



IMHOTEP

AFRICAN JOURNAL OF PURE AND APPLIED MATHEMATICS

Imhotep Mathematical Proceedings Volume 1, Numéro 1, (2014), pp. 64 – 71.

On some sequence spaces and A - statistical convergence

Ekrem Savaş


Istanbul Ticaret University ,
Department of Mathematics,
Sütlüce, Istanbul -TURKEY
ekremsavas@yahoo.com
esavas@ticaret.edu.tr

Abstract

In the present paper, we introduce and study some properties of the a new sequence space that is defined using the φ - function and de la Valee-Poussin mean.

Proceedings of the 2nd Strathmore
International Mathematics Conference
(SIMC 2013), 12 - 16 August 2013,
Strathmore University, Nairobi, Kenya.

<http://imhotep-journal.org/index.php/imhotep/>

Imhotep Mathematical Proceedings 

On some sequence spaces and A - statistical convergence

Ekrem Savaş

Abstract. In the present paper, we introduce and study some properties of the a new sequence space that is defined using the φ - function and de la Valee-Poussin mean.

Mathematics Subject Classification (2000). Primary 40H05; Secondary 40C05.

Keywords. Modulus function, Invariant Mean, Almost convergence, Lacunary sequence, φ -function.

I. Introduction and Background

Let w denote the set of all real and complex sequences $x = (x_k)$. By l_∞ and c , we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_n |x_n|$, respectively. A sequence $x \in l_\infty$ is said to be almost convergent if all of its Banach limits coincide. Let \hat{c} denote the space of all almost convergent sequences. Lorentz [4] has shown that

$$\hat{c} = \left\{ x \in l_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n \right\}$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

The space $[\hat{c}]$ of strongly almost convergent sequences was introduced by Maddox [5] as follows :

$$[\hat{c}] = \left\{ x \in l_\infty : \lim_m t_{m,n}(|x - L|) = 0, \text{ uniformly in } n, \text{ for some } \ell \right\}.$$

Let σ be a mapping of the set of positive integers into itself. A continuous linear functional ϕ on l_∞ is said to be an invariant mean or a σ -mean if and only if

1. $\phi(x) \geq 0$ when the sequence $x = (x_k)$ has $x_n \geq 0$ for all n ;
2. $\phi(e) = 1$ where $e = (1, 1, 1, \dots)$ and
3. $\phi(x_{\sigma(n)}) = \phi(x)$ for all $x \in l_\infty$.

Let V_σ denote the set of bounded sequences which have unique σ -mean (see, [16]).

It was quite natural to expect that invariant mean must give rise to a new type of convergence, namely , strong invariant convergence, just as almost convergence gives rise to the concept of strong almost convergence and this concept was introduced and discussed by

Mursaleen [8]. If $[V_\sigma]$ denotes the set of all strongly σ -convergent sequences, then Mursaleen defined,

$$[V_\sigma] = \{x \in l_\infty : \lim_m \frac{1}{m} \sum_{k=1}^m |x_{\sigma^k(n)} - l| = 0, \text{ uniformly in } n\}.$$

Here $\sigma^k(n)$ denotes the k th iterate of the mapping σ at n . Taking $\sigma(n) = n + 1$, we obtain $[V_\sigma] = [\hat{c}]$ so that strong σ -convergence generalizes the concept of strong almost convergence.

Let $\lambda = (\lambda_i)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{i+1} \leq \lambda_i + 1, \lambda_1 = 0.$$

The generalized de la Valée-Poussin mean is defined by

$$T_i(x) = \frac{1}{\lambda_i} \sum_{k \in I_i} x_k$$

where $I_i = [i - \lambda_i + 1, i]$. A sequence $x = (x_n)$ is said to be (V, λ) -summable to a number L , if $T_i(x) \rightarrow L$ as $i \rightarrow \infty$ (see [6]).

Quite Recently Malkowsky and Savaş [6] have introduced the space $[V, \lambda]$ of λ -strongly convergent sequences as follows:

$$[V, \lambda] = \left\{ x = (x_k) : \lim_i \frac{1}{\lambda_i} \sum_{k \in I_i} |x_k - L| = 0, \text{ for some } L \right\}.$$

Note that in the special case where $\lambda_i = i$, the space $[V, \lambda]$ reduces the space w of strongly Cesaro summable sequences which is defined by

$$w = \left\{ x = (x_k) : \lim_i \frac{1}{i} \sum_{k=1}^i |x_k - L| = 0, \text{ for some } l \right\}.$$

Ruckle [10] used the idea of a modulus function f to construct a class of FK spaces

$$L(f) = \left\{ x = (x_k) : \sum_{k=1}^{\infty} f(|x_k|) < \infty \right\}$$

The space $L(f)$ is closely related to the space l_1 which is an $L(f)$ space with $f(x) = x$ for all real $x \geq 0$.

