Stochastic Methods for Virtual Asset Pricing and Risk Management in Kenya

By

Serena Cherotich Rotich

Registration No: 170409

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Strathmore University
Nairobi, Kenya

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Declaration and Recommendation

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the dissertation contains no material previously published or written by another person except where due reference is made in the dissertation itself.

Rotich Serena Cherotich

Signature: ....Serena.... Date:...21/05/2025...

Approval

The dissertation of Rotich Serena Cherotich was reviewed and approved by the following:

Prof. Joseph Ivivi Mwaniki,
Lecturer, Strathmore Institute of Mathematical Science,
Strathmore University

Prof. Godfrey Madigu,
Dean, Strathmore Institute of Mathematical Science,
Strathmore University

Prof. Bernard Shibwabo,
Director of Graduate Studies,
Strathmore University

Abstract

The increasing adoption of Bitcoin (BTC) and Ethereum (ETH) in global financial markets has raised critical questions regarding their valuation, volatility, liquidity, and regulatory oversight. This study investigates the effectiveness of stochastic models including Geometric Brownian Motion (GBM), Heston, Ornstein-Uhlenbeck (O-U), and Jump-Diffusion in capturing the unique price dynamics and volatility patterns of BTC and ETH. Using historical market data, the research applies these models alongside Auto-regressive Conditional Heteroskedasticity (ARCH/GARCH) models to analyze volatility persistence and risk characteristics.

The findings indicate that while GBM provides a basic framework for price evolution, it fails to account for volatility clustering and market shocks. The Heston model captures stochastic volatility, whereas Jump-Diffusion models effectively incorporate sudden price jumps. GARCH(1,1) models confirm significant volatility clustering in both BTC and ETH.

To assess risk exposure, the study computes Value at Risk (VaR) and Conditional VaR (CVaR) at 95% and 99% confidence levels. Results show that ETH exhibits higher tail risk than BTC, implying greater vulnerability to extreme losses. Furthermore, liquidity analysis, measured through market depth reveals that BTC has stronger liquidity and lower relative volatility risk compared to ETH. The research also applies Monte Carlo simulations to price BTC and ETH derivatives, demonstrating that stochastic models significantly influence option valuation by incorporating market uncertainties.

Stress-testing scenarios highlight vulnerabilities in price stability, underscoring the need for margin requirements, volatility controls, and liquidity monitoring to mitigate systemic risks. The study's findings contribute to the growing discussions on risk management and policy formulation of the VA ecosystem, offering recommendations to enhance market stability while fostering innovation.

Keywords: *Bitcoin, Ethereum, Volatility, ARCH/GARCH, VaR, CVaR, Liquidity, Derivatives, Risk Management, Regulation, Kenya*

Contents

Abbreviations

AML- Anti-Money Laundering

CBK-Central Bank of Kenya

CFT- Combating the Financing of Terrorism

CMA-Capital Markets Authority

FRC-Financial Reporting Centre

GBM-Geometric Brownian Motion

MLE-Maximum Likelihood Estimation

SDE-Stochastic Differential Equation

VAs-Virtual Assets

VASPs-Virtual Asset Service Providers

Chapter 1
Introduction

1.1 Background of the Study

The emergence of Virtual Assets (VAs) or cryptocurrency is a cutting edge in the financial landscape. VAs are any digital representation of value that can be digitally traded, transferred, or used for payment and do not include digital representations of fiat currencies (FATF,2024). VAs have attracted growing interest in Kenya and globally due to their inherent characteristics, including anonymity/pseudonymity, availability through the internet, low cross-border transaction costs, and high speed of settlement of payments. While these assets offer potential for financial inclusion and innovation, they also come with considerable risks, including price volatility, market manipulation, cybersecurity threats, and regulatory uncertainty. Adoption is not only for speculative investment but also as alternative financial instruments that could potentially expand financial inclusion.

According to CoinMarketCap, there are over two million VAs globally, with the global VA market capitalization at approximately USD 2.29 trillion (subject to daily changes). The top ten VAs globally in terms of market capitalization include Bitcoin (BTC), Ethereum (ETH), Tether (USDT), Binance Coin (BNB), Solana, USDC, XRP, Dogecoin, Tron, and Toncoin, with Bitcoin as the largest (El-Khatib,2024). USDT and USDC are a type of VA known as stablecoins, which are pegged to the value of another currency, such as the US dollar.

In Kenya, as identified in the Virtual Assets and Virtual Asset Service Providers (VASPs) risk assessment conducted in 2023, the top ten VAs owned by Kenyans include Bitcoin, Ethereum, Binance Coin, Tether, Solana, Matic, XRP, Dogecoin, USDC, and Non-Fungible Tokens (NFTs). Unlike fungible assets like Bitcoin or Ethereum, where each unit is interchangeable, NFTs are non-fungible, meaning each token is unique and cannot be exchanged on a one-to-one basis with another token.

VAs are supported by blockchain technology, a type of distributed ledger technology (DLT) that records its data in blocks connected through cryptographic methods, making it nearly impossible to alter transactions. This decentralized technology ensures that recorded data is shared across connected computers, creating transparency and immutability.

VA pricing has progressed significantly from simplistic speculative models to more advanced, data-driven approaches. Early models largely relied on demand-supply dynamics and market sentiment, but with the growing complexity and maturity of the VA markets, there has been a shift toward applying financial theories especially stochastic modeling. Models like Geometric Brownian Motion (GBM), Jump-Diffusion, Heston's stochastic volatility model, and Ornstein-Uhlenbeck (O-U) processes are now used to better capture volatility, mean reversion, and sudden price jumps characteristic of VAs.

In parallel, GARCH-type models (including GARCH, EGARCH, and GJR-GARCH) have become popular for modeling volatility clustering, a common trait in VA markets. Additionally, the rise of machine learning techniques and token-specific valuation metrics (such as on-chain activity and tokenomics) have enriched predictive accuracy. Derivatives markets have introduced implied volatility into the pricing discourse, while regulatory developments are increasingly factored into pricing models. Together, these advances reflect a shift toward a more rigorous, multifaceted approach to understanding and forecasting virtual asset prices.

1.2 Liquidity of Virtual Assets

Liquidity of VAs refers to the ease and speed with which a VA can be bought or sold in the market without significantly impacting its price (Ahmed, 2024). High liquidity in a market means ample buyers and sellers exist, enabling transactions to occur quickly at stable prices. For VAs, liquidity is crucial because it affects price stability, transaction efficiency, and the ability of market participants to enter or exit positions.

Liquidity in VA markets is influenced by trading volume, market depth, regulatory frameworks, and institutional participation. While major VAs like Bitcoin and Ethereum benefit from high liquidity, smaller and newer tokens often struggle with limited liquidity, making them more volatile and difficult to trade. Emerging decentralized finance (DeFi) platforms and institutional involvement are shaping the future of VA liquidity, potentially leading to a more robust market.

Bitcoin's introduction of futures contracts on the Chicago Board Options Exchange (CBOE) and the Chicago Mercantile Exchange (CME) in December 2017 enhanced liquidity in VA markets. These derivatives, cash-settled in US dollars, have improved price efficiency and market quality by incorporating more available information into pricing(Chielon,2021).

A crash in the VA derivatives, can have significant impact on the economy, especially within sectors directly tied to the VA. Major derivatives exchanges such as CME and VA lending platforms would experience significant liquidity stress. A major crash could result in insolvency issues, especially if platforms are unable to meet customer withdrawals, potentially leading to bankruptcies, harming the broader VA ecosystem and may lead to regulatory scrutiny. Governments and regulatory bodies may push for stricter oversight of crypto markets, derivatives trading, and lending practices, which may include imposing leverage limits, mandatory disclosures, and requirements for insurance on deposits. Heightened regulation could change the landscape of VA markets, potentially limiting participation.

1.2.1 Problem Statement

The valuation and risk management of VAs pose significant challenges due to their high volatility, speculative nature, and lack of fundamental financial indicators such as cash flows and dividends. Unlike traditional assets, VA prices are primarily driven by market sentiment, technological developments, and regulatory shifts, making traditional valuation models ineffective. rely on fundamental metrics that are not applicable to VAs. Similarly, deterministic models fail to capture the stochastic and jump-like behavior characteristic of VA price movements.

To address these limitations, financial researchers have turned to statistical and econometric models, including time-series approaches like ARIMA, GARCH, and EGARCH, to analyze VA price dynamics. While these models can account for historical price patterns and conditional volatility, they fall short in capturing key features of VA markets, such as mean-reversion tendencies, stochastic volatility, and sudden market jumps.

Stochastic models offer a more flexible framework for modeling VA price behavior by incorporating random fluctuations and mean-reverting tendencies. Approaches such as the Ornstein-Uhlenbeck (O-U) process, can better capture the erratic yet cyclical nature of VA prices, making them more suitable for risk management and valuation (Fadahunsi,2010). The Heston model, on the other hand, offers a more dynamic representation by incorporating stochastic volatility, allowing it to reflect the changing uncertainty levels inherent in VA markets. Additionally, jump-diffusion models extend the standard stochastic framework by accounting for sudden and extreme price jumps triggered by market speculation, regulatory announcements, or macroeconomic events. However, limited research has explored the application of stochastic processes to VAs, particularly in the context of emerging economies like Kenya, where regulatory oversight remains weak and financial markets are highly susceptible to external shocks.

This research aims to bridge the gap in VA valuation and risk management by applying these stochastic models to understand the dynamics and risks of VAs. By doing so, it seeks to enhance understanding of VA price dynamics, improve risk assessment methodologies, and provide insights for investors, regulators, and financial institutions navigating the evolving VA landscape. It is important for safeguarding the stability and growth of financial systems, especially in emerging markets like Kenya where regulatory frameworks are still developing, and financial ecosystems may be less resilient to shocks.

1.3 Research Objectives

1.3.1 Main Objective

The main objective of this research is to apply a stochastic modeling framework to provide a robust mathematical foundation for VA valuation and risk management.

1.3.2 Specific Objectives

The specific objectives of this study are:

1. To evaluate the effectiveness of the stochastic models in capturing the volatility and price dynamics of BTC and ETH.
2. To analyze the market liquidity of BTC and ETH by examining the relationship between market depth and their price volatility.
3. To apply the GBM to value BTC and ETH options.
4. To utilize insights from stochastic modeling of BTC and ETH to inform regulatory strategies aimed at enhancing market stability and improving risk management practices in Kenya.
5. To identify the GARCH-type model that best captures the volatility dynamics of BTC and ETH.

1.4 Research Questions

1. How effective are stochastic models, such as jump-diffusion and stochastic volatility models, in capturing the unique volatility patterns of Bitcoin and Ethereum?
2. What is the relationship between liquidity measures, such as market depth and bid-ask spreads, and price volatility in Bitcoin and Ethereum markets?
3. How can stochastic models be applied to accurately value Bitcoin and Ethereum derivatives, such as futures and options?

