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Mathematical Modeling of House Price Dynamics and their Impact on the Cost of No Negative Equity Guarantee: Evidence from Kenya

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the dissertation contains no material previously published or written by another person except where due reference is made in the dissertation itself.

.... Date ....

Erick Omondi Mau

Approval

This dissertation of Erick Omondi Mau was reviewed and approved by the following:

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Dr. Lucy Muthoni

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Abstract

Equity Release Mortgages (ERMs) are significantly required in an ageing population with high homeownership levels. The capacity to identify the risks associated with the cost of No Negative Equity Guarantee is an essential aspect of a risk management tool for most annuity and pension providers. Therefore, the main objective of this research is to; institute a stochastic modelling framework for the No Negative Equity Guarantee (NNEG) in an Equity Release Mortgage (ERMs) loan, find the payoff structure of the NNEG, and finally price the Equity Release Mortgages. An ARMA-EGARCH model that can capture auto-correlation and volatility clustering characteristics is proposed based on the model fittings.

To analyze the regional and the national effect, we evaluate different models using the bench-marked loan data obtained from nationwide building society database in the United Kingdom, for the period between 1991-2020 and had details such as the amount borrowed, age, marital status, and sex among others during the period. House price Index (HPI) data was used to calibrate the loan data. Four baseline scenarios were used to simulate the NNEG valuation: the loan to value ratio, the roll-up rate, risk-free rate, and house price volatility.

The model forecasting power was evaluated using: root mean squared errors, mean average error, the Diebold-Mariano forest accuracy test and Occam's razor method. However, due to fluctuations in the house price data generating process and goodness of fit, the Diebold-Mariano forest accuracy test was used as the metric to evaluate the model's performance in providing superior forecasting power. The study adopts the suggestion of (Hosty et al., 2008) to investigate the model risk on the cost of NNEGs and further develops a risk-neutral valuation methodology using Conditional Esscher Transform Technique as proposed by (Bühlmann et al., 1996).

The findings indicate that ARMA (4, 3)-EGARCH (1, 1) outperformed both the Black (1976) and the Geometric Brownian Motion-risk-neutral (GBM-rn) with a score of 0.2637. The simulation results further established that, the cost of NNEG is critically sensitive and robust to; the Roll-up rate, Loan-to-Value (LTV) ratio, the volatility of the house prices, risk-free rate, and the rental yield.

Also, under current market settings, the Geometric Brownian Motion (GBM)-rn and Black' 76 may suddenly increase the NNEGs values via higher than obligatory volatilities at longer time horizons. *Key words: NNEG, Equity-Release Products, Conditional Esscher Transform.*

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I would like to express my gratitude to the Almighty God, the Creator of Heaven and Earth for the gift of life, provision, good health and strength to see me through my entire studies.

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Dedication

This is for my aunt, Catherine Akinyi Mau for trusting and believing in me and also for being a pillar of strength through my whole education. I similarly dedicate this to my late mother who planted the seed of education at much younger age. Thank you mum. Finally, I would like to appreciate my brothers and sisters for always believing in my skills and ability, an encouragement which has been a pillar for my success. May the good Lord bless you all.

List of Abbreviations

ARMA	Autoregressive Moving Average
EGARCH	Exponential Autoregressive Conditional Heteroscedasticity Model
Black' 76	is a risk-neutral measure constructed in an incomplete market.
CBK	Central Bank of Kenya
ERM	Equity Release Mortgage
CET	Conditional Esscher Transform
MLE	Maximum Likelihood Estimation
ERCs	Early Repayment Charges
GBM-rn	Geometric Brownian motion under the risk neutral world
LTV	Loan-to-Value ratio
NNEG	No Negative Equity Guarantee
PRA	Prudential Regulatory Authority
RBA	Retirement Benefits Authority
Roll-up rate	The rate at which the amount borrowed compounds.

Chapter 1

Introduction

1.1 Background To The Study

Equity Release Mortgages (ERMs) are innovative financial products that let older populations transform their initially illiquid houses into a series of cash inflows. The monies received can be used to: augment the pension benefits, regular source of income, payment for aged care, and arranging for other uses that significantly improve the quality of life for the older population (Blevins et al., 2017). (Yang, 2011), (Bloxham et al., 2011) and (Chang et al., 2012) also described it as an array of income-generating portfolio and a double crossover put option that is spontaneously applied at loan termination. This makes the modelling of ERM very significant both in financial risk management and real estate research.

It has indeed attracted vital consideration following the aftermath of the 2008 subprime crisis and the more recent regulatory changes of Basel II protracted by Basel III. This has led to the conducive climate of a tighter mortgage borrowing and an improved interest from banks, annuity providers and pension companies to moderate on the ERM losses.

Additionally, there has been a persistent rise in life expectancy in the Kenyan economy that requires an immediate need on the ways of improving the retirement income, particularly for the older population. In its 2013 report, the Retirement Benefits Authority (RBA) also confirms that approximately 75% of the living condition of retirees in Kenya has deteriorated. This is due to the weakening of financial ability, changes in the family structures, the unsustainability of the pension systems, and an upsurge in longevity (Ojijo, 2013).

Globally, older people are the most vulnerable cohort. Hence, it is widely accepted that older persons, women and children need government intervention. Numerous efforts by successive administrations in the country to address older persons plights have proved to be abortive. This, as a result, has forced the retirees to use their pension benefits for home construction, medical care and as a regular source of income resulting in unprecedented depletion of pension benefits (Ambrose and Buttimer, 2000).

The paper by (Byrne et al., 2013), therefore, recognizes that advancements within the private sector of a cutting-edge financial product with the ability to enhance the retirement proceeds would be of great benefit. (Wu and Zhang, 2018) admits that home equity, therefore, provides an alternative financial product capable of meeting the current shortfall in retirement income. Undeniably, Equity Release Mortgages are indeed products designed to bridge this gap, with borrower receiving monthly cash- inflows or an annuity in quid-pro-quo- for the transfer of part or all, of the total value of their property to the lender upon their death. According to (Authority, 2018), the absolute value of the mortgage is eventually factored in by such factors as; the risk-free rate, the house price volatility, the age of the mortgagor, and the initial house price.

(Tunaru, 2017) and (Adam et al., 2017) conducted a study in the uptake of Equity Release Mortgages in the: United States (USA), United Kingdom (UK), Australia, Canada, Japan and South Africa, and the researchers concluded that equity release mortgages are extensively provided by many institutions operating in the financial sector, such as commercial banks, insurance companies, fund managers, endowment funds, and annuity provides. However, this research failed to do a comparative study into the risks involved for such firms in the delivery of the Equity Release Mortgages. The most notable of all of these risks is the risk of Negative Equity that most of the institutions may assume if the property's sales proceeds prove to be less than the amount of the mortgage advanced to the borrower. According to (Andrews and Oberoi, 2018), the critical factor in determining the achievement of an institution's risk management is the cost of no negative equity. Therefore, new and better models that may help reshape how the annuity providers operate require attention.

1.2 The No negative equity guarantee (NNEG)

To deal with the risk of negative equity, most commercial banks and annuity providers sometimes practice no negative equity guarantee (NNEG) as the primary way of dealing with the risks associated with the equity release mortgages. NNEG is the use of insurance to protect the financier against the effects of negative equity emanating from the mortgaged amount being greater than the value of the house at the time of the loan termination (Andrews and Oberoi, 2018). Thus, (Yang, 2011) and (Tunaru, 2017) viewed it as a European put option on the house used as security.

In some developed economies such as: the United States of America (USA), The United Kingdom (UK), and Australia, equity release mortgages have become the norm in most of the lending institutions and must integrate the provisions of the NNEGs to reduce these risks and also meet the standards acceptable to the Equity Release Council (Andrews and Oberoi, 2018).

The paper by (Hosty et al., 2008) acknowledges that the effective valuation and pricing of no negative equity guarantee has turned into an enormously significant tool in developing a clear appreciation of equity release mortgages. For this motivation, NNEGs valuation framework has often presented numerous challenges to both the pension and annuity providers (Engsted et al., 2016).

For this motivation, NNEGs valuation framework has often presented numerous challenges to both the pension and annuity providers (Engsted et al., 2016). From the provider's point of view, (Pilcher and Cortazzi, 2016) notes that it is imperative to estimate the termination probability: most cases, the delayed termination results in much heavier loan accumulation. Thus forcing the financier undoubtedly to overpay the borrower at the time of mortgage contract initiation. (Chowdhury and Mallik, 2004) used an alternative log regression return model to study the behaviour of the loan termination based on key variables' behaviour. The research was based on the actual Equity Release mortgages termination data. They found out that these models categorize borrowers based on categories such as; mortality, asset classes, marital status, age, and gender.

Due to lack of robust NNEGs modelling tools, annuity providers and lenders have turned towards the use of Black-Scholes formula for NNEG put option valuation. However, according to Prudential Regulation Authority (Authority, 2018), modelling approaches are applicable provided that they can meet the four principles of; securitization, the economic value of ERPs cash flows and the present value of all the delayed ownership of the house should be less or equal to the value of the current assets.

1.3 Problem Statement

With the increase in longevity, advancement in technology, demand for decent and appropriate retirement benefits come to the foreplay. Ideally, there should be an equilibrium between the demand for the pension income and its supply. This would guarantee stability in the pension system and affording the pensioners a stable source of income (Case et al., 2000). However, this is always not the case; despite the knowledge and significance of pension management, the demand still surpasses supply (Bjork, 2009). This manifests itself through the delayed payments of the pension money, long queues, and long and tedious follow up by the pensioners. Additionally, there has been a persistent rise in life expectancy in the Kenyan economy that requires an immediate need on the ways of improving the retirement income, particularly for the older population. In its 2012/ 2013 report, the Retirement Benefits Authority (RBA) ascertain that approximately 75% of the living condition of retirees in Kenya has deteriorated. This is due to the weakening of financial ability, and changes in the family structures, which then create an extended demand for the pension payouts than the supply will sustain (Ojijo, 2013).

To assuage the pension problem: the government, researchers and institutions have been taking innovative steps through Insurers, commercial banks, annuity providers, and Retirement Benefits Authority (RBA) of innovative financial products capable of enhancing the retirement income. (Wu and Zhang, 2018) arguably admits that home equity has the capacity of meeting the current shortfall in retirement income. However, on the ongoing development for: the pricing, and valuation of the equity release mortgages, the main issue thus far, has been arguably the house price risk (Kau et al. 1995), with the supposition that house prices follow a Geometric Brownian Motion (GBM), which as a result expedites the application of the Black and Scholes (Black, 1976) option pricing model for pricing and valuing the NNEGs. Equity release mortgages valued using the postulation of the Black and Scholes formulae had equally been introduced in several prior studies, and are centred around the premise that house price dynamics follows a "standard stochastic process" as discussed by (Chen et al., 2010).

In most of the empirical discussions, two key features are associated with house price returns' dynamics. First, the log-returns of house price dynamics is always arguably believed to be auto-correlated. Finally, the log-returns volatility is found to be time-varying or experiencing conditional heteroscedasticity (Daal et al., 2007). To capture, and solve these key features in the house price return dynamics: (Commission et al., 2011) and (Fry et al., 2010) focused on the use of Auto-Regressive Moving Average-Exponential Generalized Auto-Regressive Conditional Heteroscedasticity (ARMA-EGARCH) family of models. They also noted that house price dynamics had been subjected to abnormal shocks in recent years, especially the duration of the subprime crisis of 2008.