Recently E. Savaş [13] generalized the concept of strong almost convergence by using a modulus f and examined some properties of the corresponding new sequence spaces.

Following Ruckle, a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that

- (i) $f(x) = 0$ if and only if $x = 0$,
- (ii) $f(x + y) \leq f(x) + f(y)$ for all $x, y \geq 0$,
- (iii) f increasing,
- (iv) f is continuous from the right at zero.

Since $|f(x) - f(y)| \leq f(|x - y|)$, it follows from condition (iv) that f is continuous on $[0, \infty)$. Furthermore, we have $f(nx) \leq nf(x)$ for all $n \in N$, from condition (ii) and so

$$f(x) = f\left(nx \frac{1}{n}\right) \leq nf\left(\frac{x}{n}\right) \text{ hence}$$

$$\frac{1}{n}f(x) \leq f\left(\frac{x}{n}\right) \text{ for all } n \in N$$

A modulus may be bounded or unbounded. For example, $f(x) = x^p$, for $0 < p \leq 1$ is unbounded, but $f(x) = \frac{x}{1+x}$ is bounded.

By a φ -function we understand a continuous non-decreasing function $\varphi(u)$ defined for $u \geq 0$ and such that $\varphi(0) = 0$, $\varphi(u) > 0$, for $u > 0$ and $\varphi(u) \rightarrow \infty$ as $u \rightarrow \infty$, (see, [18]).

A φ -function φ is called non weaker than a φ -function ψ and we write $\psi \prec \varphi$ if there are constants $c, b, k, l > 0$ such that $c\psi(lu) \leq b\varphi(ku)$, (for all large or small u , respectively).

A φ -function φ and ψ are called equivalent and we write $\varphi \sim \psi$ if there are positive constant b_1, b_2, c, k_1, k_2, l such that $b_1\varphi(k_1u) \leq c\psi(lu) \leq b_2\varphi(k_2u)$, (for all large or small u , respectively), (see, [18]).

A φ -function φ is said to satisfy the condition (Δ_2) , (for all large or small u , respectively) if for some constant $k > 1$ there is satisfied the inequality $\varphi(2u) \leq k\varphi(u)$.

In this paper, we introduce and study some properties of the following sequence space that is defined using the φ - function and de la Valée-Poussin mean and also some inclusion theorems are obtained.

II. Main Results

Let $\Lambda = (\lambda_n)$ be same in the above, φ be given φ -function and f be given modulus function, respectively. Moreover, let an infinite matrix $A = (a_{nk})$ be given. Then we write,

$$V_\lambda^0((A, \varphi, \sigma), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

If $x \in V_\lambda^0((A, \varphi, \sigma), f)$, the sequence x is said to be λ -strong (A, φ, σ) - convergent to zero with respect to a modulus f . When $\varphi(x) = x$ for all x , we obtain,

$$V_\lambda^0((A, \sigma), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} (|x_{\sigma^k(i)}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

If $f(x) = x$, we write

$$V_\lambda^0(A, \varphi, \sigma) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

If we take $A = I$ and $\varphi(x) = x$ respectively, then we have

$$V_\lambda^0((I, \sigma), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{k \in I_j} f \left(|x_{\sigma^k(i)}| \right) = 0, \text{ uniformly in } i \right\}.$$

If we take $A = I$, $\varphi(x) = x$ and $f(x) = x$ respectively, then we have

$$V_{\lambda}^0((I, \sigma)) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{k \in I_j} |x_{\sigma^k(i)}| = 0, \text{ uniformly in } i \right\},$$

which was defined and studied by Savaş and Savaş [12]. If we define the matrix $A = (a_{nk})$ as follows:

$$a_{nk} = \frac{1}{n} \text{ for } n \geq k \text{ and } a_{nk} = 0 \text{ otherwise,}$$

then we have the sequence space,

$$V_{\lambda}^0((C, \varphi, \sigma), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \frac{1}{n} \sum_{k=1}^n \varphi(|x_{\sigma^k(i)}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