4. How can the insights from stochastic modeling of VAs inform regulatory strategies for enhancing market stability and risk management in Kenya?
5. Which GARCH-type model best captures the volatility dynamics of VAs?

1.5 Scope of the Study

The research focuses on the application of stochastic models in the valuation and risk management of VAs in Kenya, given the country's growing adoption of VAs and its efforts to regulate them. The study explores how stochastic models can support Kenya's financial sector regulators, in managing risks in this emerging market. It examines both the theoretical and practical aspects of using mathematical models to address the unique challenges posed by the volatile and rapidly evolving VA markets. The study specifically focuses on Kenya as a representative case of an emerging market with a burgeoning virtual asset sector, driven by rising fintech adoption, digital financial innovations, and increasing interest from both retail and institutional investors. By focusing on Kenya, the study aims to address the challenges and opportunities unique to emerging markets, such as infrastructure limitations, evolving regulatory frameworks, and high consumer demand for alternative financial products. While the study is specific to Kenya, its findings and methodologies can be relevant for other emerging markets facing similar challenges with VAs. This geographic scope allows for tailored recommendations applicable to Kenya's market while also providing insights that can be adapted to other African and global emerging markets.

The study leverages the Ornstein-Uhlenbeck (O-U) process, a mean-reverting stochastic model, which is particularly suitable for modeling assets characterized by cyclical volatility, such as VAs. Other stochastic differential equations (SDEs) such as Geometric Brownian Motion (GBM) and Jump-Diffusion models may also be examined to address various aspects of VA behavior, including volatility clustering, price jumps, and external shocks.

1.6 Significance of the Research

This study contributes to the financial modeling field by developing a tailored stochastic framework that addresses the unique behaviors of VAs, particularly for emerging markets like Kenya where regulatory and market conditions differ from those in more developed economies. The use of the Ornstein-Uhlenbeck (O-U) process and other SDEs to model VA dynamics offers a new approach that can inspire future research and model development for VAs and other emerging digital financial products.

Traditional stochastic models have predominantly been applied to assets like stocks, bonds, and commodities. By adapting these models to VAs which are a unique asset class with its own characteristics, including high volatility, mean-reversion tendencies, and susceptibility to external shocks, this study pushes the boundaries of how stochastic models can be applied, thus broadening the academic applications of stochastic processes in finance.

This study provides regulators with a robust model to better understand price movements, volatility patterns, and risk factors, thereby enabling more informed, evidence-based policy development. The findings can assist regulators in determining reporting requirements, capital reserves, and other policy decisions that promote stability while supporting responsible innovation in the VA market. Given the pseudonymous nature of many VAs, they are often linked to money laundering and other financial crimes, which poses significant challenges in Kenya's AML/CFT regulatory space. This study's modeling framework provides Kenyan regulators with tools to more accurately assess and quantify risks within the VA sector, helping them better understand patterns associated with illicit activities and develop targeted policies that enhance AML/CFT effectiveness. The study's findings could support Kenya's engagement in international regulatory efforts, contributing a unique perspective on VA oversight in emerging economies to discussions at regional and global levels.

In emerging markets, the high volatility of VAs and their growing adoption among retail and institutional investors could contribute to systemic risk, especially if their instability ripples into other parts of the financial system. This study provides a foundation for understanding and mitigating these risks, which is critical to avoiding financial instability. A well-calibrated risk management framework helps regulators monitor potential systemic threats and take preemptive measures to ensure market resilience. A structured approach to valuation and risk quantification contributes to greater transparency in the VA market, which is often perceived as unpredictable. By offering a scientifically grounded valuation model, the study provides Kenyan market participants with a tool for better understanding VA risks, potentially enhancing confidence among retail and institutional investors alike. Increased investor confidence in VAs can encourage responsible engagement and discourage purely speculative trading, thereby promoting a healthier market environment.

Institutional investors in Kenya, such as banks, insurers, and fintech companies, may benefit from the study's findings as they explore the role of VAs in portfolios or financial products. The stochastic model provides a means of assessing risk and valuation, which could be foundational for institutions seeking to incorporate VAs responsibly. The study's model can also serve as a tool for these entities to design VA-backed products or investment strategies that align with their risk management policies. VAs are relatively new to many investors, and the level of understanding around their dynamics remains low in emerging markets. By disseminating findings on VA dynamics and risk management, this study contributes to broader financial education, helping market participants better understand the inherent risks and how to manage them effectively. By providing a detailed examination of stochastic modeling for VAs, this study can contribute to curriculum development in financial engineering, risk management, and fintech education, especially within Kenyan universities and professional training programs. Knowledge gained from this research may be integrated into courses on mathematical finance, financial technology, and regulatory studies, fostering an academic environment attuned to the demands of an evolving digital economy.

The study's findings offer a foundation for further research on the application of stochastic models in valuing and managing VA risks, potentially inspiring future studies on hybrid models, machine learning enhancements, or comparative studies across markets. For Kenyan academia and financial professionals, this study can encourage a deeper exploration of mathematical finance and risk modeling within emerging markets. The Kenyan market shares characteristics with other emerging markets in terms of financial inclusion, regulatory structure, and exposure to high-risk financial innovations. Thus, this study's outcomes may serve as a useful reference for researchers and regulators in other countries facing similar challenges with VAs, encouraging cross-national studies and knowledge sharing.

Chapter 2

Literature Review

2 Introduction

Valuation of VAs deviate significantly from traditional financial instruments due to their unique characteristics. Their attributes make their valuation complex, requiring advanced mathematical approaches (Hou,2020). Stochastic models, widely used in finance for asset pricing, have been increasingly adapted to understand and quantify the price dynamics of VAs. These models, such as geometric Brownian motion (GBM), Heston's stochastic volatility model, and jump-diffusion frameworks, provide robust tools for capturing the randomness and volatility inherent in VA markets.

According to Hou (2020), in addition to valuation challenges, VAs present unique risks, including price jumps, liquidity issues, and regulatory uncertainties. These risks are particularly pronounced in emerging markets like Kenya, where there is no regulatory framework for VAs, and financial systems are more vulnerable to external shocks. As VAs gain traction, the development of risk management frameworks becomes critical. Stochastic modeling offers a structured way to analyze risks, quantify volatility, and assess the implications of market events on VAs and their derivatives.

The literature on VAs has expanded to include topics such as their valuation, risk factors, and the emerging market for VA derivatives. However, most literature has focused on Bitcoin, leaving behind other coins such as Ethereum. This literature review synthesizes existing studies on stochastic modeling for VA valuation and risk management. It highlights the methodologies employed, key findings, and gaps in the current body of knowledge, particularly in the context of emerging markets. By identifying these gaps, the review provides a foundation for this research, which seeks to apply stochastic models to address the valuation and risk management needs of Kenya's virtual asset market while offering actionable insights for regulatory development.

2.1 Empirical Literature

2.1.1 Volatility and Risk in Virtual Assets

Doumenis (2021) investigates the viability as compared with other financial assets of cryptocurrencies as a currency or as an asset investment. The paper explores the potential risks associated with Bitcoin's volatility and its implications for investors and financial stability. The findings of the study reveal that Bitcoin has very high volatility compared to SP 500, Gold and Treasury Bonds. Further, the findings show that there is a positive correlation between Bitcoin's price volatility and the other three financial assets before and during COVID-19. Many of the research that have been conducted in this area have used the GARCH model. Stŕale & Tjernström (2014), for example, applied the GARCH (1,1) model to identify the drivers of bitcoin price volatility which include market sentiment, regulatory actions and technological disruptions.

In 2019, during the COVID-19 pandemic, the global financial markets were severely disrupted, leading to a drop in many of the stock market prices, affecting decisions by policymakers and other financial market players (Ghorbel & Manzli, 2022). With the growing adoption of VAs globally as highlighted by Iyer (2022), investors have been curious to understand their volatility drivers to inform them on whether to diversify their portfolio by investing in VAs and how those two asset types interact and relate.The COVID-19 pandemic underscored the interconnectedness of VA markets with global financial systems. While stock prices declined, Bitcoin's volatility increased, reflecting its status as both a speculative asset and a potential hedge (Ghorbel & Manzli, 2022).

Grissa & Abaoub(n.d.) in their research, they examined the volatility of cryptocurrency returns using time-series econometrics between the period from April 2013 to June 2022. The findings of the research, utilizing GARCH modeling indicate that cryptocurrency return volatility is not constant (heteroskedastic). It reveals that volatility shocks are permanent and significant on conditional variance, implying that shocks have a lasting impact on volatility. Grissa & Abaoub (n.d.)'s research concludes that GARCH models are important for forecasting cryptocurrency volatility, providing valuable information for portfolio and risk management for investors and portfolio managers.

2.1.2 Stochastic Models for Valuation and Risk Management

VAs exhibit extreme volatility, posing unique challenges for both valuation and risk management. Corbet et al. (2019) highlight that VA markets behave significantly differently from traditional financial markets, with higher volatility, lower liquidity, and more frequent price jumps driven by factors such as regulatory announcements and technological innovations. Such characteristics suggest that traditional models like the GBM may not be sufficient to capture the dynamics of VAs prices. The use of SDE models with jump diffusion have emerged as more suitable tools for modeling VAs. Mensi et al. (2020) explore the use of Jump-Diffusion models to account for the sharp, discontinuous movements often seen in VA markets. Their study suggests that the incorporation of jumps allows for better capture of the extreme price movements that characterize VAs. Mensi et al. (2020) state that such models improve upon the limitations of continuous GBM by allowing for sudden changes in price, which are typical in VA markets due to regulatory actions or shifts in market sentiment. Additionally, stochastic volatility models have been applied to VA markets to capture periods of increased uncertainty. Gkillas et al. (2018) highlight the role of volatility clustering in VA markets, where periods of high volatility tend to be followed by similar periods. They highlight the Heston model, which allows volatility itself to follow a stochastic process, has been successfully applied to improve risk management strategies for VAs.

When it comes to use of SDEs in financial modeling, SDEs have a well-established role in financial mathematics, particularly in the modeling of stock prices, interest rates, and derivative pricing. Models such as the Black-Scholes-Merton model use SDEs to model the dynamics of stock prices under the assumption that asset prices follow a GBM. Chan and Wong (2013)highlight that the Black-Scholes equation has been instrumental in developing modern finance, offering a way to price European options by incorporating market volatility into the equation through Brownian motion. The assumption constant volatility in models such as the Black-Scholes has been extensively criticized as it fails to capture the full range of market behaviors, especially during periods of extreme volatility or in markets prone to speculative bubbles. Extensions to the Black-Scholes model, such as Jump-Diffusion models and Stochastic Volatility models like the Heston model, introduce additional stochastic elements to better capture sudden price changes and volatility clustering Chan and Wong (2013). Although these models have proven effective for traditional assets, their applicability to the VA space is still under research. The highly speculative nature of VAs, coupled with frequent regulatory changes, demands an advanced approach to capture both market dynamics and associated risks.