These shocks are representational and depict both the systematic and non-diversifiable nature, thus resulting in numerous computational issues inside and out of the financial risk management and within the real estate research as well (Bjork, 2009). The effects of volatility clustering have equally attracted much attention over the recent past. The paper by (Chen et al., 2010) and (Wang et al., 2016) arguably used the jump-diffusion result to redefine the recent changes in the house price return dynamics with the earlier study defining the influences of abnormal shocks on the mortgage payments.

Additionally, many researchers and financial institutions have been taking necessary steps to fill this gap: one notable one is the research conducted by (Li et al., 2017), and (Pilcher and Cortazzi, 2016), which tended to extend the study conducted by (Chen et al., 2010) and (Wang et al., 2016), however, they found out that by allowing for the jump effects in the log house price returns coupled with the volatility clustering effect, greatly increases the model fit for the return house price dynamics.

Nonetheless, despite the many attempts to factor the house price data's jump effects, it seems that most of the previous research did not manage to deliberate on some vital features of: volatility clustering and auto-correlation effects within the log-returns of house price dynamics. This has been so because NNEGs valuation has not been exploited to its fullest potential. Also, the equity-release products are new concepts. The literature is still growing to address the risk factors and capital adequacy of ERMs (Ojijo, 2013). In essence, availability of quality and sufficient data has been the biggest hindrance to this kind of modelling (Tunaru, 2017). Until 2015, there were no licensed equity release mortgage providers in Kenya, and hence, no existing ERMs before that.

Therefore, this paper aims to fill the remaining gap by considering the key factors discussed in the previous literature and present a comparison of the effects of volatility clustering in the log house return dynamics arguably centred around the ARMA-EGARCH model provisions. We further assume that the jump effects follow a Poisson distribution process coupled with a time-varying conditional heteroscedasticity framework. In the data analysis, the same as the methodology adopted by the (Hosty et al., 2008), we equally emphasize the United Kingdom (UK) Equity Release Market as a benchmark for Kenya's case. At the same time basing our analysis on the national and regional effect on the log house price return dynamics. This is done in the context of benchmarked loan data obtained from the nationwide building society database in the United Kingdom.

1.4 Objectives of the Study

The main objective of this study is to develop an innovative financial product capable of boosting retirement income for the retirees to maintain acceptable living standards. The supporting objectives are to:

1. Apply the ARMA–EGARCH approach to modelling house price dynamics;
2. Perform sensitivity analysis around a predetermined base case scenarios to examine the NNEG put option's cost.
3. Investigate model risk in pricing the No-Negative-Equity-Guarantee (NNEG)

1.5 Value of the Study

The research contributes to the literature as an empirical proof of the ARMA-EGARCH modelling framework's reliability in capturing the characteristics of auto-correlation and volatility clustering effects in house price return dynamics. Since the above developments are crucial, it helps in the valuation and pricing of No-Negative-Equity-Guarantee (NNEG) (Ambrose and Buttimer, 2000). Also, given that the major risks involved are the; house price risks, risk-free rate, longevity rate, rental yield, Loan to value (LTV) ratios, and the value of the house used as collateral, the management of such inherent risks becomes an essential element for the annuity providers (Shao et al., 2015).
For the Central Banks, the model developed may establish standards for measuring and

managing various risks as stated in the Prudential Regulation framework (Authority, 2018).

1.6 Organization of the study

Chapter 1 provides a background to the study, outlining the role and status of Equity Release Products in Kenya. It spells out the problem statement and outlines the research objectives of the study. Chapter 2 reviews the existing literature related to the modelling of the ARMA-EGARCH. It also outlines the modelling approaches for the No Negative Equity Guarantee (NNEG). It concludes by providing a reason to develop tools for valuing and pricing the No Negative Equity Guarantee (NNEG). It also provides the model approach to understanding the effects of volatility clustering in the log house price return dynamics. Chapter 3 outlines the research methodology adopted in carrying out the study. In contrast, Chapter 4 scopes the data analysis and discussions conducted to address the study's objectives. Chapter 5 gives conclusions of the research findings and outlines the recommendations arrived at due to the research study. It also highlights areas of further research that the researcher recommends.

Chapter 2

Literature Review

2.1 Introduction

This chapter reviews the existing literature related to the research topic and outlines modelling approaches for the No Negative Equity Guarantee (NNEG). We further introduce a case for ARMA-EGARCH modelling framework and provide a milestone for developing tools for pricing the No negative equity guarantee.

2.2 No Negative Equity Guarantee Modelling Approaches

Equity-release mortgages must include provision for no-negative-guarantee (NNEG) to meet the Product Specifications within the Statement of Principles of Equity Release Council (Tsay et al., 2014). NNEGs protect the borrower by capping the mortgage principal at the lesser of the face amount. Thus, from the financier's point of view, the provision in the NNEG is the same as writing a European put option on the house. The option's exercise price is always equal to the loan's value at the time of loan repayment (Merton and Lai, 2016).

In the continued advancement for the pricing framework for equity release mortgages, the primary interest, thus far, has been indicated to be the house price risks (Merton and Lai, 2016). With the significant hypothesis that house prices follow a standard Geometric Brownian Motion (GBM) for all the home reversion arrangements, thereby expediting the Black and Scholes (1976) option pricing formulae for the pricing of the NNEG. The study by (Ma et al., 2007) attempts to price the NNEG using an actuarial-based model for pricing and value the Korean ERMs with a fixed periodic payment having graduated monthly payments indexed to consumer price growth rates. The findings indicate that any shock to the property prices may heavily impact the younger mortgagors. The same data is then used in a later study by (Daal et al., 2007). The study sampled 3,197 house price index from 1952-2013 that were gathered from the Nationwide House Price Index in the United Kingdom (UK). The paper studied numerous parameters that determine NNEG pricing. The results came to a similar conclusion arguing that Loan-to-Value (LTV) ratio, Interest rates, rental yield, and house price volatility are significant positive determinants of NNEG prices.

Later on, (Hosty et al., 2008) conducted another research where he developed an NNEG a predictor ensemble model with several ERM balances which grow as a claim against the house. (Engsted et al., 2016), also based their research on refining what was done (Hosty et al., 2008). They looked into the WILLIS-SHERRI'S model as a multivariate stochastic mortality model to describe the volatility improvement over time. The model further explains changes in age-specific mortality rates along the cohort direction as a function of age, initial house price and multiple stochastic discount factors. Also incorporated in the multivariate stochastic discount factor is the observed correlations between the year-to-year changes in different age groups' mortality rates. The model permits an elastic and accurate age dependence structure than the one-factor model developed by (Ambrose and Buttimer, 2000) and the two-factor model by (Bardhan et al., 2006).

(Tsay et al., 2014) looks into NNEG valuation using a Vector Auto-Regression (VAR) model to project average future house price growth rates and future risk-adjusted discount factors embedded within the NNEG framework. Five state parameters are incorporated into the model. The optimal parameter of the prototypical VAR is keenly chosen based on three information measures: Akaike Information Criteria (AIC), the Bayesian Information Criteria (BIC) and AIC corrected for small sample sizes (AICc). Though AIC proposes an ideal lag length of three factors, VAR (2) value is very close to both BIC and AIC with an optimal lag length of two, which is not by the valuation and pricing of NNEG (Tunaru, 2017). There are many comparable instances in the housing literature; however, according to the research conducted by (Chen et al., 2010), he concluded that two key characteristics become evidence, and be directly associated with the house price returns. Firstly, the log price house returns are found to be auto or serially correlated to each other. At the same time, the volatility of the log-returns is believed to be time-varying or volatility clustering. To capture these effects, (Chen et al., 2010) and (Li et al., 2017) turned to the use of Autoregressive Moving Average Exponential Autoregressive Conditional Heteroscedasticity (ARMA-EGARCH) family of models.

2.3 A case for the ARMA-EGARCH models

In many empirical studies, the Autoregressive Moving Average Exponential Autoregressive Conditional Heteroscedasticity (ARMA-EGARCH) family of models developed by (Chen et al., 2010) and (Li et al., 2017) addresses three significant problems related to house price return dynamics. The log house prices display autocorrelation (Daal et al., 2007), volatility clustering or time-varying (Fry et al., 2010), and superfluous variance relative to the underlying log house price return fundamentals (Lee et al., 2012). These features were enormously on display during the inordinate subprime mortgage crisis experienced within the; entire United Kingdom's (UK's) mortgage sector, the United States Housing Equity Market, and the World over (Campbell et al., 2009). These key macro-dynamics characterized the general log house price returns even before then. They continued to do so thereafter (Yang, 2011). Given that the effects of such a downward volatility effect are both systematic and non-diversifiable, it resulted in numerous hitches around the housing sub-sector (Fry et al., 2010).

The study conducted by (Chowdhury and Mallik, 2004) and (Chang et al., 2012) used the jump-diffusion process to re-define the alterations in the log house prices, with the latter research indicating that negative and positive innovations have significant impacts on mortgage insurance premiums. Furthermore, the paper by (Remillard, 2013) extended the double exponential jump-diffusion model of (Chowdhury and Mallik, 2004) to integrate the asymmetric jump risk in pricing and to value the insured mortgages. On the other hand, (Fry et al., 2010), Chen et al. (2010), (Byrne et al., 2013) find that accommodating the volatility effects in the log return and housing jumps considerably improved the model fit for the house price return data for pricing the NNEG.

(Lee et al., 2012) from the American Risk and Insurance Association published a paper on NNEG pricing using a dataset of around 11,000 loan and house price data given in a housing challenge. Their findings found that housing price data exhibit serial correlation and an infinitely increasing variance that the Black '76 and GBM failed to capture. Several authors in some past papers have addressed the above problems. (Booth and Marcato, 2004), and (Tsay et al., 2014) presented a generalized ARMA-EGARCH process with the conditional Esscher transform challenged the many setbacks raised against the Black-Scholes and the GBM processes. The results indicate that in the absence of a unique martingale measure, NNEG embedded option is a stochastic process and can therefore be priced based on the generalized ARMA-EGARCH modelling approach. Following (Chang et al., 2012) and (Merton and Lai, 2016), we assume the jump distributions is to Poisson with a time-changing conditional intensity criterion. In the empirical investigation, similar to the path taken by (Li et al., 2017), we focus on the UK equity-release market as a bench-mark for Kenya's case.

