coefficient of relative risk aversion. (3 marks)

(b) An investor is comparing two assets with the following returns across three states:

State	Probability	Asset 1 Return (%)	Asset 2 Return (%)
1	$\frac{1}{3}$	12	4
2	$\frac{1}{3}$	2	14
3	$\frac{1}{3}$	12	18

- (i) Does one asset dominate the other in a state-by-state comparison? (2 marks)
- (ii) Which asset exhibits mean-variance dominance? (3 marks)
- (iii) Which asset has the highest Sharpe ratio? (3 marks)

(c) Provide an intuitive explanation of the following asset pricing concepts:

- (i) Two-fund theorem (separation theorem) (6 marks)
- (ii) Complete Markets (5 marks)
- (iii) Roll's critique (of CAPM) (6 marks)

Question 2

(a) An investor with initial wealth $Y_0 = 120$ invests in stocks and a risk-free bond.

- Stock returns: $r_G = 18\%$ (good state), $r_B = 4\%$ (bad state)
- Probabilities: 0.5 each
- Risk-free rate: $r_f = 6\%$

The investor maximizes:

$$\max_a E[\ln Y]$$

Determine the optimal investment a^* in stocks. (8 marks)

(b) A risk-averse investor has utility:

$$u(Y) = \frac{Y^{1-\gamma}}{1-\gamma}, \quad \gamma = 1$$

The investor faces a risky asset:

$$Z_G = 90, \quad Z_B = 30, \quad P = 0.5$$

- (i) Compute the certainty equivalent $CE(\tilde{Z})$. (8 marks)
- (ii) Compute the risk premium $\Psi(\tilde{Z})$. (4 marks)

Question 3

(a) A portfolio manager forms a portfolio with three assets:

- Asset 1: $\mu_1 = 10$, $\sigma_1 = 3$
- Asset 2: $\mu_2 = 5$, $\sigma_2 = 3$
- Asset 3: $\mu_3 = 7$, $\sigma_3 = 3$

The portfolio is constructed to achieve $\mu_p = 7$ with minimum variance.

Use the Lagrangian method to derive the optimal weights w_1^*, w_2^* . (8 marks)

(b) Explain the interpretation of the CAPM test equation:

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \epsilon_i$$

What values should γ_0 and γ_1 take if CAPM holds? (6 marks)

(c) Explain three advantages of the Arrow-Debreu approach to asset pricing compared to CAPM. (6 marks)

Question 4

Suppose an investor solves:

$$\max_{\alpha} U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]$$

subject to:

$$c_t = e_t - \alpha p_t$$

$$c_{t+1} = e_{t+1} + \alpha x_{t+1}$$

where utility is:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

(i) Show that:

$$p_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]$$

(3 marks)

(ii) Provide an intuitive interpretation of the stochastic discount factor. (3 marks)

(iii) Show that:

$$1 = E(mR)$$

(2 marks)

(iv) Show that:

$$R_f = \frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma}$$

Discuss three factors influencing interest rates. (6 marks)

(v) Show that:

$$E(R) = R_f - \frac{\text{Cov}(u'(c_{t+1}), R)}{E(u'(c_{t+1}))}$$

Interpret this equation. (6 marks)

Question 5

(a) You are given the following information:

State	Stock Payoff	Bond Payoff	Arrow-Debreu Securities
Upstate	1400	1100	(1,0)
Downstate	900	1100	(0,1)
Current Price	1100	1000	

Show how to construct pure state securities. (8 marks)

(b) There are three states with probabilities:

$$(q_1, q_2, q_3) = (0.4, 0.4, 0.2)$$

Utility:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma = 0.4$$

Endowments:

$$(y_1, y_2, y_3) = (120, 90, 60)$$

Treating state 1 as numeraire, find equilibrium prices for state 2 and 3 goods. (12 marks)

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