Miralles-Quirós, in their paper, "Mathematics, Cryptocurrencies and Blockchain Technology" discuss the use of stochastic processes and mathematical models to analyze the price behavior of VAs, they however do not delve into the application of SDEs. They focus on the broader mathematical tools like cryptography, consensus mechanisms, and stochastic processes related to price volatility and risk. Their study describes the jump detection technique that captures the Bitcoin price dynamics and calculates the intensity of the jumps.

2.1.3 Derivatives and Hedging in Virtual Asset Markets

There have been a few studies such as those by Scaillet et al., 2020, Siu & Elliot, 2021 and Jalan & Saqib, 2021 that combine the Bitcoin literature with that on option pricing to construct Bitcoin option pricing models with dynamic jumps. However, these papers do not provide a specific measure by detecting jumps for implied volatility in jump-diffusion models, and the option is hedged with the underlying Bitcoin. The research by Scaillet et al., (2020) provides valuable insights into the volatile nature of the Bitcoin market by highlighting the prevalence of jumps. These abrupt price changes reflect the market's reaction to news and liquidity conditions, which investors and regulators should consider when analyzing Bitcoin's risk profile. The paper emphasizes the importance of developing more sophisticated models that incorporate jump dynamics for better risk assessment in the VA market.

In the paper by Siu & Elliot, 2021, the authors apply the SETAR-GARCH model and demonstrate that it is a more effective tool for pricing Bitcoin options due to its ability to capture both regime-switching behavior and volatility clustering. This model offers improved accuracy over traditional methods, making it a valuable tool for traders and financial analysts working with Bitcoin derivatives. The model, however has its limitations, especially in handling of extreme jumps or crashes. The SETAR-GARCH model struggles with capturing extreme tail events such as sudden market crashes or volatile price movements that occur in the Bitcoin market. Such events might require additional modeling techniques, such as jump diffusion models or stochastic volatility models, to be fully accounted for.

The paper by Jalan & Saqib, 2021, aims to assess whether traditional option pricing models like Black-Scholes are suitable for pricing Bitcoin options and evaluate the risks associated with trading Bitcoin options. The authors provide an important first step in understanding the pricing and risk dynamics of the Bitcoin options market. The paper uses empirical data from the Bitcoin options market to test the effectiveness of standard models such as the Black-Scholes model. They also assess risk measures, including Value at Risk (VaR) and Expected Shortfall (ES), to understand the risk exposure of market participants. The study compares the Bitcoin options market to traditional financial markets to highlight the unique challenges of pricing and risk management in VA options. The study shows that models like Black-Scholes underestimate volatility in the Bitcoin options market, leading to mispricing of options while the VaR and ES indicate that the Bitcoin options market is exposed to extreme risks, with tail events being more frequent compared to traditional markets. The paper however, does not explore the impact of regulatory changes on the Bitcoin options market, which is an important factor given the growing interest in regulating VAs. Regulatory changes could significantly alter market behavior, volatility, and risk, and this is a missing element in the analysis. The authors suggest the need for conducting further research using Stochastic Volatility, or Jump Diffusion models, which might provide more accurate pricing for Bitcoin options due to their ability to capture the complexities of the Bitcoin market.

Araújo and Barbosa (2023) in their paper, explore the use of Markov Chains as a mathematical tool to model and reconstruct the dynamics of VA processes. The authors propose using Markov Chain models as a powerful stochastic process to better understand the behavior of VAs, particularly in terms of price fluctuations and market trends and capture the state-dependent transitions observed in VA price movements. The authors use empirical VA market data to fit the Markov Chain models. They analyze the transition probabilities between different states, such as price increases and decreases, to construct a Markov process that can simulate future VA behavior. The study finds that Markov Chains are highly effective in modeling short-term price changes and volatility patterns in VAs. Araújo and Barbosa (2023), highlight potential applications of this model in areas such as risk management, portfolio optimization, and VA trading strategies. By reconstructing market processes, Markov Chains could help investors and regulators better understand and anticipate market behaviors.

Matic, Packham, and Hárdle (2023) in their study, underscore the challenges and opportunities in hedging VA options. They suggest that adopting stochastic models and dynamic strategies, market participants can improve hedging effectiveness. They also highlight the need for improved market conditions, regulatory clarity, and cost-effective solutions to address the unique risks of VA markets. Söylemez (2019) explores cryptocurrency derivatives, focusing on Bitcoin. The paper explores their role in the financial markets, their pricing mechanisms, and the broader implications for market efficiency, risk management, and regulatory considerations. The authors highlight how the introduction of Bitcoin futures on regulated exchanges like the CME marked a significant milestone, attracting institutional investors and increasing market legitimacy. The study discusses how models, such as the Black-Scholes model, may need adjustments to accommodate the characteristics of Bitcoin, including fat-tailed distributions and price jumps. Söylemez (2019) highlights that the futures contracts allow for better risk management tools for market participants but require robust risk mitigation frameworks to prevent market manipulation. Furthermore, the study by Söylemez does not deeply explore pricing models tailored for VA, especially those that incorporate jumps, volatility clustering, or mean reversion. The gap will be addressed by this paper as it will evaluate stochastic models for better derivative pricing.

The study by Cindicator (2018) offers valuable insights into Bitcoin derivatives and their impact on financial markets, however, it primarily focuses on Bitcoin derivatives, leaving out a VA like Ethereum, which has distinct market dynamics. This research, in a bid to explore the possibility of developing VA derivatives will conduct a comparative analysis of Bitcoin derivatives and Ethereum derivatives to understand their unique pricing, market impacts, and risk profiles. Additionally, it will explore how VA derivatives could affect financial stability, inclusion, and risk management in developing economies like Kenya.

2.2 Theoretical Literature

2.2.1 Regulatory Implications and Emerging Markets

The unpredictable price movements and lack of oversight in many VA markets increase the systemic risks associated with them. One of the major concerns for regulators is the ability to stress test and quantify the risks posed by VAs in financial systems. According to Ammous (2018), the adoption of stochastic models could provide regulators with better tools to simulate extreme market scenarios and assess the impact of VAs on the broader financial system. By incorporating stochastic processes into regulatory stress tests, regulators can better account for the volatility and unpredictability of VA markets, especially when making decisions regarding capital requirements and liquidity buffers. Feng et al. (2021) stresses that incorporating SDE models with jump components into risk management frameworks can help regulators anticipate the effects of extreme events, such as regulatory crackdowns or technical failures, on market stability. They argue that the integration of SDEs into regulatory frameworks can provide more data-driven decision-making, allowing for better control of systemic risks. Their argument is particularly relevant in emerging markets like Kenya, where VAs are gaining traction, but regulatory oversight is still developing.

In terms of VA Markets in emerging economies such as Kenya, VAs offer new opportunities for financial inclusion and investment diversification, however, they also pose significant risks due to regulatory uncertainty and market volatility. Njeri and Kinuthia (2022) in their paper, discuss the growing interest in VAs and tokenized assets in Kenya, driven by a lack of access to traditional banking services and the rapid rise of mobile money platforms like M-Pesa. Their paper also highlights the challenges faced by regulators in managing risks associated with VAs particularly in terms of preventing fraud, ensuring investor protection, and maintaining market stability.

According to Odhiambo (2023), the adoption of SDE-based risk management frameworks can help Kenyan regulators better assess the risks posed by VAs particularly because of the limited regulatory infrastructure in emerging markets. The use of Jump-Diffusion models could allow for early identification of market risks, enabling regulators to take proactive measures to stabilize the market (Odhiambo, 2023). By incorporating stochastic elements into regulatory frameworks, Kenya could better manage the volatility and risk inherent in VA markets, while also promoting responsible innovation and financial inclusion (Feng et al., 2021).

Applying SDE models to VAs proves to be beneficial in regulation of VAs, however they have a challenge particularly in the calibration of model parameters. High volatility and short history of VAs make it difficult to estimate parameters like drift, volatility, and jump intensity with high confidence. Additionally, data on VA prices is often limited or noisy, complicating the calibration process (Liu et al., 2020). To address the challenge, Chen and Hsieh (2021) explore the use of Bayesian estimation techniques to improve parameter estimation in markets with limited data. The techniques allow for the incorporation of prior knowledge and can help mitigate the data scarcity problem. Additionally, Monte Carlo simulation and maximum likelihood estimation (MLE) are methods that can be used in calibrating SDE models in VA markets.

The paper by Caporale & Zekokh (2018) acknowledges the ongoing discussions around cryptocurrency regulation and positions its findings as potentially valuable for regulatory bodies in their efforts to understand and manage the risks associated with them. The study fitted over 1,000 GARCH models to the log returns of the exchange rates for four VAs including Bitcoin, Ethereum, Ripple, and Litecoin to generate one-step ahead predictions of Value-at-Risk (VaR) and Expected Shortfall (ES) using a rolling window basis with data from 2010 to 2018. The research highlights that structural breaks, which are likely to occur in VA series due to factors such as policy changes, can bias the results of standard GARCH models.

Caporale & Zekokh (2018)demonstrate the limitations of standard GARCH models in capturing the volatility dynamics of major VAs and highlights the benefits of using other models such as Markov-switching and mixture of GARCH models that account for regime changes and leverage effects for more accurate risk assessment that can benefit both investors and regulators. The paper suggests that more accurate risk metrics derived from appropriate volatility models can help regulators in their efforts to regulate VAs.

Akin(2024) in their study " Valuation, Accounting Principles, and Classification of Assets in the Metaverse," explores the methods for valuing VAs, the accounting principles needed, and classification systems for accurate financial reporting in the Metaverse.The study provides insights into the multifaceted nature of asset valuation in the Metaverse, highlighting various influencing factors. It emphasizes the need for tailored Generally Accepted Accounting Principles (GAAP) focusing on legal recognition, tracking, jurisdictional issues, evaluation, auditing, and risk management. Compliance, transparent reporting, continuous monitoring, and integrating legislation, GAAP, and technology are essential for robust financial reporting.

2.3 Gaps in Literature

Whereas there has been progress in applying SDE models to VA markets, there are still several gaps in the literature. The integration of regulatory factors into SDE models remains an area of ongoing research, as VA markets continue to evolve and face new regulatory challenges. Most research on stochastic models for asset valuation and risk management concentrates on traditional financial assets and little has been done to adapt these models to VAs.

Most of the existing models often fail to account for the unique traits of VAs, such as extreme volatility and sudden market shocks like regulatory announcements. Additionally, regulatory research in VAs often emphasizes qualitative approaches, such as surveys or comparative analyses of legal frameworks. Few studies leverage quantitative models to assess regulatory implications for valuation and risk management. Hence the need for developing a framework that captures these characteristics through advanced stochastic techniques and simulations and providing a quantitative basis for policymakers in Kenya and developing economies to enhance regulatory oversight and alignment with international standards.

