STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

Bachelor of Business Science Actuarial Science, Finance & Financial Economics

END OF SEMESTER EXAMINATION

BSA 3108:- THEORY OF FINANCE

Date: July 2017   Time: 2 Hours

Instructions

1. This examination consists of FIVE questions.

2. Answer Question ONE (COMPULSORY) and any other TWO questions.

3. Question One carries 30 marks.

4. All other question carry 20 marks each.

Question 1 - Compulsory

1.   (a) Explain the following concepts as used in Finance:

      i. No arbitrage   *(2 marks)*

      ii. Complete Markets   *(2 marks)*

(b) Differentiate between Loss aversion and Risk aversion. What are the implications of Loss aversion for investment decision making. *(4 marks)*

(c) Identify which forms of market efficiency (if any) do the following observations about financial markets violate. Explain why.

     i. "The price of a stock can trace out a pattern that looks like a left shoulder (small rise and fall), followed by a head (larger rise and fall in price), followed by a right shoulder (another small rise and fall). In order for it to be a proper head-and-shoulders pattern, technical analysts require the trading volume during the left shoulder period should be higher than during the right shoulder period. Research indicates that abnormal stock returns after such a pattern is observed are negative and significant." *(3 marks)*

     ii. Recent finance literature has shown that stocks with high idiosyncratic risk (estimated using past year of daily data) earn significantly lower abnormal returns according to the CAPM than do stocks with low idiosyncratic risk. *(3 marks)*

(d) Define a vector of expected returns for N assets as $\mu_p = \mathbf{X}'\mathbf{r}$ where $\mathbf{X}$ is an $N \times 1$ vector of portfolio weights and $\mathbf{r}$ is an $N \times 1$ vector of asset returns. Solve for the optimal portfolio weights if, in addition to the usual constraint of $\mathbf{X}'\mathbf{1} = 1$, the investor requires the portfolio to yield an expected return threshold $\bar{r}$. *(7 marks)*

(e) Suppose that a investor has Constant Absolute Risk Aversion (CARA) utility. This means that the Absolute Risk Aversion Coefficient (denoted $\alpha(w)$) is constant at every level of wealth.

Consider the following utility function:

$$u(w) = -exp(-\alpha w)$$

where w denotes the consumer's wealth level, α is the absolute risk aversion coefficient. The individual has a certainty equivalent of $w - \pi$ where π is the risk premium he is willing to pay to avoid a gamble ε, i.e

$$u(w - \pi) = E[u(w + \varepsilon)]$$

Verify that the fraction of wealth he will pay to avoid a gamble that is proportional to wealth is independent of initial wealth w_0 *(5 marks)*

(f) An analyst has a dataset of monthly returns of the following securities: SPY, representing large cap stocks; IWM, representing small cap stocks; VWO, representing emerging markets; LQD, representing investment-grade, taxable, fixed income

securities and HYG, representing non-investment-grade, taxable, fixed income securities.

He is interested in creating the optimal asset allocation from the assets above, and comes up with the data in the following table:

Portfolio X								
	A	C	D	E	F	G	H	I
Average return	0.20%	0.40%	0.50%	0.60%	0.70%	0.75%	0.80%	0.90%
Std Deviation	2.55%	2.49%	3.21%	3.61%	4.08%	4.36%	4.66%	5.33%
Slope	0.0785	0.1608	0.1557	0.1660	0.1716	0.1720	0.1719	0.1689
SPY	9%	0%	0%	0%	0%	0%	0%	0%
IWM	7%	4%	0%	0%	0%	0%	0%	0%
VWO	0%	13%	20%	27%	38%	44%	50%	61%
LQD	10%	43%	60%	73%	62%	56%	50%	39%
HYG	74%	41%	20%	0%	0%	0%	0%	0%

From the TABLE, which should he choose? Select one from A to I and justify your answer using arguments from Modern Portfolio Theory

(4 marks)

Total for Question 1: 30 marks

Question 2

2. (a) A non-dividend paying stock currently trades at KES. 100. Your market forecasts lead you to conclude that after three months the stock price may either increase to KES. 110 with a 60% probability or decrease to KES. 85 with a probability of 40%. The continuously compounded risk free rate is 12% p.a. A European Call option could be written on the stock with maturity is 6 months and the strike price KES. 99

    i. Explain whether there is arbitrage in the market   *(3 marks)*

    ii. Using the Binomial Option Pricing Model (BOPM) and the concept of Put-Call Parity Calculate the price of a standard European Put option written on the underlying with a similar maturity and strike price to the Call Option described above.   *(7 marks)*

(b)   i. Prove that if $f(u)$ is a strictly increasing and concave function, then the utility function $g(w) = f(u(w))$ is more risk averse than the utility function $u(w)$. *(6 marks)*

    ii. Comment on the risk preference attributes of an investor who has the following utility function using the Arrow Pratt framework

$$u(w) = \frac{\rho}{1 - \rho} \left(\frac{w - \xi}{\rho} \right)^{1-\rho}$$

for a constant ξ and a constant ρ with $\rho \neq 0$ and $\rho \neq 1$ *(4 marks)*