2.4 Conclusion

The literature reviewed in this study highlights the milestones made in developing tools for valuing and pricing the No Negative Equity Guarantee (NNEG). Nevertheless, despite the volatility effects having been factored in the modelling of house price dynamics, it appears that each of these prior researches has failed to consider the important properties of volatility persistence and autocorrelation in the log house price returns. We, therefore, aim to fill this gap within the extant literature by considering these factors. More specifically, we study the volatility clustering effects in house price returns based upon an ARMA-EGARCH model specification that allows for both the fixed and the variable house prices and risk neutralize the model parameters to price the NNEG. Following the research done by (Hosty et al., 2008) and (Chen et al., 2010), we further assume that the distributions are Poisson process with a time-varying conditional Esscher transform process. In the following chapter, similar to the methodology in (Hosty et al., 2008), we simulate the ARMA-EGARCH, and GBM-rn using Monte Carlo simulation the risk-neutral measure. We shall refer to this Monte Carlo simulation as the ARMA-EGARCH. Finally, we will test the model with the bench-marked monthly average house price data calibrated with an out of sample loan portfolio for 10,000 subscribers for the reference period from January 1991 to January 2020, obtained from a nationwide building society database United Kingdom (UK).

Chapter 3

Methodology

This chapter outlines the models used to analyse the NNEG data set, the study design and how these models will be evaluated for performance.

3.1 Research Design

The study is an experimental-based work involving simulation of NNEG loadings using ARMA-EGARCH and GBM-rn models.

The baseline scenarios simulations will be tested. The developed model will be evaluated using the bench-marked loan data obtained from the nationwide building society database in the United Kingdom, for the period between – 1991- to- 2020. House price Index (HPI) data was also used to calibrate the loan data.

3.2 Population and Sampling

We model the house price dynamics at two different levels; we first identify the Payoff structure of the NNEG, specify the models under the real-world measure. And then we finally risk neutralising the ARMA-EGARCH model.

The statistical analyses conducted utilises the bench-marked monthly average house price calibrated with an out of sample loan portfolio for 10,000 subscribers gathered from Nationwide Building Society database which we use to validate both the ARMA-EGARCH and GBM-rn models. The selections of sampling frequency and sample periods are constrained by data availability. For the NNEG analysis, we focus on regional and national analysis, where the most dramatic increases in housing prices have been observed over recent periods.

3.3 Data Collection Methods

We first use the Monte-Carlo simulation to generate ARMA-EGARCH and GBM-rn models of historical bench-marked average house price monthly data for the reference between January 1991 and January 2020. The data is then used to generate the NNEG at different rates.

Secondly, the Secondary data for validating the model is obtained from the Nationwide Building Society database and includes an out of sample portfolio for 10,000 loan contracts for 35 years. The analysis further considers 10,000 data points from which, 5073 are male, and 4927 are female loan subscribers, all of which have the data points over the whole period under consideration.

3.4 NNEG Modelling Framework

3.4.1 The Payoff of NNEG

Given that for some considerable time now, it has widely established that all equity release mortgages must include a plan for the inclusion of No Negative Equity Guarantees (NNEG). Thus the most practical way of pricing and valuating the no negative equity guarantee has become very important both within the equity release market and real estate finance as a whole. Ideally, NNEG protects the mortgagors against the risks of no negative equity should the redemption value prove less than the principal amount borrowed. Therefore, the NNEG pricing framework's provisions are equivalent to writing a European put option on the house.

In essence, we adopt the studies conducted by (Chowdhury and Mallik, 2004) and (Shao et al., 2015) and further letting K_T be the outstanding loan balance at time,T,and H_T be the value of the mortgaged property. It therefore, results from the perspective of (Adam et al., 2017) that, the amount of loan repayable at time, T, is ideally, the total sum of all the principal amount, K, and the accumulated interests valued at a fixed rate,

V_T, therefore;

$$K_T = Ke \sum_{k=0}^{\tau=T-1} V_T \tag{3.1}$$

Based on the assumption of (Bjork, 2009), at the time the ERMs becomes due for repayment, if in any case the value H_T <K_T then the mortgagor is likely to pay an amount equivalent to H_T and if otherwise the value is H_T >K_T then the mortgagor pays an amount of K_T. After the mortgage has been repaid fully, the lender receives an amount equivalent to K_T, plus the payoff of the NNEG given by the following formula (Campbell et al., 2009);

$$V(T) = max(K_T - H_T, 0) \tag{3.2}$$

where $V(T)$ is the investor's (lender) actual net payoff at maturity, on the other hand, K_T indicate the remaining ERM loan balance after maturity date while H_T represent market value of the collaterised house.

Adopting suggestion by (Daal et al., 2007) and (Commission et al., 2011) “that NNEG becomes due when the borrower dies.” ”Thus for a mortgagor who is aged x years at the commencement of the contract, the expected total cost of the NNEG can implicitly be expressed as a sequence of European put options with diverse dates of maturity” (Ambrose and Buttimer, 2000). (Winkelmann, 2008) considered the logic that the no –arbitrage value of NNEG under discrete time scale is similar to the ”fair value of the expected cost of NNEG put option”. Essentially, (Chen et al., 2010) supports that the numerical value satisfy the equation:

$$V(0,x) = \sum_{k=0}^{\eta-x-1} P_x^Q(0,x) q_x^Q(0,x) V(0,s) \tag{3.3}$$

where η is considered to be the longest ”age” of the mortgagor as defined by (Winkelmann, 2008), ”$P_x^Q(0,x)$ is the projected probability of a mortgagor who is currently aged x at contract commencement surviving to age $x+s$” (Fry et al., 2010) and ” $q_x^Q(0,x)$ is the probability that a mortgagor aged x at the ERM inception dying during the time horizon from s to $s+1$ under the risk adjusted probability measure Q or otherwise called the risk-neutral measure" (Tunaru, 2017).

To deal with the no-arbitrage value of $V(0, x)$, (Merton and Lai, 2016) proposes that we ideally need to discount the existing payoff of the NNEG at a time limit x under the equivalent martingale measure Q (also known as the risk-neutral measure), given by the following equations;

$$V(0, x) = E^{Q}\left[exp(-\int_{0}^{x} r_t dt)max(K_x - H_x, 0)\right] \tag{3.4}$$

where the "r_t is ideally the prevailing risk-free rate of interest applicable in the market at the time, t, and $K = L_0 e^{RT}$." From here, we solve for the equivalent martingale measure Q of the underlying average house price returns under generalized "ARMA-EGARCH" modelling process. To do this, (Booth and Marcato, 2004) and (Tsay et al., 2014) proposes the use of "Conditional Esscher Transform (CET) process" to formulate the equivalent martingale measure under risk-neutral world. Ideally, we can also not overlook the effects of mortality and interest rates because of the long and uncertain feature of the NNEG.

3.4.2 Econometric Methods for ARMA-EGARCH

The ARMA-EGARCH model consists of two significant steps. The first step is to specify the model under the real-world measure. In the second step, we risk neutralising the ARMA-EGARCH.

3.4.2.1 Model Specification under real-world measure

An experimental investigation into the classical features of "volatility clustering" and the ideal "effects of auto-correlation" using the dynamics of average house price returns was first done by (Chen et al., 2010) and (Li et al., 2017)". (Chen et al., 2010) and (Li et al., 2017) constructed an average house price return model that could capture the features of volatility clustering and auto-correlation effects. Thus, based on the study conducted by (Hosty et al., 2008), we first study the effects of jumps and volatility clustering in the average house price data. Basing our analysis on some time-series returns, we then proceed to derive the corresponding risk-neutral measure under the suggested ARMA-EGARCH model. To introduce the risk adjusted probability measure Q as in (Hosty et al., 2008), we define a total probability space (η,ϕ, Ω, (θ_t)). Ω is ideally the data generating process for the probability measure, having model specifications for the conditional mean and variance (Li et al., 2017). Based on the study by (Bardhan et al., 2006), we further let H_t denote the current period house price, while Y_t is expressed as the house price return at some implicit time t. Thus under discrete time steps, Y_t can be defined as $ln(\frac{H_t}{H_{t-1}})$ that gives a response to the ARMA-EGARCH process controlling the house return framework given by

$$Y_t = ln\left[\frac{H_t}{H_{t-1}}\right] = \mu_t + \vartheta_t \tag{3.5}$$

Where H_{t-1} is the previous house price returns. The mean return are usually randomly adjusted to follow an "ARMA (w, W) process" as;

$$\mu_t = C + \sum_{i=1}^{w} \Phi_i Y_{t-i} + \sum_{j=1}^{W} \theta_j \vartheta_{t-j-i} + \vartheta_t \tag{3.6}$$

Where C is the drift of the model, w the order of the Auto-Correlation term(s), W is the order of the Moving-Average term(s), Φ_i and θ_j are ideally the weights on the previous ith log return, Y_{t-i} and the previous jth innovation, ϑ_{t-j-i}, for $j = 1, \cdots, W$, ϑ_t represent the aggregate new innovations flowing to the house and impacting the house prices at the current time, t (Bardhan et al., 2006). Thus extending the study by (Chang et al., 2012), we fix two stochastic parameters ($\vartheta_{1,t}$) and ($\vartheta_{2,t}$) from the error term from which the first part exclusively captures growing variations in the conditional variance. In contrast, the remaining part deals with infrequent but large jumps in the general return data. Thus the general framework of the error term can then be given as;

$$\vartheta_t = \vartheta_{1,t} + \vartheta_{2,t} \tag{3.7}$$

This process then allows us to set $\vartheta_{1,t}$ as a mean zero innovation with an expectation of ($E\left[\vartheta_{1,t}|\Phi_{i-1}\right] = 0$), with a standard stochastic forcing method as

$$\vartheta_{1,t} = \sqrt{h_t}\theta_t \tag{3.8}$$

and $\theta_t \sim \mathcal{N}(\mu, h_t)$. More precisely, we further assume a normal distribution of $\vartheta_{1,t}$ with mean zero and variance h_t as in (Chang et al., 2012).

To increase our flexibility and quantify the leverage effect that we found in the Nationwide House Price Index, (Adam et al., 2017) extends the paper by (Li et al., 2017) for ARMA (w, W) model by adding the serial dependence of the conditional variance h_t for an Exponential GARCH (EGARCH (M, R)) model, the standard logarithmic notation for conditional variance, ln(h_t), at time $t > 0$ can thus be set as in (Chen et al., 2010);

$$ln(h_t) = K + \sum_{i=1}^{w} \alpha_i ln(h_{t-1}) + \sum_{j=1}^{R} \beta_j |\acute{\vartheta}_{t-j}| - E|\acute{\vartheta}_{t-j}| + \sum_{j=1}^{R} \varphi_j |\acute{\vartheta}_{t-j}| \tag{3.9}$$

Where $\acute{\vartheta}_t = \frac{\vartheta_t}{\sqrt{h_t}}$ is the standardized innovation at time t, "α_i and β_j" are the thus the weights of the previous ith log conditional variance, $ln(h_{t-1})$ and the jth previous standardized innovation $\acute{\vartheta}_{t-j}$.

An optimal leverage effect is mandatory within the real estate sector, (Commission et al., 2011) proposes parameter estimates φ_j, $j = 1, \cdots, R$ that enables the conditional variance to react asymmetrically to any innovations within the house prices, we thus need no parameter constraints to ensure positive conditional variances (Hosty et al., 2008).