CHAPTER 3
Research Methodology

3.1 Introduction

The research methodology will outline the logical framework that we will adopt in undertaking this research. The chapter will evaluate the research design, data collection methods, development methodology and ethical considerations that we will look at when conducting research. Additionally, it will cover the target audience of the research, quality and reliability techniques. This section is important as it will highlight the specific procedures and techniques that will be used in the study and using this, researchers can evaluate the studies reliability and validity in future studies.

The primary contribution of this study lies in its application of advanced stochastic models to the Kenyan VA market, an emerging financial landscape characterized by unique regulatory challenges. By exploring the applicability of the GBM,Heston,O-U and jump-diffusion models, the research provides a more comprehensive framework for assessing VA price behavior and associated risks. Through this approach, the research contributes to the growing body of knowledge on VA valuation and risk management, offering a more robust modeling framework suited to the volatility and market dynamics of VAs in emerging economies.

3.2 Research Design

The research design outlines the structure, strategy, and overall plan for conducting the study. It combines both theoretical modeling and empirical analysis to address the research objectives, focusing on the application of SDEs in managing risks and regulatory compliance in Kenya's VA sector. Research design are the techniques and strategies that a researcher uses in answering research questions.

The study will adopt a quantitative research approach, leveraging SDE models to represent and simulate the behavior of VA prices. The SDE-based models will be developed and calibrated using historical data on VA prices, volatility, and jumps (sudden price changes). The research type will be an exploratory and applied research which explores the feasibility and effectiveness of using SDEs to enhance VA market regulation and aims to provide actionable insights that could be applied to the financial system in Kenya. Exploratory research investigates the relatively new area of applying stochastic models to VA regulation, a sector that is still in development. The aim of applied research is to develop models that can be directly implemented by regulators for stress testing, risk management, and ensuring compliance in the VA sector. This mixed-method approach that will be applied by the study combines both theoretical model development and quantitative analysis of real-world data, to create a comprehensive framework for applying stochastic modelling in the understanding of the dynamics and risks of VAs. The methodology will ensure that the research is both mathematically rigorous and practically relevant.

The research strategy involves different steps with the first one being the development of theoretical framework. Based on existing financial models including GBM, Jump-Diffusion models, and Stochastic volatility models, a customized stochastic model will be developed for VA markets. The second step will be data analysis and calibration of the models. It will involve using real-world data from VA markets. Maximum Likelihood Estimation (MLE) and Monte Carlo simulations will be employed for model calibration and testing the accuracy of the model. The number of simulation paths will be 10,000 to ensure a statistically robust representation of possible price evolution over time. Number of time steps will be 30, which is daily over 30 days and we will assume a a 5% annual risk-free rate. The last step will involve stress testing and simulation. The calibrated models will be used to simulate different market conditions such as regulatory changes, price crashes, extreme volatility and their impact on VA prices. The analysis will be essential in demonstrating how stochastic models can predict market behavior under various scenarios. The models will calculate key risk metrics, such as Value at Risk (VaR) and Expected Shortfall (ES), to determine how vulnerable the VA market is to extreme events. To get regulatory insights, we will simulate extreme market conditions for BTC and ETH using Merton's Jump-Diffusion Model and compare it with their normal scenarios. For the stress scenario, we will increase volatility and jump parameters to simulate extreme conditions.

3.2.1 Stochastic Differential Equation Model

An SDE model that can be applied to the valuation and risk management of VAs is the GBM model. The GBM model is often used in financial markets to describe the stochastic behavior of asset prices. The dynamics of a VA price could be represented similarly, with certain modifications to account for its specific volatility and market characteristics.

Let S_t represent the price of a virtual asset at time t. The price dynamics can be modeled using the following Stochastic Differential Equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Where:

- S_t is the price of the virtual asset (VA) at time t.
- μ is the drift term, representing the expected rate of return of the VA.
- σ represents the volatility of the VA price.
- W_t is the Wiener process, introducing randomness.
- dt represents an infinitesimal time increment.
- dW_t is the increment of Brownian motion, accounting for random shocks.

Some VAs, especially those that are highly influenced by market fundamentals or interventions such as government regulations, can exhibit mean-reverting behavior, and to account for that, the model can be extended to an Ornstein-Uhlenbeck (O-U) process where prices tend to revert to a long-term mean. According to Fadahunsi (2010), the O-U process is the most widely used mean reverting stochastic process in financial modelling. With properly chosen drift and diffusion parameters, an

O-U process can be used to mathematically model trading activities in a financial market.

The O-U process, S, is modelled as follows:

$$ds = \lambda(\mu - S)\, dt + \sigma\, dW_t$$

Where:

- σ, is the measure of volatility.
- λ, is the measure of speed of mean reversion.
- W_t, is the Brownian Motion, hence $W_t \sim \mathcal{N}(0, t)$.
- μ, is the long-term mean to which the process tends to revert.

Additionally, VAs may experience sudden jumps in price. That can be included by incorporating a jump component to the SDE, resulting in a Jump-Diffusion Model;

$$dS_t = \mu S_t dt + \sigma S_t dW_t + J S_t dN_t$$

Where:

- dN_t is a Poisson process representing the occurrence of jumps.
- J represents the size of the jumps.

The model assumes that the price of the asset follows a log-normal distribution, therefore, it is essential to convert the price data into log returns (r_t), which can be calculated as follows:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

The drift, which is the average rate of return of the VA over time, can be estimated by calculating the mean of the log returns:

$$\mu = \frac{1}{T}\sum_{t=1}^{T} r_t$$

Where T is the number of time periods.

The volatility can be estimated as the standard deviation of the log returns:

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2}$$

After estimating the parameters μ and σ, the asset price path can be simulated using the GBM or the chosen SDE model. To evaluate how well the model fits the data, a comparison of the model's simulated price paths with the actual historical prices will be done, followed by assessing the fit using statistical measures such as the Root Mean Squared Error (RMSE), which is given by:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{S}_i - S_i\right)^2}$$

Where:

- $\hat{S}_i$ is the model's simulated price at time i.
- S_i is the actual historical price at time i.
- n is the number of time points.

3.2.2 Simulating the Stochastic Differential Equation Model

Simulating the SDE involves generating numerical approximations for its solutions. For this research, simulating the SDE will allow us to model VA price dynamics and assess risk measures. After choosing the SDE model as mentioned above, the next step will involve discretizing the SDE. We will use the Euler-Maruyama method to approximate SDE solutions over small time steps δt as shown below:

$$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma S_t \sqrt{\Delta t} Z$$

- $Z \sim \mathcal{N}(0,1)$: Standard normal random variable.

For jump-diffusion models:

$$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma S_t \sqrt{\Delta t} Z + J_t S_t \Delta q_t$$

It will be followed by simulating multiple paths of VA prices to account for randomness and jumps.

3.2.3 Maximum Likelihood Estimation (MLE)

In undertaking model calibration and testing the accuracy of the model using the MLE, it will involve finding the parameters drift (μ), volatility (σ) and jump intensity (λ) that maximize the likelihood of observing the historical data given the model. For GBM, the likelihood function L based on the log returns can be derived from the assumption that log returns are normally distributed with mean $\mu - \frac{\sigma^2}{2}$ and variance σ^2.

The log-likelihood function $\ell(\mu, \sigma)$ is:

$$\ell(\mu, \sigma) = -\frac{T}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} (r_t - \mu)^2$$

For Jump-Diffusion, the likelihood involves combining the probability distribution functions for the diffusion and jump components. The jump sizes and intensities are incorporated into the likelihood function. After writing the likelihood function, the next step will involve maximizing the Log-Likelihood and statistical software will be used to optimize the log-likelihood.

3.3 GARCH Model

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, introduced by Bollerslev (1986), extends the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982) through incorporating lagged conditional variances. This model is effective in capturing volatility clustering. A standard GARCH(1,1) model is defined as follows:

3.3.1 Mean Equation

The following equation shows the mean equation

$$r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t$$

where r_t represents the asset return at time t, μ is the mean return, ϵ_t is the error term, and z_t is a white noise process with zero mean and unit variance. The conditional standard deviation σ_t models the time-varying volatility.

3.3.2 Variance Equation

The following shows the variance equation

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where:

- $\omega > 0$ is a constant term ensuring positive variance,
- α (ARCH term) captures the impact of past squared shocks ϵ_{t-1}^2,
- β (GARCH term) measures the persistence of past variances σ_{t-1}^2.

For stationarity and stability, the parameters must satisfy $\alpha + \beta < 1$, ensuring that volatility does not explode over time. A comparison of AIC, BIC and Log-Likelihood will be conducted to help select the best model, whereby, the least AIC and BIC and the highest log-likelihood will indicate the best balance of goodness-of-fit.

3.4 Data Collection Methods

Data collection will involve gathering historical data of Bitcoin and Ethereum from the spot and derivative markets from reliable sources such as CoinMarketCap, CoinGecko, CryptoCompare, TradingView and Coinbase. The data will include information on price, volume, and volatility. Since VAs are cross-border in nature, the data will be global.

The timeframe will be five years, that is since 2019, allowing us to capture relevant market movements. This period too will capture the market pre and post COVID pandemic times which could give the study more insights on the different market periods. The development of SDE-based models will be grounded in established financial theories, but tailored to address the unique characteristics of VAs, such as extreme volatility and regulatory sensitivity. GBM will be used for continuous price modeling, Merton Jump-Diffusion models for accounting for large, sudden price changes and Heston Stochastic Volatility model for modeling changing volatility patterns, which are common in VA markets.

3.5 Data Analysis

The data analysis for this study will involve both quantitative statistical techniques and mathematical modeling methods, primarily focusing on the application of stochastic models to understand and simulate the behavior of VA prices and their risks.

The first step in the analysis will be to summarize the quantitative data obtained from VA markets, focusing on the price distributions of VAs which include mean, median and standard deviation, trading volume analysis to identify patterns or anomalies and volatility analysis using standard deviation and variance of asset prices to provide insight into market risk. These statistics will offer a foundational understanding of market trends and help identify periods of extreme volatility, which are essential for calibrating the stochastic models.

Given that the study deals with historical data on VA prices, time series analysis will be an important component. To assess the time-dependent structure of price changes and volatility, Augmented Dickey-Fuller (ADF) tests will be used to determine if the asset price series is stationary which is a vital requirement for stochastic modeling. Further, statistical hypothesis testing will be done to determine if regulatory interventions have a substantial impact on market volatility and risk. Applying the stochastic and quantitative analysis techniques in the study will offer a comprehensive view of the market dynamics of VAs and provide data-driven recommendations for regulatory compliance and risk management.

In this study, GARCH-type models, including EGARCH, GJR-GARCH, GARCH and ARCH will be applied to better understand the volatility structure of VAs and assess their risk characteristics. Tests such as the Ljung-Box test, ARCH-LM test and histogram of the residuals for auto correlation will be used to test and verify the presence of auto-correlation, heteroscedasticity and distributional properties of the residuals.