If we take $\sigma(i) = i + 1$, the sequence spaces $V_{\lambda}^0((A, \varphi, \sigma), f), V_{\lambda}^0((A, \sigma), f), V_{\lambda}^0((A, \varphi, \sigma)), V_{\lambda}^0((I, \sigma), f)$ and $V_{\lambda}^0((C, \varphi, \sigma), f)$ reduce to the following sequence spaces respectively.

$$\hat{V}_{\lambda}^0((A, \varphi), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{k+i}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

$$\hat{V}_{\lambda}^0(A, f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} (|x_{k+i}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

$$\hat{V}_{\lambda}^0(A, \varphi) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{n \in I_j} \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{k+i}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

$$\hat{V}_{\lambda}^0(I, f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{k \in I_j} f(|x_{k+i}|) = 0, \text{ uniformly in } i \right\}.$$

which was defined and studied by Malkowsky and Savaş [6].

$$\hat{V}_{\lambda}^0((C, \varphi), f) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \sum_{k \in I_j} f \left(\left| \frac{1}{n} \sum_{k=1}^n \varphi(|x_{k+i}|) \right| \right) = 0, \text{ uniformly in } i \right\}.$$

We now have,

Theorem II.1. *Let $A = (a_{nk})$ be an infinite matrix and let the φ - function $\varphi(u)$ satisfies the condition (Δ_2) . Then the following conditions are true.*

(a) *If $x = (x_k) \in V_{\lambda}^0((A, \varphi, \sigma), f)$ and α is an arbitrary number, then $\alpha x \in V_{\lambda}^0((A, \varphi, \sigma), f)$.*

(b) If $x, y \in V_\lambda^0((A, \varphi, \sigma), f)$ where $x = (x_k)$, $y = (y_k)$ and α, β are given numbers, then $\alpha x + \beta y \in V_\lambda^0((A, \varphi, \sigma), f)$.

Proof. (a) Let $x \in V_\lambda^0((A, \varphi, \sigma), f)$. First let us remark that for $0 < \alpha < 1$ we write

$$\frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|\alpha x_{\sigma^k(i)}|) \right| \right) \leq \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right).$$

Furthermore, if $\alpha > 1$ then we may find a positive number s such that $\alpha < 2^s$ and we have

$$\begin{aligned} \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|\alpha x_{\sigma^k(i)}|) \right| \right) \\ \leq \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(d^s \left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right), \\ \leq \frac{L}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right), \end{aligned}$$

where d and L are constants connected with the properties of φ and f . Therefore we obtain the condition (a).

(b) In the following let the numbers α, β and the elements $x, y \in V_\lambda^0((A, \varphi, \sigma), f)$ be given. From the part (a) it follows that the following inequality is true

$$\begin{aligned} \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|\alpha x_{\sigma^k(i)} + \beta y_{\sigma^k(i)}|) \right| \right) \\ \leq L_1 \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right) \\ + L_2 \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|y_{\sigma^k(i)}|) \right| \right), \end{aligned}$$

where the constants L_1 and L_2 are defined as in (a). Hence $\alpha x + \beta y \in V_\lambda^0((A, \varphi, \sigma), f)$. ■

Theorem II.2. Let f be an modulus function. Then

$$V_\lambda^0(A, \varphi, \sigma) \subset V_\lambda^0((A, \varphi, \sigma), f).$$

Proof. Let $x \in V_\lambda^0(A, \varphi, \sigma)$. For a given $\varepsilon > 0$ we choose $0 < \delta < 1$ such that $f(x) < \varepsilon$ for every $x \in [0, \delta]$. We can write

$$\frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right) = S_1 + S_2$$

where $S_1 = \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right)$ and this sum is taken over

$$\sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \leq \delta$$

and

$$S_2 = \frac{1}{\lambda_j} \sum_{n \in I_j} f \left(\left| \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \right| \right)$$

and this sum is taken over

$$\sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) > \delta.$$

By definition of the modulus f we have $S_1 = \frac{1}{\lambda_j} \sum_{n \in I_j} f(\delta) = f(\delta) < \varepsilon$ and furthermore

$$S_2 = f(1) \frac{1}{\delta} \frac{1}{\lambda_j} \sum_{n \in I_j} \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|).$$

Hence we have $x \in V_{\lambda}^0((A, \varphi, \sigma), f)$.