Total for Question 2: 20 marks

Question 3

3. (a) Suppose there are three states and three securities have the following payoff vectors in state S; $X_S = [x_1, x_2, x_3]$ where $X_1 = [6, 6, 2]$, $X_2 = [3, 0, 0]$, and $X_3 = [0, 3, 1]$. Assume you can buy or sell fractions of a share. Prove that this market is complete or otherwise.   *(4 marks)*

(b) Suppose that the CAPM holds. Expected annual returns and betas a=of assets A and B are: $E[R_a] = 14\%$, $E[R_b] = 7.25\%$, $\beta_A = 2$, $\beta_B = 0.6$. The Sharpe ratio of the market portfolio is 0.25.

   i. Calculate the risk free rate and the market risk premium?   *(2 marks)*

   ii. Calculate the standard deviation of the market portfolio?   *(2 marks)*

   iii. Stock C has a beta of 1.2 and is expected to pay a liquidating dividend of $500 in a year. There are no other cash flows that the company will generate. Calculate the current price of C.   *(2 marks)*

   iv. You are considering forming a portfolio of stocks D and E. You notice that a portfolio with equal weights in the two stocks has the lowest variance of any portfolio of D and E. The two stocks have positive betas and there returns are uncorrelated, but Stock D is more correlated with the market than Stock E i.e $\rho_{DM} > \rho_{EM}$, where M denotes the market portfolio. Explain which stock has a higher expected return.   *(3 marks)*

(c) Two assets have the following distribution of returns:

Return (1) (%)	Probability (1)	Return (2)	Probability (2)
-1	8.33	0	50
11	91.67	20	50

   i. Calculate the following for each asset. Where necessary assume a benchmark return of 0%

      α) Variance   *(1 mark)*

      β) Semi-variance   *(1 mark)*

      γ) Shortfall probability   *(1 mark)*

   ii. Which asset would an investor with a quadratic utility function choose. Explain.   *(2 marks)*

   iii. Explain why variance of return is frequently used as a measure of risk.   *(2 marks)*

Total for Question 3: 20 marks

Question 4

4. (a) Suppose you have the following utility function $U(C_0, C_1) = (C_0 C_1)$, where C_0 is today's consumption and C_1 is tomorrow's consumption. You are endowed with Y_0 = 12 today and nothing tomorrow. There is a production opportunity that if I_0 is invested today, then the output tomorrow would be $f(I_0) = 40 I_0^{\frac{1}{2}}$. There is also a capital market where lending and borrowing rate of interest is $r = \frac{1}{3}$, i.e., if you borrow (or lend) 1 unit of good today, you will pay (or be paid) $\frac{4}{3}$ units of goods tomorrow.

    i. Suppose you have access to the production opportunity but do not have access to the capital market,

        α) How much should you invest in the production? *(3 marks)*

        β) What is your optimal consumption plan? *(3 marks)*

    ii. Now suppose you have access to both the production opportunity and the capital market, what is your optimal consumption plan? *(6 marks)*

(b) In their famous "Financial Modelers Manifesto" of 2009 Emanuel Derman and Paul Wilmott made the following comments about financial modeling:

"Models are at bottom tools for approximate thinking; they serve to transform your intuition about the future into a price for a security today. Our experience in the financial arena has taught us to be very humble in applying mathematics to markets, and to be extremely wary of ambitious theories, which are in the end trying to model human behavior. We like simplicity, but we like to remember that it is our models that are simple, not the world. We do need models and mathematics you cannot think about finance and economics without them but one must never forget that models are not the world. Whenever we make a model of something involving human beings, we are trying to force the ugly stepsister's foot into Cinderella's pretty glass slipper. It doesn't fit without cutting off some essential parts. And in cutting off parts for the sake of beauty and precision, models inevitably mask the true risk rather than exposing it. The most important question about any financial model is how wrong it is likely to be, and how useful it is despite its assumptions. You must start with models and then overlay them with common sense and experience"

Using the **Arbitrage Pricing Theorem** as an example critically discuss the foregoing comments. Be careful to define all the assumptions and notations used. *(8 marks)*

Total for Question 4: 20 marks

Question 5

5. (a) Cornell and Roll (1981) provide rules for analyzing rational behavior of investors when information is useful but costly.

      i. Derive the two necessary conditions for the existence of a stable equilibrium under their proposed analytical framework. *(8 marks)*

      ii. What is the implication of this model to market efficiency and security analysis? *(4 marks)*

(b) The expected returns and variance of three risky assets are given in the table below:

Asset	Expected return	Variance	Correlations		
			A	**B**	**C**
A	11.0%	9%	1.00	0.30	0.15
B	14.5%	20.25%	0.30	1.00	0.45
C	9.0%	9%	0.15	0.45	1.00

      i. Compute the Sharpe Ratio of a portfolio with an investment spread in as 30% in A, 30% in B and 40% in C. The risk free rate is 4% *(4 marks)*

      ii. Using a mean variance framework determine the optimal weights in assets A& C for a portfolio of assets A and C. *(4 marks)*

Total for Question 5: 20 marks