3.6 Research Quality – Validity, Reliability, and Objectivity of the Research.

When conducting a study, it is imperative to ensure research quality as it delivers credible, actionable, and reproducible conclusions. Validity is defined as how well the research measures what it is intended to measure. It looks at whether the findings truly reflect the reality they are meant to capture. Reliability means that there is consistency of the research results over time and across different conditions. If the research was to be repeated under the same conditions, it should produce similar findings. Objectivity refers to the research being free from bias or researcher influence, ensuring that the findings are impartial.

To ensure the validity, reliability and objectivity of the study, the research will ensure that the concepts being studied are properly defined and measured and the findings are relevant to other sectors or regions with similar regulatory frameworks or market structures. Further, it will ensure that it uses statistical tools and mathematical models that are transparent and verifiable, where in our case, the parameters are derived from the data and not imposed based on preconceived notions.

3.7 Ethical Considerations and Limitations

When it comes to handling quantitative data for research in sectors like VAs, various ethical considerations must be taken into account. This study understands the implication an unethical conduct will have on Strathmore University's reputation and the reliability of the product in question and with that, it will seek to get approval of the school before any data is collected and any models are developed. Taking into account ethical considerations, ensures that the work upholds the highest standards of professional integrity and social responsibility.

The methodology will also consider limitations, such as the availability and reliability of data on VAs in Kenya, and the potential ethical implications of using complex mathematical models in financial regulation, particularly in emerging markets.

Chapter 4
Results

4.1 Introduction

This chapter presents the findings from the analysis of BTC and ETH daily price data from 2019 to 2024. In this section we use Python for data exploration and also to achieve the objectives of the study. The results are aligned with the research objectives, which focus on the effectiveness of stochastic models in capturing volatility, liquidity assessment, valuation of VA derivatives, and regulatory implications. By utilizing established mathematical techniques, such as the Ornstein-Uhlenbeck (O-U) process and other stochastic processes, the study seeks to assess their utility in providing accurate insights into VA pricing and risk assessment. Further, this section involves simulation from actual data and also looking at parameter estimation from actual data.

These results offer a comparative analysis of the behavior of VAs under different modeling frameworks, focusing on their potential applications in policy formulation, regulatory oversight, and investment strategies within the local context. The findings are discussed in light of the theoretical models introduced in earlier chapters, with an emphasis on how these models perform can be applied in the Kenyan financial ecosystem. In the following sections, the results of the stochastic simulations, asset valuations, and risk assessments are presented, accompanied by detailed analysis and interpretation. These findings are critical for understanding the role of stochastic models in improving the accuracy of VA market predictions, as well as their limitations and areas for future research.

4.2 Data Summary

The following is the basic summary of the Bitcoin and Ethereum data;

Bitcoin Dataset Summary

	Price	**Market Cap**	**Total Volume**
Count	2142	2142	2142
Mean	29954.48	5.73369e+11	2.92458e+10
Std	19955.37	3.90763e+11	1.07856e+10
Min	3394.01	5.94824e+10	4.80486e+09
25%	12880.04	1.81808e+11	1.83132e+10
50%	26636.34	5.19800e+11	2.56969e+10
75%	44236.72	8.30000e+11	3.17900e+10
Max	84086.72	1.59000e+12	7.90000e+10

Table 4.1: BTC Dataset Summary

From BTC's summary above, the average price of BTC over the dataset period is USD 29,954.48, indicating a generally high valuation. BTC's standard deviation of USD 19,965.37 shows that the coin has experienced significant price fluctuations. The minimum Price of USD 3,394.01, suggests that BTC was at a much lower valuation in the past, possibly during early stages or after a market crash. The maximum price of USD 84,066.72 indicates the highest recorded BTC price in this dataset, reflecting market peaks during bullish periods. In summary, BTC has had a volatile price history, with large swings between its lowest and highest values. The high mean and large standard deviation indicate significant price movements over time.

The mean Market Cap of USD 573.36 billion shows that, on average, BTC had a substantial market capitalization, reinforcing its dominance in the VA space. The standard deviation of USD 390.76 billion shows high fluctuations in market cap indicating large price swings and changing investor sentiment. The minimum Market Cap of USD 59.48 billion suggests a much lower valuation during market downturns or early adoption phases. The maximum Market Cap of USD 1.59 trillion shows that BTC reached an enormous valuation during its peak, likely during major bull runs such as towards the end of 2021. In summary, BTC's market cap correlates with its price movements. The high variance in market cap signifies cyclical bull and bear markets, affecting investor confidence and adoption.

The average trading volume of USD 29.45 billion suggests strong liquidity and active trading. The standard deviation of USD 10.78 billion which shows moderate fluctuations indicate periodic surges and declines in trading activity. The minimum trading volume of USD 4.08 billion shows that there were periods of low trading activity, possibly during market stagnation. The maximum trading volume of USD 179 billion shows that extremely high volume was recorded at certain points, likely during market crashes or euphoric buying sprees. In summary, BTC's high average volume suggests that it remains the most liquid and widely traded VA. The spikes in volume likely correspond to major market events such as price rallies or crashes.

Ethereum Dataset Summary

	Price	**Market Cap**	**Total Volume**
Count	2142	2142	2142
Mean	1639.28	1.49748e+11	1.56029e+10
Std	1227.36	1.46932e+11	1.23715e+10
Min	104.27	1.00325e+10	1.43311e+09
25%	560.88	4.46008e+10	4.28533e+09
50%	1250.88	1.08000e+11	1.28532e+10
75%	2574.75	2.28000e+11	2.30000e+10
Max	4815.00	5.72000e+11	1.41000e+11

Table 4.2: ETH Dataset Summary

ETH's average price of USD 1,639.28 is significantly lower than BTC's, which aligns with its smaller market cap. The standard deviation of USD1,227.14 indicates high volatility, though less extreme than BTC in absolute dollar terms. The minimum price of USD 104.27 suggests that ETH was once extremely undervalued compared to today. The maximum price of USD 4,815 reflects ETH's all-time highs, likely during the 2021 bull run. In summary, ETH has also experienced high volatility, with price swings similar to BTC but at a lower absolute price level.

The average market cap of USD 194.79 billion shows that ETH holds a large but significantly smaller market cap compared to BTC. The standard deviation of USD 169.92 billion indicates that ETH's market valuation has fluctuated widely. The minimum market cap of USD 3.44 billion indicates that ETH was once a much smaller asset but has grown significantly over the years. The maximum market cap of USD 572 billion indicates that ETH has approached BTC's valuation at times but remains behind in total market dominance. In summary, ETH's market cap growth over time shows its increasing adoption. However, it remains more volatile compared to BTC.

The mean trading volume of USD 15.56 billion shows that ETH has a lower average trading volume than BTC, though still significant. The standard deviation of USD 12.37 billion shows that trading activity fluctuates widely, likely driven by market cycles and DeFi trends. The minimum trading volume of USD 1.43 billion indicates that some periods had much lower trading interest while the maximum trading volume of USD141 billion shows that huge spikes indicate major speculative activity or market events. In summary, ETH has a high but fluctuating liquidity, often influenced by the DeFi ecosystem, NFT trends, and smart contract adoption.

4.2.1 Price Trends

In comparing the two assets, Bitcoin has greater stability, higher absolute valuation, and acts as the market leader in terms of liquidity and price strength. Ethereum is more volatile, has lower absolute prices, but has seen massive growth, especially due to DeFi, NFTs, and smart contracts. Liquidity patterns suggest that BTC is used more as a store of value, whereas ETH is actively traded and utilized in applications. The results are consistent with previous findings that appeared in the literature by (De Sousa, 2021) which, in relation to other traditional assets, pointed out an extreme high value of the volatility.

The following shows the price trends for BTC and ETH over time.

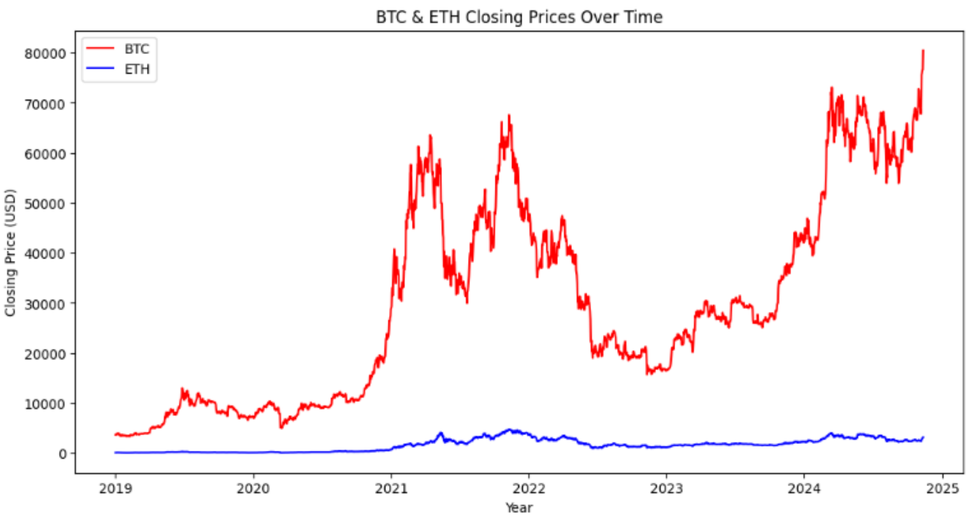

Figure 4.1: BTC & ETH Closing Prices from 2019 to 2025

In looking at the OLS regression for the two assets,the dependent variable in this case is the price of ETH while the independent variable is the BTC price. The R-squared of 0.827 model explains 82.7% of the variance in ETH prices using BTC prices while the adjusted R-squared of 0.827, similar to R^2, confirms a strong linear relationship.