This completes the proof. ■

III. A - statistical convergence

The idea of convergence of a real sequence had been extended to statistical convergence by Fast [1] (see also Schoenberg [17]) as follows :

A sequence $x = (x_k)$ of real numbers is said to be statistically convergent to L if for arbitrary $\varepsilon > 0$, the set $K(\varepsilon) = \{n \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ has natural density zero, i.e.,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \chi_{K(\varepsilon)}(k) = 0,$$

where $\chi_{K(\varepsilon)}$ denotes the characteristic function of $K(\varepsilon)$.

Statistical convergence turned out to be one of the most active areas of research in summability theory after the work of Fridy [2] and Šalát [11]. Di Maio and Kočinac [7] introduced the concept of statistical convergence in topological spaces and statistical Cauchy condition in uniform spaces and established the topological nature of this convergence.

In another direction, a new type of convergence called λ - statistical convergence was introduced in [8] as follows.

A sequence (x_k) of real numbers is said to be λ - statistically convergent to L (or, S_{λ} -convergent to L) if for any $\varepsilon > 0$,

$$\lim_{j \rightarrow \infty} \frac{1}{\lambda_j} |\{k \in I_j : |x_k - L| \geq \varepsilon\}| = 0$$

where $|A|$ denotes the cardinality of $A \subset \mathbb{N}$. In [8] the relation between λ - statistical convergence and statistical convergence was established among other things.

Recently E. Savas [15] defined almost λ -statistical convergence by using the notion of (V, λ) -summability to generalize the concept of statistical convergence.

Let the infinite matrix $A = (a_{nk})$, the sequence $x = (x_k)$, the φ - function $\varphi(u)$ and a positive number $\varepsilon > 0$ be given. We write, for all i

$$K_{\lambda}^j((A, \varphi, \sigma), \varepsilon) = \{n \in I_j : \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \geq \varepsilon\}.$$

The sequence x is said to be (A, φ, σ) - statistically convergent to a number zero if for every $\varepsilon > 0$

$$\lim_j \frac{1}{\lambda_j} \mu(K_\lambda^j((A, \varphi, \sigma), \varepsilon)) = 0, \text{ uniformly in } i$$

where $\mu(K_\lambda^j((A, \varphi, \sigma), \varepsilon))$ denotes the number of elements belonging to $K_\lambda^j((A, \varphi, \sigma), \varepsilon)$. We denote by $S_\lambda^0((A, \varphi, \sigma))$, the set of sequences $x = (x_k)$ which are uniform (A, φ, σ) - statistical convergent to zero. We write

$$S_\lambda^0((A, \varphi, \sigma)) = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} \mu(K_\lambda^j((A, \varphi, \sigma), \varepsilon)) = 0, \text{ uniformly in } i \right\}$$

If we take $A = I$ and $\varphi(x) = x$ respectively, then $S_\lambda^0((A, \varphi, \sigma))$ reduce to $S_{\sigma, \lambda}^0$ which was defined as follows:

$$S_{\sigma, \lambda}^0 = \left\{ x = (x_k) : \lim_j \frac{1}{\lambda_j} |\{k \in I_j : |x_{\sigma^k(i)}| \geq \varepsilon\}| = 0, \text{ uniformly in } i \right\}.$$

Finally we conclude this paper by stating the following theorem.

Theorem III.1. *If $\psi \prec \varphi$ then $S_\lambda^0((A, \psi)) \subset S_\lambda^0((A, \varphi))$.*

Proof. By assumption we have $\psi(|x_{\sigma^k(i)}|) \leq b\varphi(c|x_{\sigma^k(i)}|)$ and we have for all i ,

$$\sum_{k=1}^{\infty} a_{nk} \psi(|x_{\sigma^k(i)}|) \leq b \sum_{k=1}^{\infty} a_{nk} \varphi(c|x_{\sigma^k(i)}|) \leq L \sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|)$$

for $b, c > 0$, where the constant L is connected with properties of φ . Thus, the condition $\sum_{k=1}^{\infty} a_{nk} \psi(|x_{\sigma^k(i)}|) \geq \varepsilon$ implies the condition $\sum_{k=1}^{\infty} a_{nk} \varphi(|x_{\sigma^k(i)}|) \geq \varepsilon$ and in consequence we get

$$\mu(K_\lambda^j((A, \varphi, \sigma), \varepsilon)) \subset \mu(K_\lambda^j((A, \psi, \sigma), \varepsilon))$$

and

$$\lim_j \frac{1}{\lambda_j} \mu(K_\lambda^j((A, \varphi, \sigma), \varepsilon)) \leq \lim_j \frac{1}{\lambda_j} \mu(K_\lambda^j((A, \psi, \sigma), \varepsilon)).$$