3.4.2.2 Risk Neutralisation of ARMA-EGARCH

To price and value the NNEG, we need to obtain the equivalent risk-neutral measure based on the planned ARMA-EGARCH framework by applying the "Conditional Esscher Transform (CET) process" (Ambrose and Buttimer, 2000). We thus define a range of probability space (Ω,F,$\mathbb{P}$), with the $\mathbb{P}$ as the house price data generating parameter indexed to all economic multidimensional variables occurring at time t, where t $\in \tau$, and τ is the index time set $\{0, 1 \cdots\cdots, \tau\}$, it then follows that, $\theta = \theta\{\theta_t\}_{t\in\tau}$ is the data filtration such that for t $\in \tau$, θ_t comprises all the available information in the market up to and including time t and $\theta_\tau = F$. Estimating the CETP parameters can therefore be viewed as an extension of the (Lee et al., 2012) convergence for a locally risk neutralised Gaussian ARMA-EGARCH models the bivariate diffusion process. It is therefore, important to accept that under the probability measure $\mathbb{P}$, the indexing of the log-house price return(s) $\{Y_t\}_{t\in\tau}$ ideally follows an ARMA (w, W)-EGARCH(M, R) with Gaussian innovations (Siu-Hang Li et al., 2010). Corresponding to the real world measure ($\mathbb{P}$), the log house price return can be given by the following equation (Lee et al., 2012);

$$Y_t|\theta_{t-1} \sim \mathcal{N}(\mu, h_t) \tag{3.10}$$

where $\mu = C + \sum_{i=1}^{w} \Phi_i Y_{t-i} + \sum_{j=1}^{W} \theta_j \vartheta_{t-j-i} + \vartheta_t$. Agreeing with the notation by (Chen et al., 2010), we specify the sequence $\{Z\}_{t\in\tau}$ with $Z_0 = 1$ and for $t \geq 1$ adapted stochastic process (Bühlmann et al., 1996):

$$Z = \Pi_{k=1}^{t} \frac{e^{\lambda_k Y_k}}{E(e^{\lambda_k Y_k}|\theta_{k-1})} \tag{3.11}$$

for some continuous random variables $\lambda_1, \lambda_2, \cdots\cdots, \lambda_t$ and Y_k represent the house price return dynamics.

Following (Siu-Hang Li et al., 2010) definition of the martingale measure under risk neutral world, the resulting function can be concluded that $\{\mathbb{Z}_t\}_{t\in\tau}$ is a martingale. Adopting (Bühlmann et al., 1996) definition of a martingale measure, we define some restrictions $\mathbb{F}$ on the probability measure $\mathbb{P}$ and some set of information θ_t to each of the individual data set $t \in \tau$ to create a group of measures $\{\mathbb{P}\hat{_{t\in\tau}}\}$ such that $d\hat{\mathbb{P}}_t = Z_t d\mathbb{P}_t$ and $\hat{\mathbb{P}} = \mathbb{P}_{t+1}|\theta_t$ and probability measure $\hat{\mathbb{P}} = \hat{\mathbb{P}}\mathcal{T}$ on the sample space $(\Omega,\, F)$. We follow (Bühlmann et al., 1996) to set further that A be an open interval on a real time scale, and thus allowing that $I(Y_t \in A)$ be a real pointer function with a conditional market event satisfying the parameter $\{Y_t \in A\}$, is ideally, the Conditional Esscher Transform (CET) expressed as in (Lee et al., 2012);

$$\hat{\mathbb{P}}_t(Y_t \in A|\theta_{t-1}) = \mathbb{E}\left[I(Y_{t\in A})\frac{e^{\lambda_t y_t}}{\mathbb{E}_{|t}(e^{\lambda_t y_t})}|\theta_{t-1}\right] \tag{3.12}$$

From equation (3.5) it follows then that the probability distribution of y_t given some filtration $\theta t - 1$ under the martingale measure $\hat{\mathbb{P}}_t$ after replacing the $(-\infty, y)$, where y is a real number, can be obtained by

$$F_{\tilde{p}t(y:\lambda_t|\theta_{t-1})} = \frac{\int_{-\infty e^{\lambda x} dF_{Pt}(x|\theta_{t-1})}^{y}}{E_{pt}(e^{\lambda_t Y_t|\theta_{t-1}})} \tag{3.13}$$

Using the paper by (Chen et al., 2010), we estimate that the moment generating Function (MGF) of Y_t given θ_{t-1} under the risk neutral measure $\hat{\mathbb{P}}$ embedded in the (Lee et al., 2012) model is given by

$$E_{\tilde{P}t(e^{zYt;\lambda_t|\theta_{t-1}})} = \frac{E_{Pt}e^{(z+\lambda t)Y_t|\theta_{t-1})}}{E_{Pt}(e^{\lambda tY_t|\theta_{t-1}})} \tag{3.14}$$

Similarly, using equation (3.7) and from the intuition that $Y_t|\theta_{t-1}\sim \mathcal{N}(\mu, h_t)$. We can approximate the parameters using the following equation:

$$E_{\tilde{p}t(e^{zYt;\lambda_t|\theta_{t-1}})} = e^{(\mu_t+h_t\lambda_t)z+\frac{1}{2}h_tz^2} \tag{3.15}$$

(Chang et al., 2012) decided to enhance the precision of the (Bühlmann et al., 1996) model by constructing an equivalent martingale measure $\mathbb{Q}$ which ideally is similar to house price data generating parameter $\mathbb{P}$. From (Chang et al., 2012) we select an arrangement of Conditional Esscher Transform (CET) parameters $\{\lambda_t^q\}_{t\in\tau}$ such that the new risk- neutral probability function is set as:

$$E_{\tilde{p}t(e^{Y_t;\lambda_t^q|\theta_{t-1}})} = e^{r_t-g} \tag{3.16}$$

Where (r_t) is the market risk free rate while g is the monthly rental growth rate. Based on the preposition by (Bühlmann et al., 1996), we substitute $\{\lambda_t^q\}$ into equation (3.8) resulting into the risk-neutralising constant given by

$$E_{\mathbb{Q}_t}(e^{zY_t}|\theta_{t-1}) = e^{(r_t-g-\frac{1}{2}h_t)z+\frac{1}{2}h_tz^2} \tag{3.17}$$

where $\mathbb{Q}_t$ is a real time restrictions of $\mathbb{Q}$ on house price information θ_t, and $\mathbb{Q}_\tau$= $\mathbb{Q}$. Thus, under $\mathbb{Q}$, $Y_t|\theta_{t-1} \sim \mathcal{N}(r_t - g - \frac{1}{2}h_t, h_t)$ or equally, $Y_t = \mu_t + \varepsilon_t$ and $\varepsilon_t \sim \mathcal{N}(r - g - \frac{1}{2}h_t, h_t)$. That is, the dynamics of Y_t under the comparable risk-neutral measure is equivalent to that under the real-world measure, with an exception of the distribution of ε_t which is interpreted by an equivalent extent of the $-\mu_t + r - g - \frac{1}{2}h_t$. Finally, under the equivalent martingale measure $\mathbb{Q}$, the time t, where t $\in \tau$, the price of an option that gives a payoff V_τ a at maturity is given by the following equation (Tsay et al., 2014):

$$V_t = E_{\mathbb{Q}_t}\left[e^{-r}(T-t)V_\tau|\theta_t\right] \tag{3.18}$$

3.4.3 Cost of No Negative Equity Guarantee

Even though a substantial percentage of ERMs are lent to couples, we assume a single (male or female policyholder) and joint-life (where both husband and wife are involved in the ERM borrowing). We also do not take into consideration the effects of voluntary pre-payments. Ideally, the ERM contract's termination can thus be determined by two significant events: the policyholder's death and early entry into long-term care. We further analyze the impacts of risks involved in the No negative equity guarantee (NNEG). Thus, the risk neutralized value of NNEG is dependable upon average house price return dynamics, the prevailing risk-free rate, rates of mortality, roll-up rate, the volatility of house prices, and rental yield (Authority, 2018). Considering all these dynamics substantially rises the exertion in pricing and valuation of no negative equity guarantee as in equation (3.3).

To solve these problems systematically, we use Monte Carlo simulations as defined by (Bühlmann et al., 1996). We logically "generate 10,000 sample paths of the risk- neutralized average house price returns, interest rates, mortality rates, based on equations (3.5) separately" and finally computing the cost of NNEG ($V(0, x)$) centred around equations (3.3), and (3.4). Also, we further adopt that all policyholders die at half-year and therefore, "δ is the average delay in the actual sale of the house in determining the cost of the NNEG" (Shitote et al., 2006).

We further adopt the suggestion by (Bühlmann et al., 1996) and consider a floating roll up loan with a floating interest rate of (V_t). Thus, the floating rate (V_t) can be equated to be the same to the risk free rate (r_t) plus a fixed rental spread of (v_{r-g}) that is, $(V_t) = r_t + (v_{r-g})$.

For purposes of comparison, we follow the study by (Hosty et al., 2008) and set-up important risk analysis conventions for the pricing of the NNEG and provide these critical conventions are as shown in the preceding Table.

3.4.4 Parameter Estimation

When pricing the NNEG, it is imperative to divide the data into two groups: the house price data and the loan data. This enables us to simulate the ARMA-EGARCH-rn using the house price data and get more relevant forecasting parameters; the simulated average house-prices are then used to calibrate the loan data. Once the NNEG has been generated, it is essential to evaluate the model fit and test how good the model is in its prediction. The ARMA-EGARCH parameters can be assessed using the following techniques, namely Maximum- Likelihood- Estimation (MLE) method, Method- of- Moments (MM), and Generalized-Method- of-Moment (GMM).

3.4.4.1 Maximum Likelihood Estimation (MLE) Method

The specifications of the proposed ARMA-EGARCH family of models will thus be assessed using the Maximum Likelihood Function (MLE). To construct the MLE, we follow (Chen et al., 2010) estimation method, First, we let $F_n(\gamma)$ represent the Log-Likelihood function and γ be the set of parameter(s) that controls the projected ARMA-EGARCH, implying that the sets in γ=(C, θ_1, θ_2, α, β, λ_0) are controlled by the functionality of (Li et al., 2017) . We aim to extend the concepts above and find the optimal parameter ($\gamma*$) that maximizes the Log-Likelihood function. The log-likelihood function may be thus expressed as in (Blevins et al., 2017);

$$F_n(\gamma) = \sum_{t=1}^{N} \ln f(Y_t|\theta_{t-1}, \gamma) \tag{3.19}$$

The conditional volatility clustering occurring on the conditional density of returns is Gaussian, if and only if;

$$f(Y_t|N_{t-1} = j, \theta_{t-1}, \gamma) = \frac{1}{\sqrt{2\pi(h_t + j\theta^2)}} \exp\left[-\frac{(Y_t - u_t + \lambda_t - j\lambda)^2}{2\pi(h_t + j\theta^2)}\right] \tag{3.20}$$

From the equation (3.16), the conditional density function of return at some specific time, t, is ideally ($f(Y_t|\theta_{t-1}, \gamma)$, for calculating Log-Likelihood-function can thus be found by integrating out the number of volatility clustering affecting the jump diffusion (Adam et al., 2017) thus;

$$f(Y_t|\theta_{t-1}, \gamma) = \sum_{j=0}^{\infty} f(Y_t|N_{t-1} = j, \theta_{t-1}, \gamma) p(Y_t|N_{t-1} = j, \theta_{t-1}, \gamma) \tag{3.21}$$

$$= \sum_{j=0}^{\infty} \frac{1}{\sqrt{2\pi(h_t + j\theta^2)}} \exp\left[-\frac{(Y_t - u_t + \theta_t - j\lambda)^2}{2\pi(h_t + j\theta^2)}\right] \frac{\exp(-\lambda_t)\lambda_t}{j!} \tag{3.22}$$