The following shows the OLS regression results;

```
                            OLS Regression Results
==============================================================================
Dep. Variable:              price_eth   R-squared:                       0.827
Model:                            OLS   Adj. R-squared:                  0.827
Method:                 Least Squares   F-statistic:                 1.023e+04
Date:                Wed, 12 Feb 2025   Prob (F-statistic):               0.00
Time:                        16:45:50   Log-Likelihood:                -16395.
No. Observations:                2142   AIC:                         3.279e+04
Df Residuals:                    2140   BIC:                         3.281e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -34.9562     19.895     -1.757      0.079     -73.972       4.059
price_btc      0.0559      0.001    101.127      0.000       0.055       0.057
==============================================================================
Omnibus:                       21.316   Durbin-Watson:                   0.011
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               32.916
Skew:                          -0.055   Prob(JB):                     7.12e-08
Kurtosis:                       3.597   Cond. No.                     6.49e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.49e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Figure 4.2: BTC & ETH OLS Regression

4.2.2 Stationarity Tests

The stationarity tests for BTC and ETH are as follows;

```
ADF Statistic for BTC: -0.3930520063601275
p-value: 0.9112523960787167
Non-Stationary
ADF Statistic for ETH: -1.5022288847045822
p-value: 0.5324084547688349
Non-Stationary
```

Figure 4.3: BTC & ETH Stationarity

The Augmented Dickey-Fuller (ADF) test is used to determine whether a time series is stationary, i.e., its statistical properties do not change over time. The null hypothesis (H_0) assumes that the series is non-stationary.

The ADF stationarity tests show that BTC and ETH prices are non-stationary because their ADF statistics are not sufficiently negative and their p-values are greater than 0.05. Since the p-values are higher than 0.05, we fail to reject the null hypothesis, confirming that both series exhibit trends, seasonality, or random walks.

4.3 Volatility

Since BTC and ETH are both volatile, we compare their historical rolling standard deviations to see which asset fluctuates more over time. This helps to check if ETH exhibits higher relative volatility than BTC and if BTC and ETH have synchronized volatility cycles for example during market crashes.

The chart below shows the 30-day rolling standard deviation (volatility) of BTC and ETH returns over time.

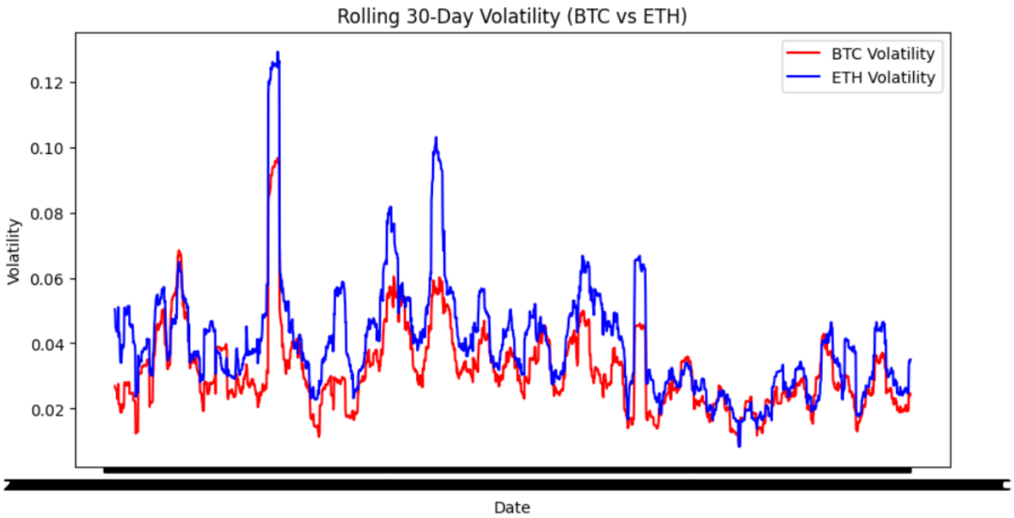

Figure 4.4: BTC & ETH Volatility

Higher peaks indicate periods of high price fluctuation, while lower values suggest relative stability. From the results, both BTC and ETH exhibit similar volatility patterns, meaning that their price fluctuations tend to be correlated, implying that they respond to similar market conditions. In most periods, ETH appears to have slightly higher volatility than BTC. The volatility of ETH is expected as BTC is the more mature and widely adopted asset.

The high volatility periods could correspond to major market events such as bull runs, crashes, regulatory announcements, or macroeconomic events. The most significant spikes suggest strong market reactions, possibly due to liquidity squeezes, high trading activity, or external shocks.

4.4 Simulating BTC and ETH using GBM

4.4.1 BTC Simulated Price Paths

The following plot shows multiple simulated BTC price paths using the GBM model over a 1-year period (365 days). Each colored line represents a different realization of the stochastic process, modeling possible future BTC price movements.

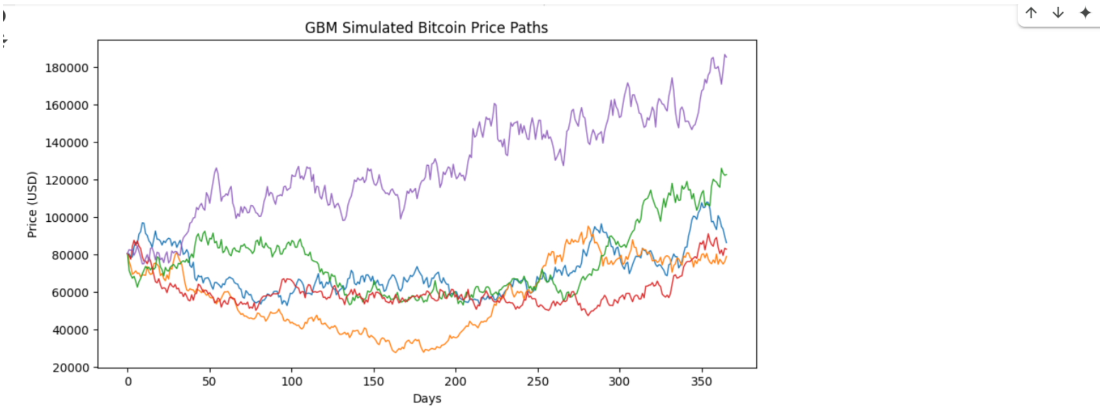

Figure 4.5: BTC Simulated Price Paths

The general trend from the above results is that the simulated BTC price paths exhibit random fluctuations, consistent with a stochastic model. Some paths increase significantly, while others experience temporary or prolonged declines, capturing the uncertainty in BTC price movements. Some paths trend upwards, suggesting prolonged bullish conditions while others decline initially before recovering, indicating potential mean-reversion effects. A few paths exhibit a sharp increase in volatility, showing potential speculative bubbles or price shocks. The spread between different paths increases as time progresses, which aligns with the nature of GBM where uncertainty compounds over time, reflecting higher price uncertainty in the long run, making long-term predictions riskier.

4.4.2 ETH Simulated Price Paths

The chart below shows multiple simulated ETH price paths over a 1-year period (365 days) using the GBM model. Each colored line represents a different realization of the stochastic process, modeling possible ETH price movements.

The simulated ETH paths show high volatility, with some realizations trending strongly upwards, while others decline or stay range-bound. Some paths spike dramatically, suggesting scenarios where ETH could experience significant growth. The divergence between price paths widens as time progresses, reflecting growing uncertainty in future ETH prices. This is expected in GBM simulations, where price variance increases with time due to the compounding effect of volatility.

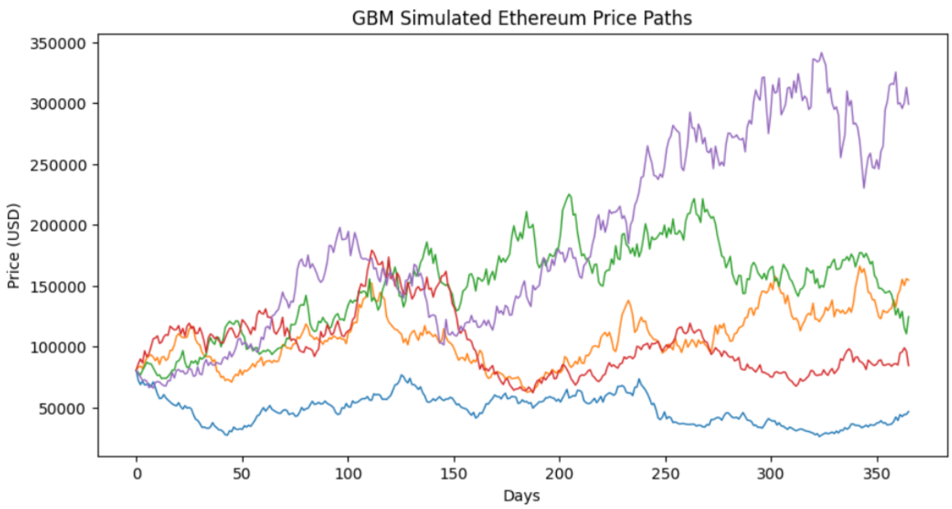

Figure 4.6: ETH Simulated Price Paths

When comparing the ETH and BTC simulation, the ETH simulation seems to exhibit higher price fluctuations than the BTC simulation, suggesting higher volatility in ETH compared to BTC. The upper bound of ETH simulations reaches over USD 300,000, whereas BTC's upper bound in the previous chart was around USD 180,000, possibly indicating greater relative price swings.

The GBM parameters are as follows;

Assets	**Drift (μ)**	**Volatility (σ)**
BTC	0.001439	0.034165
ETH	0.001490	0.044066

Table 4.3: Drift and Volatility of BTC and ETH

These parameters define the GBM used for simulating BTC and ETH price paths. The drift term (μ), which represents the expected average daily return of each asset, for BTC is 0.001439 (or 0.1439% daily return), while that of ETH is 0.001490 (or 0.1490% daily return). ETH has a slightly higher drift than BTC, suggesting a marginally higher expected daily growth rate. The volatility (σ), which represents the daily fluctuations in price, for BTC is 0.034165 (3.42% daily standard deviation) and for ETH is 0.044066 (4.41% daily standard deviation). ETH has a higher volatility than BTC, meaning its price movements are more erratic and unpredictable.

4.5 Simulating BTC and ETH using Heston Model

The Heston model helps in capturing volatility clustering with stochastic variance. It uses mean-reverting variance process. Correlated Brownian motions control price and volatility.

The results of simulating BTC and ETH using Heston model is as follows;

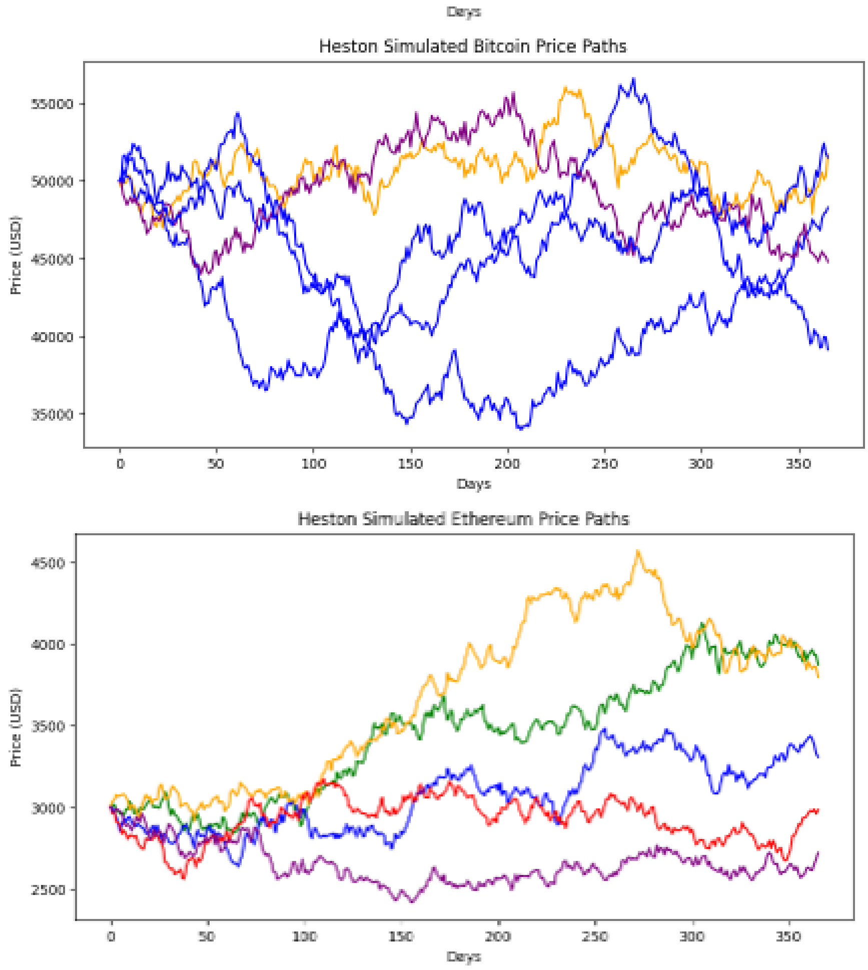

Figure 4.7: BTC & ETH Heston Simulated Price Paths

The two plots above illustrate simulated BTC and ETH price paths under the Heston model, which accounts for stochastic volatility rather than assuming constant volatility like the GBM model.

From the plots, in comparison with the GBM simulations which exhibit the smoother paths, these the Heston simulation trajectories exhibit periods of high and low volatility. This is more aligned with real-world VA markets, where volatility tends to cluster (high volatility follows high volatility, and low volatility follows low volatility). Additionally, the GBM model had more uniform trends, assuming constant volatility over time while in this case, stochastic variance introduces more diverse price paths, making the simulation more realistic. For price dispersion and mean-reversion effects, some paths show sudden surges and drops, capturing random volatility shocks, while some paths steadily increase, others tend to mean-revert, a behavior introduced by the variance mean-reversion parameter in the Heston model.

In conclusion, the Heston model better reflects realistic VA price dynamics, especially for assets like BTC and ETH, which are highly volatile. It is particularly useful in risk management and options pricing, where volatility dynamics matter significantly.

4.6 Simulating BTC and ETH Using Ornstein-Uhlenbeck (O-U) Process

The O-U model is useful for modeling BTC as a mean-reverting asset, particularly in cases where regulatory interventions or market corrections keep prices from drifting indefinitely. It is also useful in capturing ETH's short-term fluctuations and market corrections while preventing unrealistic exponential growth. It is often applied in pairs trading and statistical arbitrage, where relative price stability is assumed.

4.6.1 Simulated Bitcoin Price Trajectories Using O-U Process

The following plot visualizes simulated BTC price trajectories over time using the O-U process.

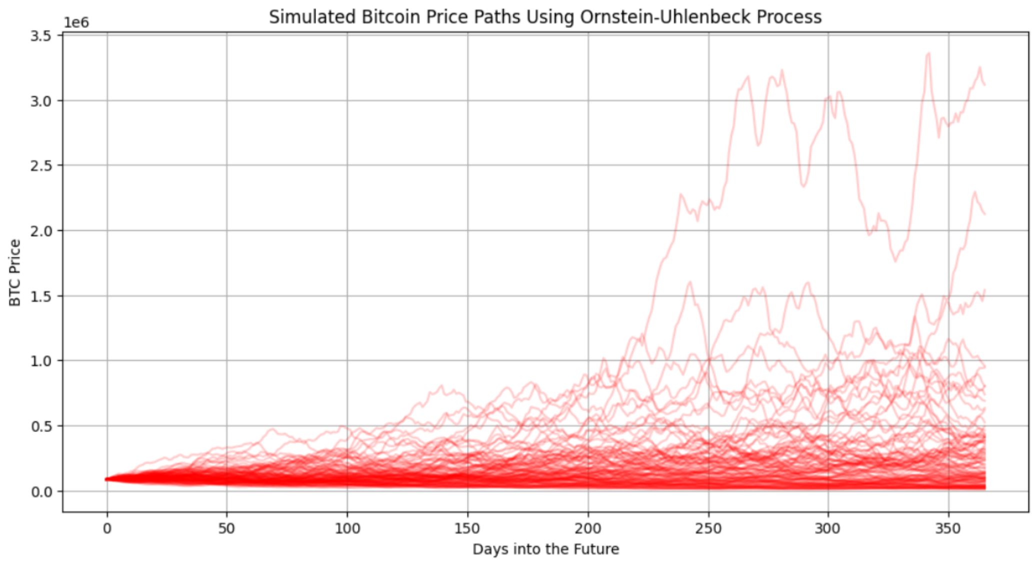

Figure 4.8: BTC O-U Simulated Price Paths

From the above plot, unlike GBM, which follows an exponential random walk, the O-U process tends to revert toward a long-term mean, it is evident as many paths stay near a central value despite short-term fluctuations. While most simulated BTC prices oscillate around the mean, some paths deviate significantly, indicating possible extreme price swings, this aligns with VA market behavior, where sharp price spikes and crashes occur. Since the O-U process is often used in interest rate and commodity pricing models, it suggests that BTC prices could stabilize over time instead of drifting infinitely as seen in GBM. However, the parameters (mean-reversion speed, volatility) significantly affect this behavior.