This completes the proof. ■

References

- [1] H. Fast, *Sur la convergence statistique*, Colloq Math. **2**, (1951), 241 – 244.
- [2] J. A. Fridy, *On statistical convergence*, Analysis, **5** (1985), 301 – 313.
- [3] E. Kolk, *matrix summability of statistically convergent sequences*, Analysis, **18**, (1993), 77 – 83.
- [4] G. G. Lorentz, *A contribution to the theory of divergent sequences*, Acta. Math. **80** (1948), 167 – 190.
- [5] I. J. Maddox, *Spaces of strongly summable sequences*, Quart. J. Math., **18**(1967), 345 – 355.
- [6] E. Malkowsky and E. Savaş, *Some λ - sequence spaces defined by a modulus*, Archivum Math. **36**, (2000), 219 – 228.
- [7] G. D. Maio and L. D. R. Kocinac, *Statistical convergence in topology*, Topology Appl., **156** (2008) 28 – 45.

- [8] M. Mursaleen, *Matrix transformation between some new sequence spaces*, Houston J. Math. **9**, (1993), 505 – 509.
- [9] M. Mursaleen, *λ -statistical convergence*, Math. Slovaca, **50** (2000), 111 – 115.
- [10] W. H. Ruckle, *FK Spaces in which the sequence of coordinate vectors is bounded*, Cand. J. Math. **25** (1973), 973 – 978.
- [11] T. Šalát, *On statistically convergent sequences of real numbers*, Math. Slovaca, **30** (1980), 139 – 150.
- [12] E. Savaş, and R. Savaş, *Some λ - sequence spaces defined by Orlicz functions*, Indian J. Pure. Appl. Math. **34**(12), (2003), 1673 – 1680.
- [13] E. Savaş, *On some generalized sequence spaces defined by a modulus*, Indian J. Pur. Appl. Math. **30**(5), (1999), 459 – 464.
- [14] E. Savaş, *On some new sequence spaces defined by infinite matrix and modulus*, Advances in Differences Equations 2013, **274** (2013).
- [15] E. Savaş, *Strong almost convergence and almost λ -statistical convergence*, Hokkaido Math. J. **24**(3), (2000), 531 – 536.
- [16] P. Schaefer, *Infinite matrices and invariant means*, Proc. Amer. Math. Soc. **36** (1972), 104 – 110.
- [17] I.J. Schoenberg, *The integrability methods*, Amer. Math. Monthly, **66** (1959), 361 – 375.
- [18] A. Waszak, *On the strong convergence in sequence spaces*, Fasciculi Math. **33**, (2002), 125 – 137.

Ekrem Savaş

Istanbul Ticaret University , Department of Mathematics, Sütlüce, Istanbul -TURKEY

e-mail: ekremsavas@yahoo.com

e-mail: esavas@ticaret.edu.tr

INFORMATION TO READERS: We encourage readers to sign up for the publishing notification service for this journal. Use the Register link at the top of the homepage for the journal. This registration will result in the reader receiving the Table of Contents by email for each new issue of the journal. This list also allows the journal to claim a certain level of support or readership. See the journal's Privacy Statement which assures readers that their name and email address will not be used for other purposes

INFORMATION TO AUTHORS: Interested in submitting to this journal? We recommend that you review the About the Journal page for the journal's section policies, as well as the Author Guidelines.

Authors need to register with the journal prior to submitting, or if already registered can simply log in and begin the 5 step process.

Manuscripts must be submitted in pdf-format through the journal website. Authors are advised to keep a copy of their manuscripts since the publisher cannot accept responsibility for lost copy. Manuscripts received are assumed to be final. If excessive changes are requested after an article has been finalized, the author will be charged with the cost of the alterations other than the printer's errors.

Authors are advised to study the printing style of the journal before making the final version of the manuscript. Styles files may be requested to the editorial assistant.

Authors of papers accepted for publication will be asked to supply electronic version of the paper together with a paper manuscript. The file should be in \LaTeX , and it should be sent by attached mail to the editorial assistant.

INFORMATION FOR LIBRARIANS: We encourage libraries to list this open access journal among its electronic journal holdings. This publishing system is also designed to be hosted and operated by research libraries in support of the journal publishing work of its associated faculty. For more information on using Open Journal Systems to manage and publish peer-reviewed journals,