Where the conditional density function is expressed as $N_t(p(Y_t|N_{t-1} = j, \theta_{t-1}, \gamma))$ which is indicated in our previous equation (3.11). Assuming that the time varying conditional volatility parameter (λ_t) that follows a proposed ARMA-EGARCH model, we ideally need to solve for the existence of the shock (β_{t-1}) that impacts on the conditional volatility variance for any counting process thus,(β_{t-1}) may be defined as in (Tunaru, 2017);

$$\beta_{t-1} = E\left[N_{t-1}|\theta_{t-1}, \gamma\right] - \lambda_{t-1} \tag{3.23}$$

$$= \sum_{j=0}^{\infty} jp\left[N_{t-1} = j|\theta_{t-1}, \gamma\right] - \lambda_{t-1} \tag{3.24}$$

Where $E\left[N_{t-1}|\theta_{t-1}, \gamma\right]$ may be given by the equation (3.11). This expression may hence be assessed if and only if $p(N_{t-1} = j|\theta_{t-1}, \gamma)$ are well known. Following (Ma et al., 2007) and (Shao et al., 2015), the actual returns probability for the existence of J at discrete time $t-1$ and may then be inferred with the help of Bayes formula (Shao et al., 2015);

$$E\left[N_{t-1}|\theta_{t-1}, \gamma\right] = \sum_{j=0}^{\infty} jp\left[N_{t-1} = j|\theta_{t-1}, \gamma\right] \tag{3.25}$$

$$= \sum_{j=0}^{\infty} j\frac{f(Y_{t-1}|N_{t-1} = j, \theta_{t-2}, \gamma)p(N_{t-1} = j, |\theta_{t-2}, \gamma)}{f(Y_{t-1}|\theta_{t-2}, \gamma)} \tag{3.26}$$

$$= \sum_{j=1}^{\infty} \frac{\frac{1}{\sqrt{2\pi(h_t+j\theta^2)}} \exp\left[-\frac{(Y_t-u_t+\theta_t-j\lambda)^2}{2(h_{t-1}+j\theta^2)}\right] \frac{\exp(-\lambda_t\lambda_t^j}{j!}}{f(Y_{t-1}|\theta_{t-2}, \gamma)} \tag{3.27}$$

Based on the assumption of (Tunaru, 2017) and (Blevins et al., 2017), we, therefore, simulate equation (3.15), (3.21) and (3.23) based on the ARMA-EGARCH framework.

Chapter 4

Data Analysis, Results and Discussions

4.1 General Data Overview

The housing data required to validate the NNEG under ARMA-EGARCH and GBM-rn was not available in the Kenyan context. Despite the many housing incentives, Kenya has failed to develop a robust house ownership system leading to fairly low housing data. To solve this, we bench-marked monthly nation-wide average house price data by (Hosty et al., 2008) calibrated with an out of sample loan portfolio of 10,000 subscribers for the reference period between January 1991 and January 2020 and had details such as the amount borrowed, age, marital status, and sex among others during the period. House price Index (HPI) data was also used to calibrate the loan data.

Careful selection of modelling technique had to be done, which is an integral part of this research. "Before the data was used for analysis", it had to be cleaned and organized in a way such that analysis was possible. The categorical values such as gender were classified according to their termination probabilities based on their mortality, morbidity and prepayment. Loan to Value (LTV) ratios were categorized into Flexible, Flexible Max and Flexible Max Plus.

4.1.1 Empirical Analysis of Model Fit

The metrics that were highlighted in chapter 3 were used to evaluate the model for performance and accuracy. We assess the general outlook of the proposed ARMA-EGARCH framework while at the same time applying the time-series data from the bench-marked monthly nation-wide average house price data, "paying special attention to the exploration into whether the conditional volatilities are time-varying or volatility clustering". Finding a universal measure for measuring performance is often problematic and tends to be subjective. In practice, the optimal valuation model would be a data generating process, which would be a function that provides superior forecasting compared to other paired models.

The summary statistics on the estimated drifts and volatility parameters for the "Log-Likelihood", "Method of Moments (MM)" and "Generalized Method of Moments (GMM)" are all described in Table 4.1", from which there is an unambiguous indication of volatility clustering using Diebold-Mariano (DM) accuracy-test (Li et al., 2017).

	mu	sigma
Log-Likelihood	0.054	0.040
Generalized Methods of Moments	0.029	0.040
Method of Moment	0.054	0.040

Table 4.1: Parameter estimates applied to the bench-marked average house price monthly data for the reference between January 1991 and January 2020.

We examine the volatility clustering for both the predictive and fixed stochastic discrete-time models, using the proposed ARMA-EGARCH family. The parameter estimates of the ARMA-EGARCH models are projected by the "maximization" of the provisional "log-likelihood function". The model selection in the proposed ARMA-EGARCH is based upon a trade-off between a forward-looking "Akaike Information Criteria (AIC)", and the "Bayesian Information Criteria (BIC)," from which we capture their effects using the "Occam's razor", that is, looking for the least possible number of values that not only has a substantial estimate but also afford a precise fit to other data generating processes. Details on the valuation of ARMA-EGARCH family of models and their applicable approximations are as presented in the Table 4.2 below.

Parameter	Estimates	Standard Error	t-Value	Pr(>|t|)
mu	0.006	0.00002	413.519	0
ar1	0.946	0.003	348.275	0
ar2	−0.811	0.002	−371.436	0
ar3	−0.003	0.0004	−7.536	0
ar4	0.337	0.001	470.767	0
ma1	−0.716	0.002	−366.641	0
ma2	1.047	0.001	1, 338.811	0
ma3	0.005	0.0005	11.649	0
omega	−0.810	3.149	−0.257	0.797
alpha1	−0.091	0.493	−0.185	0.853
beta1	0.914	0.337	2.708	0.007
gamma1	0.177	0.832	0.213	0.831

Table 4.2: Parameter estimates and Model fit for the fitted ARMA (4,3)-EGARCH(1,1) on the HPI Data

Our empirical findings show the robustness of the ARMA (4, 3)-EGARCH (1, 1) model over other existing models. The ARMA (4, 3)-EGARCH (1, 1) also exhibits supplementary improvements centered around the goodness of fit estimations. Further, the model has the highest accuracy score. “The in-sample fit is exceptional and the conditional volatilities series are in the expected range, varying with a 23 in a mean-reverting fashion around the value of 1%”.

4.1.2 Forecasting and Comparison

To model house price dynamics, as opposed to employing the static mortality models, we ideally consider the house price volatility risk in the pricing and valuation of NNEG (Bühlmann et al., 1996) to be able to project future house changes over the long horizon. Ideally, to be able to have a more robust examination into the importance of serial correlation and volatility clustering, we follow (Ambrose and Buttimer, 2000) guidelines and match the performance of the ARMA (4,3)-EGARCH (1,1) with another conditional volatility group of models such as the Geometric Brownian Motion (GBM-rn) (Tunaru, 2017). The models are then fitted with the same time-series data, and the results are observed over a long period. The fitted results are presented under Diebold Mariano test (Lee et al., 2012) for each of the different models presented in Table 4.3.

Estimation Method	RMSE	MAE
GBM−Log-Likelihood	0.007629534	0.0006037901
GBM−GMM	0.007622328	0.006025631
GBM−MM	0.007780410	0.006198896
ARMA(4,3)−EGARCH(1,1)	0.07100348	0.05913294

Table 4.3: Model Comparison across different estimation techniques

The results indicate forecasting superiority of the “ARMA (4, 3)-EGARCH (1, 1) over the standard GBM-rn model”, the model further provides an improvement on the GBM-rn model constructed based upon the requirements of the “Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) values”.

We reserved the GBM-rn model with the earlier discussed techniques of estimation and the proposed “ARMA (4, 3)-EGARCH (1, 1) that also provides an excellent model fit.” Even at much longer horizons, forecasting under the “ARMA (4, 3) - EGARCH (1, 1) framework” is ideally much better than full range forecasting under the GBM-rn technique.

“As a check of robustness, we further investigate the model fit by considering different periods of the bench-marked monthly nation-wide average house price data. The results for the fourth quarter from 1991 to 2001 and quarters from 2002 to 2012 are as presented in Table 4.4, for both the sub-intervals, the ARMA (4, 3)-EGARCH (1, 1) model is still found to perform much better than the GBM-rn model.”

Model 1	Model 2	Statistic	P-value
GBM−Log-Likelihood	GBM−GMM	−2.3477	0.0278
GBM−Log-Likelihood	GBM−MM	−2.1684	0.0407
GBM−Log-Likelihood	ARMA(4,3)−EGARCH(1,1)	0.2327	0.8180
GBM−GMM	GBM−MM	0.4038	0.6900
GBM−GMM	ARMA(4,3)−EGARCH(1,1)	0.2649	0.7934
GBM−MM	ARMA(4,3)−EGARCH(1,1)	0.2637	0.7943

Table 4.4: Diebold-Mariano Forecast Accuracy testing over the bench-marked monthly nationwide house price data.

Although the effect of volatility clustering is taken into account while using the diffusion models such as those suggested by (Chang et al., 2012), however, their performance are nonetheless considered to be more lower than those presented by the time-series models in which the effects of auto-correlation and volatility clustering are all taken into account, “thus it proves very clearly that a log average house price return model capable of instantaneously taking into consideration all the two important features would represent a significant contribution to this research.”