4.6.2 Simulated Ethereum Price Trajectories Using O-U Process

The chart below represents simulated Ethereum price trajectories over time using the O-U process.

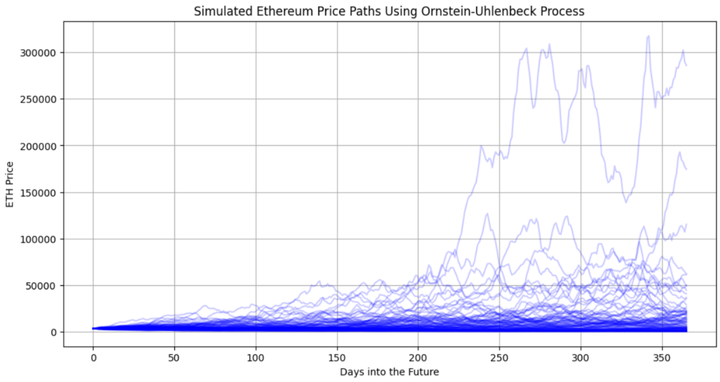

Figure 4.9: ETH O-U Simulated Price Paths

From the above plot, majority of simulated ETH price paths oscillate around a central mean level, GBM where prices tend to drift indefinitely. This suggests that under the O-U model, ETH prices exhibit some level of correction toward a fundamental value over time. For volatility and price spikes, although most paths remain relatively close to the mean, some show extreme upward deviations, indicating periods of high volatility. The behavior aligns with ETH's historical price action, where rapid price surges and corrections are common. Many paths appear to stabilize near the mean, reinforcing the O-U process's tendency to pull prices back toward a central value. However, some trajectories show explosive growth, possibly due to the stochastic noise and model parameters (volatility and mean-reversion speed).

4.7 Simulating BTC and ETH Using Jump Diffusion Model

A Jump-Diffusion Model, such as the Merton Jump-Diffusion Model, extends the GBM by incorporating random price jumps, making it more realistic for modeling asset prices, especially in volatile markets like VAs.

4.7.1 BTC Simulated Price Paths using Merton Jump-Diffusion Model

The plot below represents simulated Bitcoin price paths over 365 days using the Merton Jump-Diffusion Model.

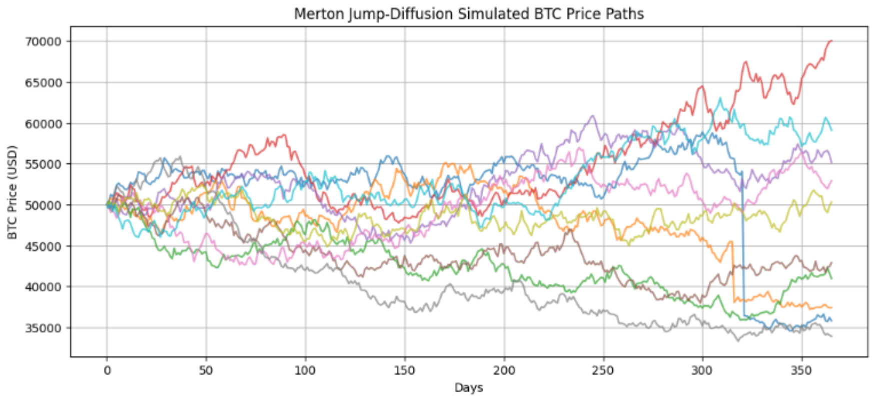

Figure 4.10: BTC Simulated Price Paths

From the plot, the price paths exhibit a general random-walk behavior with an upward trend, driven by drift (expected return) and volatility. Sudden large jumps (both upward and downward) occur sporadically, capturing real-world BTC market shocks. Compared to a standard GBM, this model introduces discrete jumps, making the paths more erratic. These jumps better replicate the extreme price movements observed in BTC markets, such as price surges or crashes. The other observation is diverse simulations, whereby, some paths trend upwards significantly, indicating a bullish BTC market in those scenarios while others decline sharply, showing bearish possibilities. In conclusion, the Merton Jump-Diffusion Model effectively captures the sudden spikes and crashes seen in BTC, making it a more realistic model for pricing and risk analysis compared to pure GBM.

4.7.2 ETH Simulated Price Paths using Merton Jump-Diffusion Model

The plot below represents simulated Ethereum price paths over 365 days using the Merton Jump-Diffusion Model.

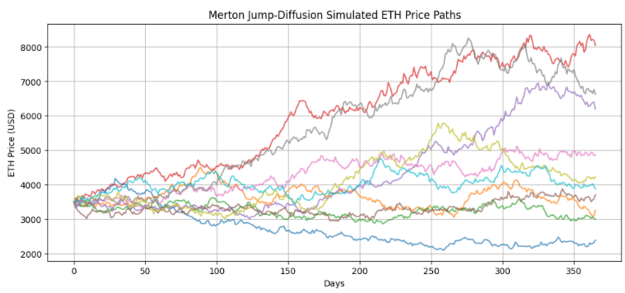

Figure 4.11: ETH Simulated Price Paths

From the plot, the jump-diffusion dynamics observed include, the paths exhibiting random fluctuations with a general upward trend due to the drift component, sudden large jumps appearing at various points, reflecting extreme price changes in the ETH market and some paths showing sharp crashes, capturing possible downward jumps (negative shocks).

The Merton model introduces random jumps, making the price paths more erratic compared to a pure GBM, reflecting the real-world nature of ETH, which is prone to sudden price surges and dips due to news, liquidations, or market sentiment shifts. Additionally, the plot shows divergence in simulations, whereby, some scenarios depict ETH reaching above USD 8,000, indicating strong bullish trends while others trend downward, illustrating potential bearish conditions. This variability highlights the uncertainty in predicting ETH prices, even with stochastic modeling. In conclusion, the Merton Jump-Diffusion Model better captures the high-volatility, event-driven nature of ETH compared to models like GBM, making it more suitable for risk management and derivatives pricing in VA markets.

4.7.3 A Combined Plot of BTC and ETH Simulated Price Paths using Merton Jump-Diffusion Model

When combining the two plots, the following is observed;

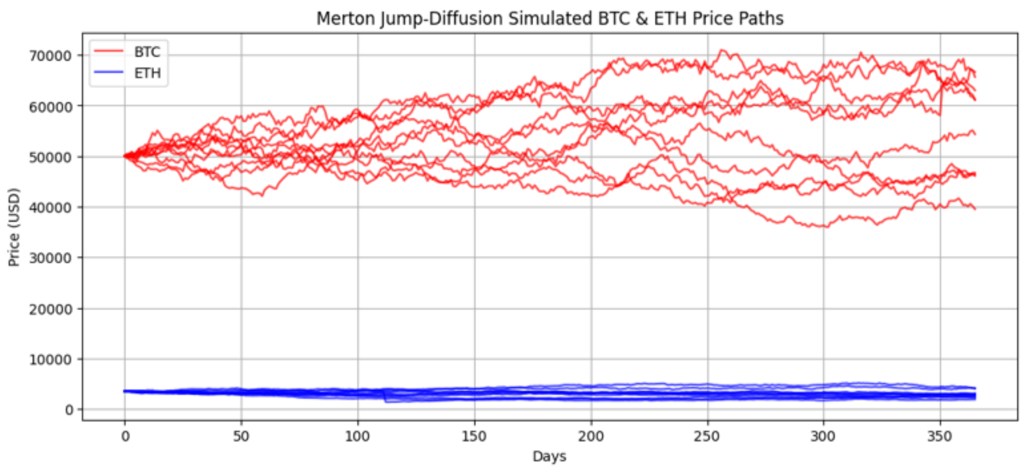

Figure 4.12: BTC & ETH Simulated Price Paths

From the plot, it is observed that BTC prices are significantly higher, ranging between USD 30,000 and USD 70,000 and ETH prices are much lower, staying below USD 10,000, reflecting real-world differences in price levels between the two assets. Both assets exhibit sudden price jumps and drops, capturing real-world VA volatility, which may be a result of extreme market events, news, or liquidations. BTC paths exhibit moderate jumps but maintain a more stable upward trend while ETH paths seem more clustered and have a lower range of fluctuations, indicating relatively lower volatility in this simulation compared to BTC.

BTC appears to be following an upward trajectory with occasional drops showing higher long-term price growth and volatility. ETH remains relatively stable with less pronounced jumps, which could be due to model parameter differences. In conclusion, this Merton Jump-Diffusion simulation effectively captures the stochastic nature of BTC & ETH price movements, incorporating both continuous Brownian motion and random jump risks.

4.7.4 Root Mean Squared Error

To evaluate how well the models we used fits the data, we used the Root Mean Squared Error (RMSE). It compares the models' simulated price paths with the actual historical prices and the model with the lowest RMSE provides the best fit to the actual data.

An analysis of the four models yielded the following RMSEs;

Model	**BTC**	**ETH**
Heston	19,462.0258	26,299.3279
Jump-Diffusion	20,174.1878	26,906.6277
O-U	20,309.3025	27,095.8688
GBM	20,671.7168	27,474.0263

Table 4.4: RMSE Results for Different Stochastic Models

For both BTC and ETH, the Heston model produces the lowest RMSE, which indicates that it most accurately replicates the actual historical price behavior of these assets. This suggests that the Heston model's incorporation of stochastic volatility, capturing volatility clustering and long memory in price fluctuations better fits the market data compared to the simpler GBM, and the Jump-Diffusion and O-U. It provides a more precise estimation of future price behavior, which is critical for derivative valuation and risk management in VA markets.

4.8 Liquidity Analysis and Risk Implications

4.8.1 Trading Volume Trends

The average and variation of daily trading volume is as follows;

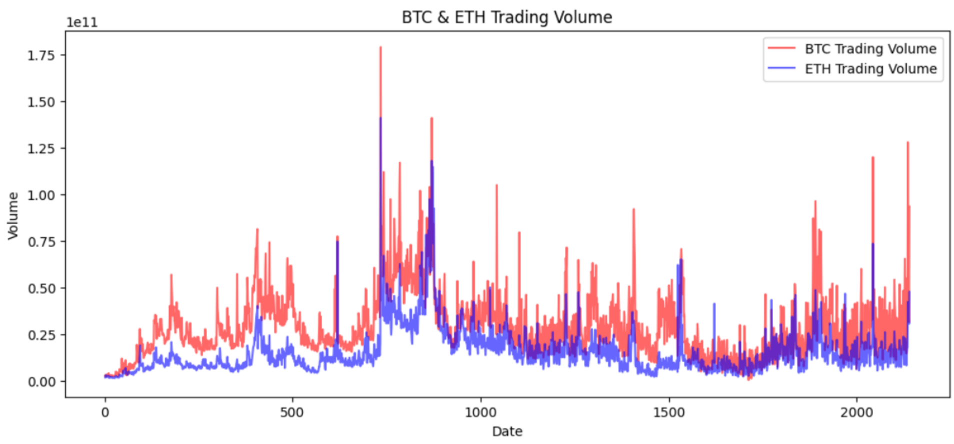

Figure 4.13: BTC & ETH Trading Volume Over Time

The plot shows historical trading volumes of BTC and ETH, where volume spikes represent periods of heightened market activity. BTC generally has higher trading volumes than ETH, indicating deeper liquidity in BTC markets. Both assets exhibit cyclical volume patterns, likely tied to market cycles, regulatory news, and macroeconomic events.

The high-volume periods suggest strong liquidity, meaning large trades can be executed with minimal price impact while the lower-volume periods may result in higher bid-ask spreads and increased slippage, affecting trade execution costs.

For the volatility and liquidity risk, spikes in trading volume often correlate with price volatility, suggesting that liquidity-driven market moves may be a key risk factor. Sudden drops in trading volume may indicate liquidity drying up, increasing risks of price manipulation and flash crashes. For systemic market risk, if volume spikes coincide with major price moves, it suggests herd behavior whereby investors follow the actions of the majority, often ignoring their own analysis or rational decision-making and potential systemic risks in the VA market. Liquidity crunches during market downturns can exacerbate price declines, leading to cascading liquidations.

VA Volume surges may be driven by market sentiment shifts, major announcements such as regulatory crackdowns or institutional trading activities. Monitoring volume trends can help in assessing when market liquidity is at risk due to external shocks.

In conclusion, BTC exhibits stronger liquidity than ETH, reducing price impact risks. Liquidity risk increases during volume downturns, potentially leading to higher volatility. Risk mitigation strategies should consider volume-based signals for trade execution and market stability assessment.

To assess liquidity, relative to market size, we can compute the volume-to-market-cap ratio (VMC) for both BTC and ETH. The volume-to-market-cap ratio measures how much a VA is traded in a day compared to its total value. A higher VMC ratio indicates a more liquid market where assets trade frequently relative to their total market value while a lower VMC ratio suggests lower liquidity, meaning large trades may significantly impact prices.

The image below shows the BTC and ETH VMC over time. As illustrated above,

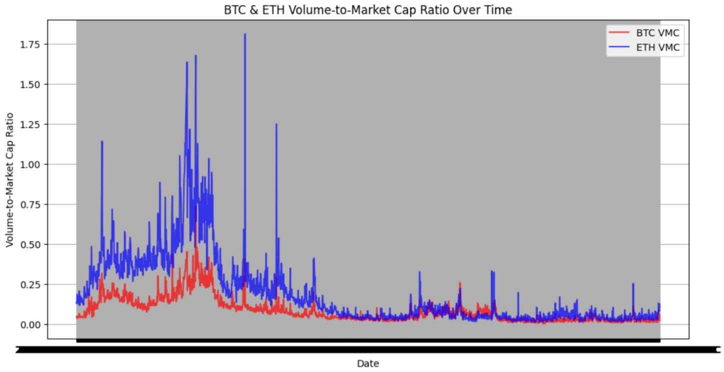

Figure 4.14: BTC & ETH Volume-to-Market Cap Ratio Over Time

ETH shows generally higher VMC peaks in earlier periods, suggesting that, at certain times, ETH's trading volume was disproportionately large relative to its market cap. This could be due to DeFi activity, NFT booms, or other ETH-centric market events driving higher turnover.

BTC has a lower VMC ratio, indicating that, relative to its much larger market cap, BTC's trading volume was proportionally smaller. However, BTC still has deeper absolute liquidity (higher dollar volume), but relative to its large market cap, the ratio remains smaller. In conclusion, monitoring VMC over time can help gauge market liquidity, investor sentiment, and potential speculative activity in the VA markets.

4.9 VA Derivative Valuation Using Stochastic Models

The following graphs present a Monte Carlo simulation of BTC and ETH price paths using a GBM model, along with the estimated BTC and ETH call option prices.