4.2 Results and Discussion

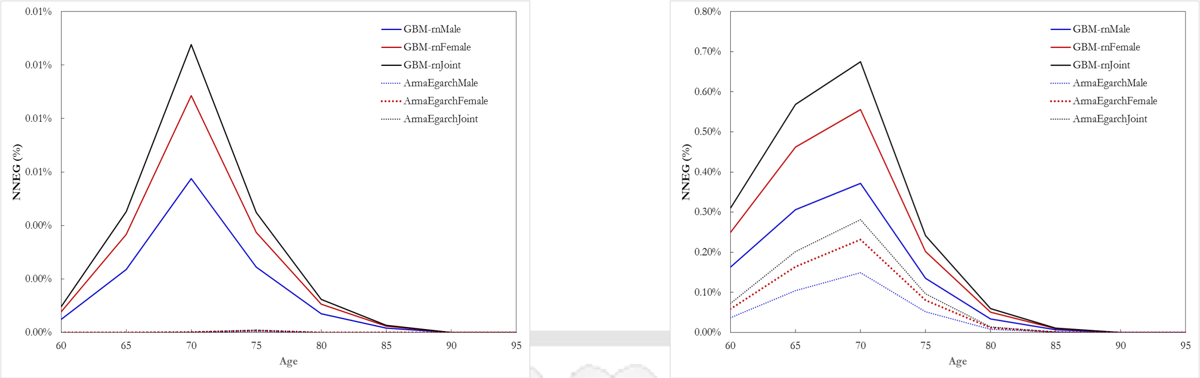
4.2.1 Feature Analysis

To better understand the assumptions used, the table that follows presents a synopsis of the key baseline scenarios used;

Baseline Scenario			
	Notation	Value	Sensitivity
Initial House price	H_0	=3,000,000	={ 60, 70, 80, 90}
Loan advanced	L_0	=3,000,000	=1,800,000
Average delay time (in year)	δ	=0.5	=0.5
Loan to Value Ratio	LTV	= 4.43%	={4.99%, 5.80%}
Roll-up Rate	R	=4.15% and 5.25%	={ 6.15%, 7.15%, 3.5%}
Rental Yield	g	=1%	={ 2%, 3%,0.5%, 0% }
Risk Free Rate	r	=1.75%	={2.0%,2.5%,1.25%,0.75%}

4.2.2 Sensitivity Analysis

In the absence of market bench-market house prices, we conduct a sensitivity analysis around a pre-determined scenario (Authority, 2018). Thus, we carry out a sensitivity investigation to examine the real cost of NNEG under different scenarios. All our simulations are under multiple decrements table, thereafter as a percentage of lump sum advanced. For an initial baseline modelling, both the GBM-rn and the ARMA (4,3)-EGARCH (1,1) results in low NNEG values which could be as a result of small volatility levels representing an ever-increasing house price enough to curtail the outstanding loan balance at a roll-up rate R of 4.15%. However, for the following baseline scenarios, the NNEG values under ARMA (4,3)-EGARCH (1,1) are practically half the values simulated under the GBM-rn model. Their sets are much larger than the initial baseline scenarios. This could be as a result of rather an aggressive roll-up rate $R_2 = 5.25\%$.



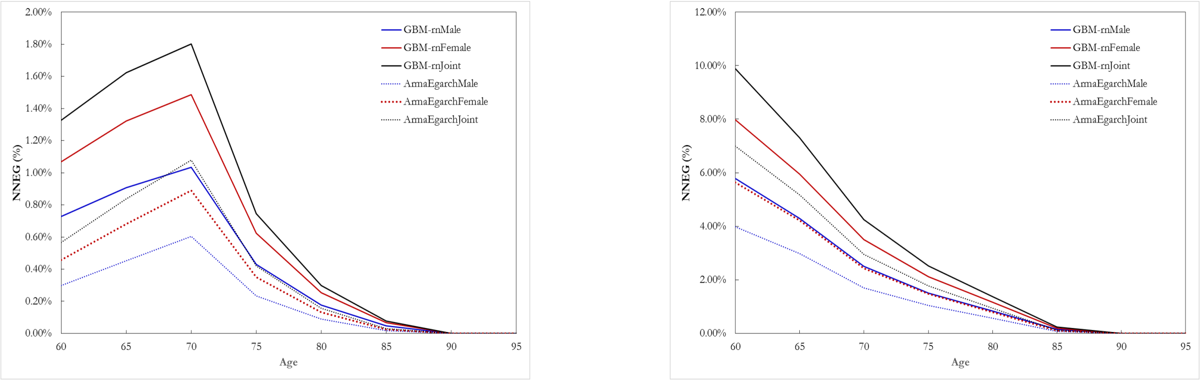
(a) “R1 = 4.15%” (b) “R2 = 5.25%”

Figure 4.1: NNEG valuation under Multiple-Decrements.

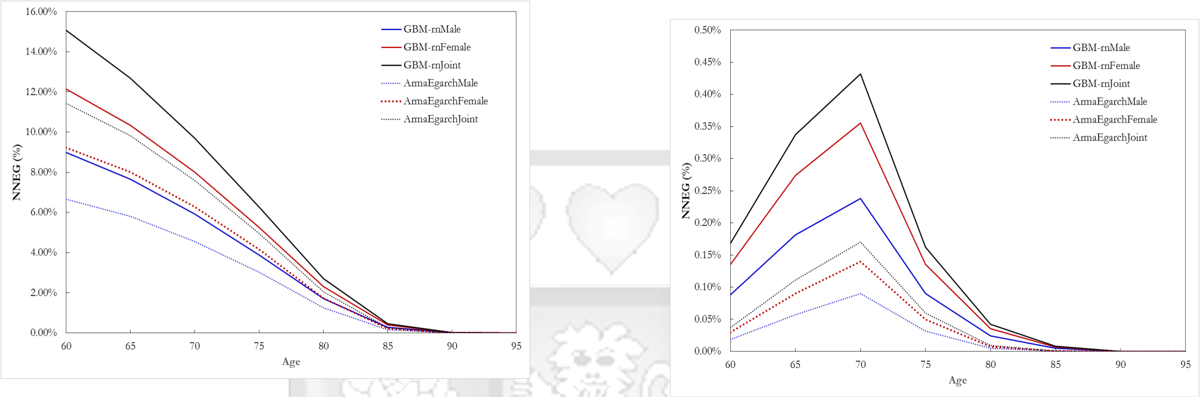
From figure 4.1, we observe that NNEG values are higher at earlier ages of 65th year, 70th year, and 75th year, this is contingent on the type of market calibration actually applied, the value of Loan-to-value ratio (LTV) applied on the loan, among other key parameters ideally used for valuation purposes. However, the values begin to fall rapidly after the 75th year. We also noticed that under current market standardization, “ARMA (4, 3)-EGARCH (1, 1) have a much lower NNEGs value than that of GBM-rn.” This comparative assembling is the opposite of what was discussed by (Li et al., 2017), which may be as a result of many assumptions used. We thus agree that an increase in the age of mortgagors will always result in a decrease in the cost of NNEG, even though, this varies by gender (Hosty et al., 2008). We further conclude that the NNEG costs will be much superior for females than it is for male borrowers, basically because ladies will always have a longer life expectancy than their male counterparts.

4.2.3 Sensitivity to the Loan-to-Value (LTV) Ratio

Figure 4.2 list the relative NNEG valuation under different Loan-to-value (LTV) ratio loadings.



(a) “Flexible Plus” LTV (b) “Flexible Max” LTV



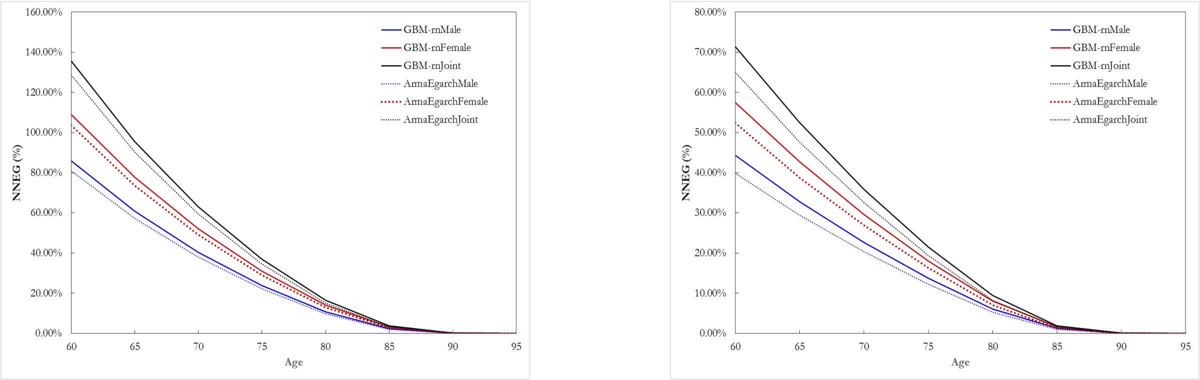
(c) “Flexible Max Plus” LTV (d) ERC

Figure 4.2: NNEG valuations under different LTVs with “r = 1.75%, R = 4.15%, g = 1% and $\sigma = 3.90\%$.”

From figure 4.2, we observe that when all the LTV ratios considered, then, there is an insignificant rise in the cost of NNEG. For male mortgagors who are currently aged 60 years, the cost as a fraction of the loan advanced rises from 5.11% to 5.28% under the proposed “ARMA (4, 3)-EGARCH (1, 1) model”, and from 5.20% to 5.38% under the GBM-rn. It can therefore, be deduced that the most effective way of managing NNEG valuation risks is to carefully align the effects of LTV ratios on the amount of mortgage advanced. We thus assume that the higher the LTV ratios, the higher the steepening of the NNEG rates.

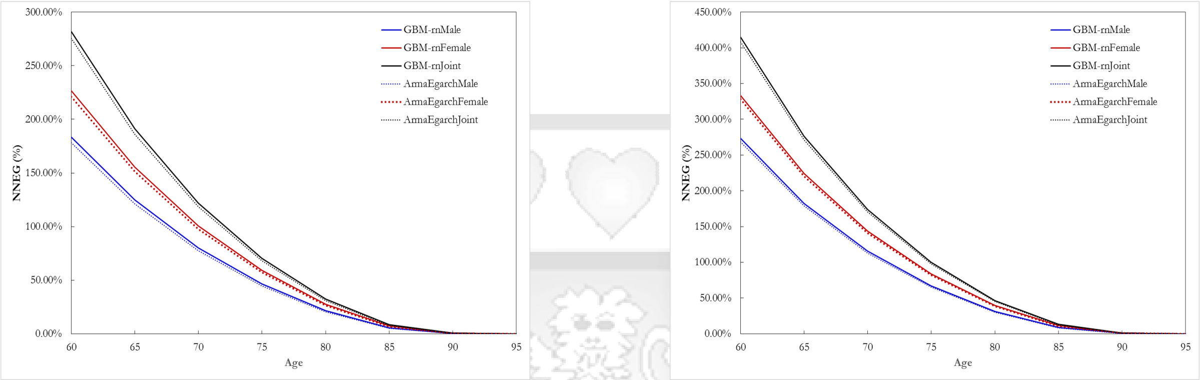
4.2.4 Sensitivity to the Risk-Free Rate

Figure 4.3 lists the summary values of the NNEG under Flexible Max Plus LTV simulated under diverse risk free rates.



(a) Flexible Max Plus LTV at risk free rate of 2%

(b) Flexible Max Plus LTV at a risk free rate of 2.5%



(c) Flexible Max Plus LTV and a risk free rate of 1.25%

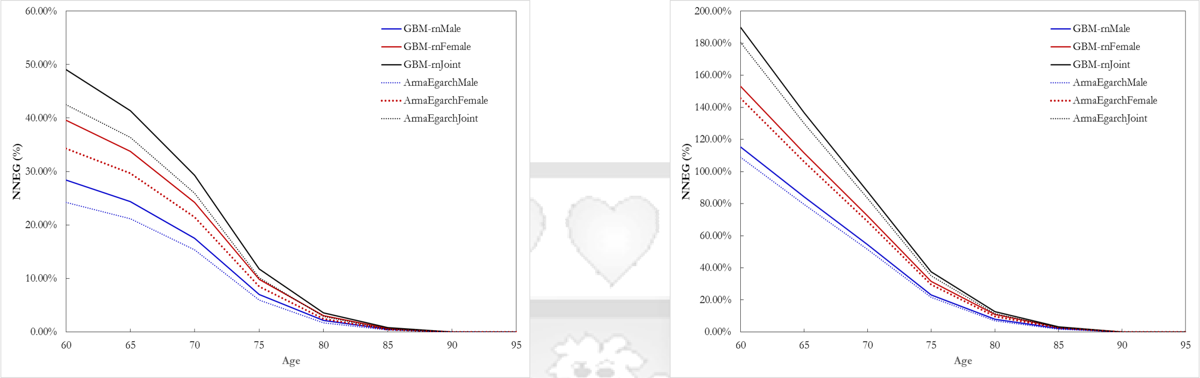
(d) Flexible Max Plus LTV and a risk free rate of 0.75%

Figure 4.3: NNEG valuation under Flexible Max plus, where "R_{fmp}= 5.8%, g = 1%, σ = 3.90%."

From figure 4.3, it is observed that all estimated values for NNEG decrease with a slight increase in risk free rate and increases with an equivalent decrease in r_t "for both the GBM-rn and ARMA (4, 3)-EGARCH (1, 1)," which therefore indicate that a larger r_t moves the planned log future house price return upwards. Besides, in Flexi Max Plus loan-to –value (LTV) loading framework, a value of r_t = 1.25% or 0.75%, provides almost two identical values under the two valuation methodologies. At the same level of risk- free rate, ARMA (4, 3)-EGARCH (1, 1)-rn values are always much lower than the GBM-rn valuation method. This means that the risk-free rate affects the calculation of NNEG in two diverse ways. The most common one is through the discount factor model, "a much lesser risk-free rate that keeps the back- end of NNEG put payoffs at much higher level that is, a high risk-free rate diminishes the back end value of the NNEG put option." And the subsequent possibility is through the calibration of the "conditional Esscher martingale measure (CET) ($r_t - g$)." which also appear in the GBM-rn. This risk-neutralization of the "local drift function" allows the values of the NNEG put option to move in the opposite direction as a risk-free rate (Shao et al., 2015).

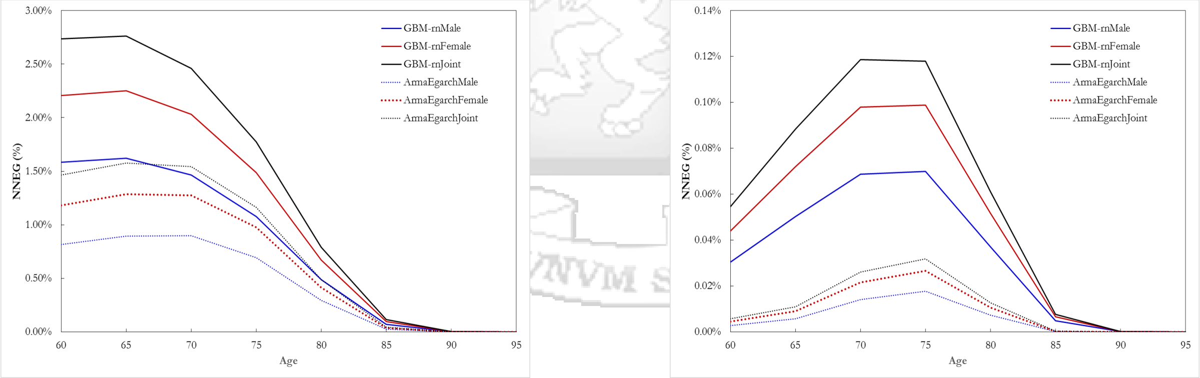
4.2.5 Sensitivity to the Roll-up Rate

Figure 4.4 shows the calculations of the NNEG sensitivities with respect to roll-up rate variations.



(a) ERC LTV and roll-up rate at 6.15%

(b) ERC LTV and roll-up rate at 7.15%



(c) Flexible MaxPlus LTV and roll-up rate at 3.50%

(d) Flexible MaxPlus LTV and roll-up rate at 2.5%

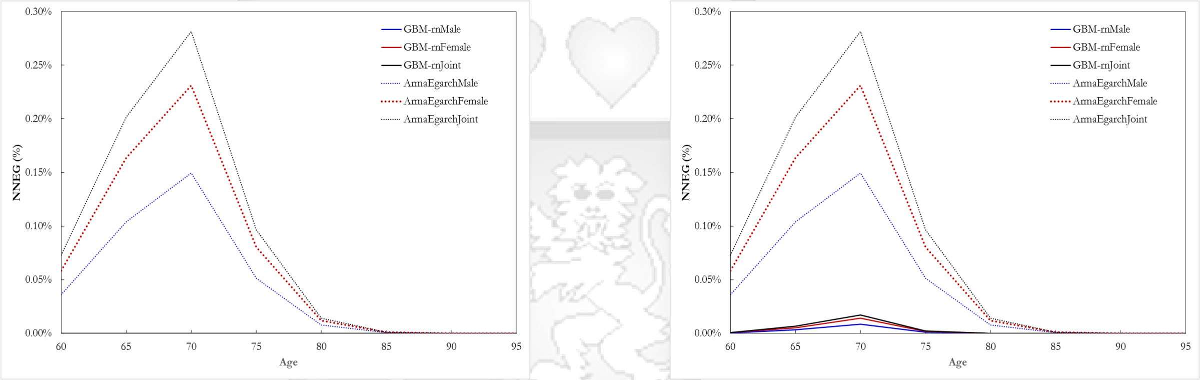
Figure 4.4: "NNEG valuation w.r.t Risk free rate under ERC and Flexible MaxPlus LTV and r= 1.75%, g = 1%, σ = 3.90%."

From figure 4.4, it can be observed that the cost of the NNEG is very "sensitive to the assumptions on the roll-up rate of interest," for instance, an increase in the roll-up rate by say 1%, increases the value of the NNEG for a 60-year-old male mortgagor by more than 200%. For large LTV and large roll-up rates of interest, valuations between GBM-rn and ARMA (4, 3)-EGARCH (1, 1) become indistinguishable. Increasing the roll-up rate broadens the risk spread between the roll-up(R) rate and the risk free rate of interest

$(R - r)$ for profit and expenses but intensely increase the burden from the NNEG. From our study, "it is so obvious that a slight rise in the roll-up rate, compounded monthly to 55 years inflates the accumulated loan balance to very high values thus manage the NNEG levels attached to ERMs, we maintain the roll-up rates as low as possible" (Wang et al., 2016).

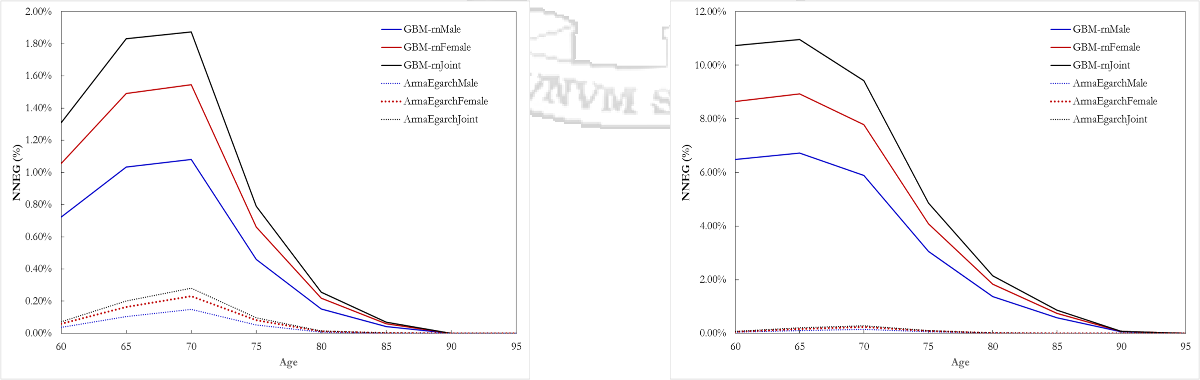
4.2.6 Sensitivity to the house price volatility

Figure 4.5 shows the NNEG values with respect to variations in the volatility of house prices.



(a) Flexible LTV and a volatility of 1%

(b) Flexible LTV and a volatility of 2%



(c) Flexible LTV and a volatility of 5.0%

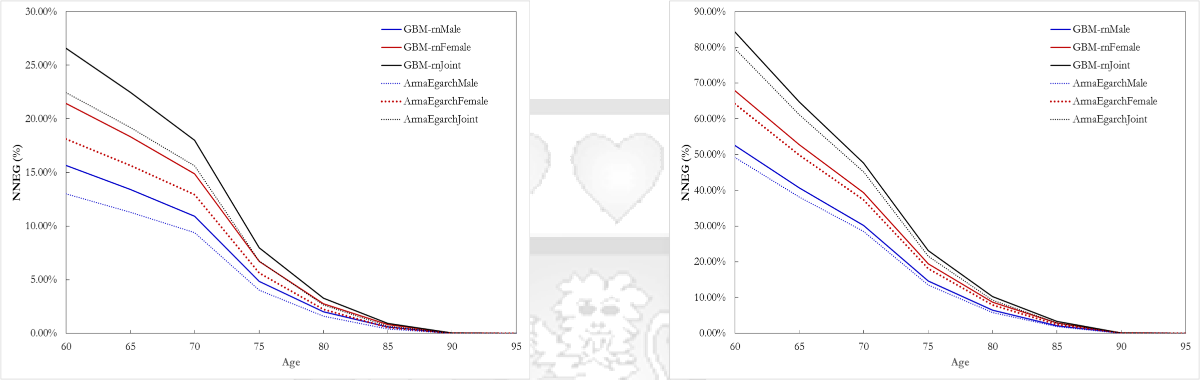
(d) Flexible LTV and a volatility of 8%

Figure 4.5: " NNEG valuation w.r.t.g under baseline loading and $r = 1.75\%$,$R = 5.25\%$."

It is observed that "NNEG values increase with an increase in volatility and decline with the fall in volatility. Increasing the volatility levels tend to double the NNEGs by approximately 75%." "That is the marketability and liquidity of the house would affect the price of the NNEG. An extended delay between property construction and sale raises the cost of the NNEG. An added 1-year delay would otherwise increase the value of the guarantee, for instance, the NNEG cost for a 60-year-old male borrower by approximately 50%."

4.2.7 Sensitivity to the rental yield

Figure 4.6 shows the sensitivity of NNEG valuations with respect to the rental yield g



(a) Flexible Plus LTV and g of 2%

(b) Flexible Plus LTV and g of 3%

Figure 4.6: Sensitivity to the rental yield and r= 1.75%,R= 4.43%.

Rental yield (g) plays a reverse role to the risk free rate of interest (r_t. A higher rental yield computationally results in a much lower risk-neutral drift term such that the house price pathways is downward trending, boosting the NNEG put values. The values of the NNEG put option continues to rise with every increase in rental yield and declines with the fall in rental yield, for both the GBM-rn and ARMA-EGARCH-rn.
"Certainly this is accurate, subsequently a larger value for g implies a low or even negative drift in the risk-neutral world so the projected house prices will be lower in the future, implying a higher NNEG value. The opposite is true, a smaller value for g or even zero, as some insurers are using, leads to a more positive drift in the GBM-rn model that will give increased house prices in the future and hence lower NNEGs."

Chapter 5

Conclusion and Recommendations

5.1 Conclusion

This research's key objective was to ideally develop an innovative financial product capable of boosting retirement income for the retirees to maintain acceptable living standards. This task required:

1. The existence of No Negative Equity Guarantee (NNEG) pricing patterns;
2. Continuous flow of Equity Release Products (ERP) data from the industry players; and
3. Existence of market benchmarked average house prices.

The required information was neither adequate nor available in the market. Presently, Equity Release Mortgages (ERMs) market is not well established in the Kenyan context. To solve this, we first had to carefully consider a case where the original house prices follow a standard "Geometric Brownian Motion (GBM)," since the GBM always result into a" closed-form solution for the NNEG put option prices." We then performed a sensitivity analysis around a pre-determined base case scenario to examine the NNEG put option cost. Careful selection of sensitivity parameters had to be done, which is an essential component of this study.

The housing data used in this study established that ARMA-EGARCH had a better performance compared to GBM-rn and among the time series models. For both, the sequence, "the ARMA (4, 3) - EGARCH (1, 1)" showed the best efficiency in terms of forecasting comparison over a long horizon. "Efficiency in this study was taken to be the capacity of the models to have a good force at one- year horizons, mean reversion at five-year horizons, and excess longer-term volatility relative to the underlying house price fundamentals." The measure that was used to capture this was the root mean squared error (RMSE) which was how well the model fits or explains the data, and "mean average error (MAE) defined as the average of the absolute values of forecasting errors." "A lower RMSE or MAE is an indication of a better forecasting model." However, Diebold-Mariano accuracy test which is an improved approach for comparing forecasting models (Diebold Mariano, 1995) performed well when it came to evaluating the model's performance in providing the superior forecasting power, with a score of 0.2637.

The study also established that the borrower's age at contract inception, coupled with the mortality improvements plays a crucial role in determining the NNEG costs and subsequent cost-effectiveness. "Sensitivity results" also indicate that the profits are ideally much higher for the investors for younger borrowers. However, higher longevity rate enables the property's value to appreciate and means that higher cost of funding.

"Higher Loan to value (LTV) ratios present higher earnings. Sensitivity results to loan interest rate changes were produced in an economic context of low-interest rates, so the model is susceptible to rental yield and volatility. Lower borrowing rates offer higher returns, and more valuable properties also yield more earnings to the lender."

"From the study, we establish that in the absence of market prices or recognized benchmark prices, it is difficult to identify the best pricing model concerning the cost of the NNEG. The best that can be done is to look for a model with a good forecasting power for the house prices; and compare various models across a large set of scenarios, from standard baseline to stressed scenarios. ARMA-EGARCH outperforms the GBM-rn model in terms of forecasting both short and long term horizons. This is not surprising since the theoretical properties of the GBM-rn model is in contradiction with the empirical features of house price time series."

5.2 Recommendations for Further Research

This study has used the proposed ARMA (4, 3)-EGARCH (1, 1) to model and determine the payoff structure of NNEG. Our analysis has also shown that it ideally offers a more improved model fit than other previous models suggested in the preceding literature. The model specification under real-world measure further allows for the analysis of; house price volatility, LTV, and interest rate risk.

Advances in technology and the upsurge in the longevity risks have greatly influenced Equity Release Mortgages (ERMs) in the global ageing society. Therefore, an insurer issuing such loans need first to analyze and consider the effects of different risk factors on the cost of the NNEGs. Also, a more comprehensive multivariate time series approach should be used where Kenyan data could be used alongside data from other markets such as South Africa, Singapore and Australia.

Also, in our discussions, we equivalently considered the effects of equivalent martingale measure that allows for the time variation in; the HPI data, mortality estimates, intensity of the jump parameters, and as well as the changes in the LTVs, but ideally, there need to prolong this to different jump setups such as the well-known stochastic volatility clustering in the innovations introduced by (Wang et al., 2016). It could also be quite interesting to discount the cash flows arising from the NNEG using the risk-adjusted stochastic discount factors as the one discussed in (Chang et al., 2012).

An impending line of research may also include an "understanding of the effect of mortgagor's age, gender, marital status, income and home address in the contract's termination." As stated earlier, "the risk of termination can also be triggered by borrowers' mobility, which can also be discussed in future studies."

Concerning the profitability investigation, impending studies may pay much attention to studying the importance of having fixed income streams and inflation-indexed income streams payments and making a comparison across the lump sum payments.

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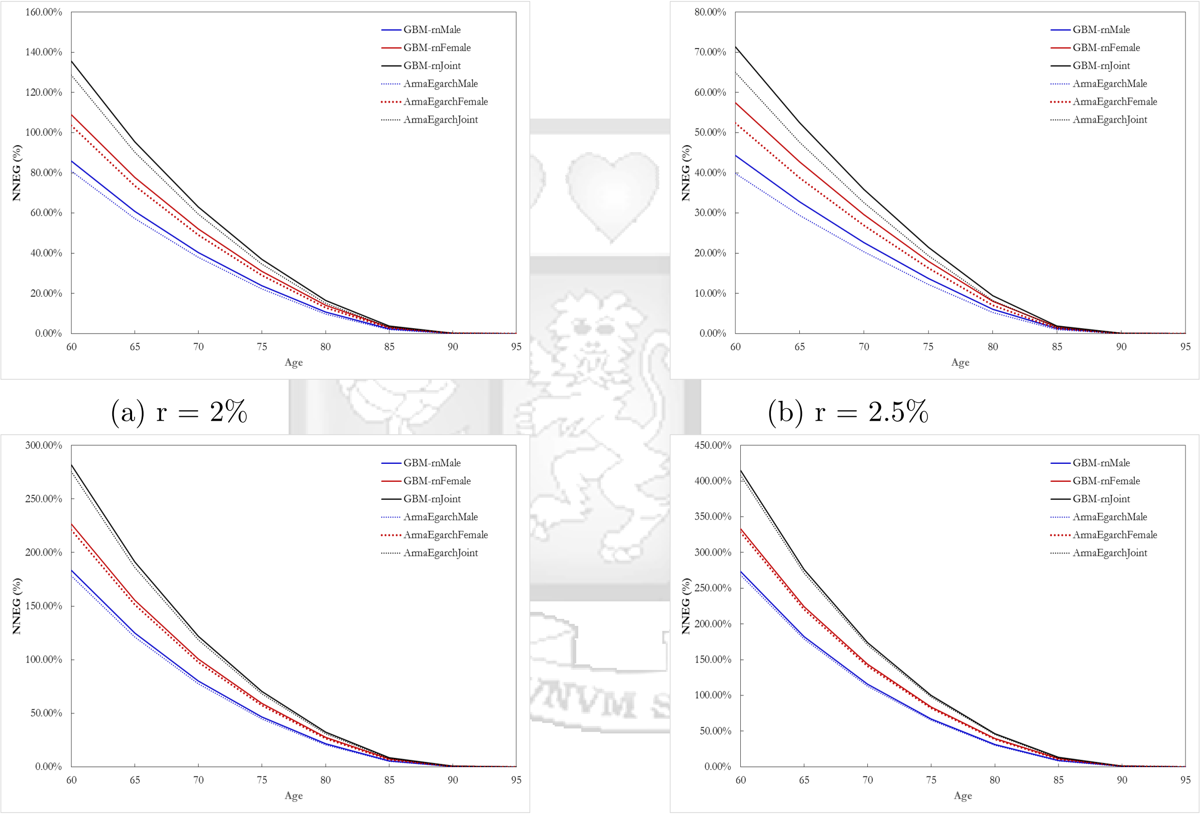
Appendix A

Flexible Max LTV loading

A.0.1 Additional Simulations Results



(a) r = 2%

(b) r = 2.5%

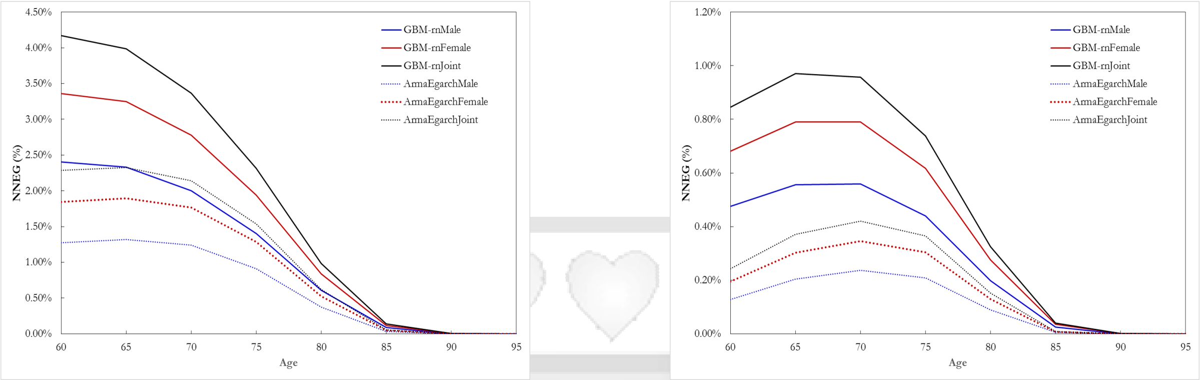
(c) r = 1.25%

(d) r = 0.75%

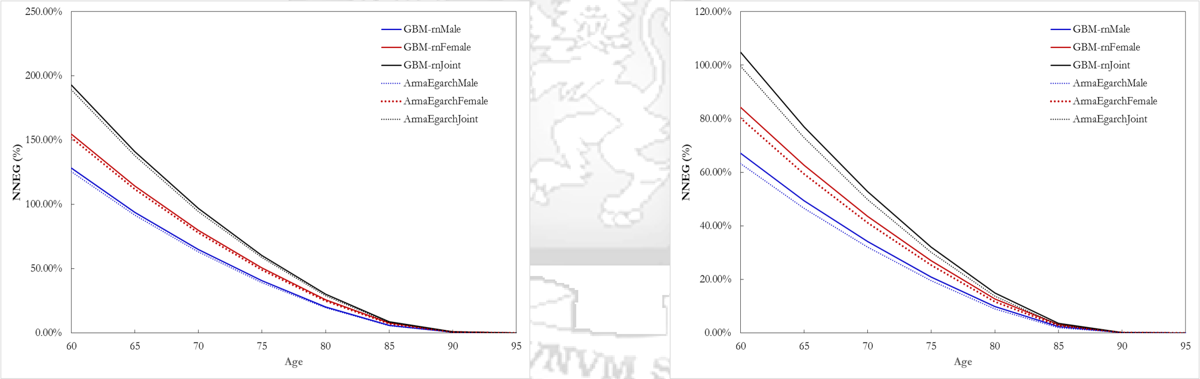
Figure A.1: Sensitivity Analysis of NNEG valuation w.r.t under Flexible Max LTV Loadings, where R = 4.99%, g = 1% and $\sigma = 3.90\%$.

Appendix B

Max ERC LTV loading



(a) g = 0% (b) g = -0.5%



(c) g = 3% (d) g = 2%

Figure B.1: Sensitivity Analysis of NNEG valuation w.r.t under Max ERC LTV, where R$_{Max}$ = 4.56%, r = 1.75% $and\sigma$ = 3.90%.

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