```
Current BTC Price (S0): $80466.72
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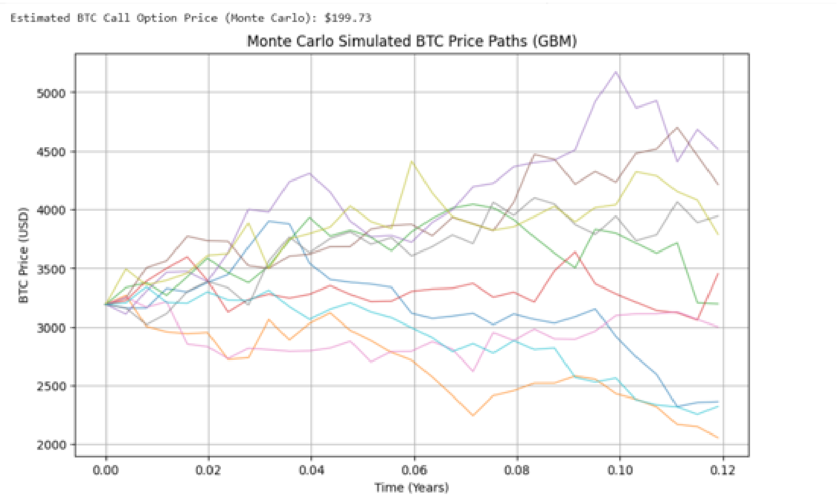


Figure 4.15: Monte Carlo Simulated BTC Price Paths (GBM)

The current spot price of BTC (USD 80,466.72) is quite high, reflecting strong market valuation. The GBM model simulates several potential future price trajectories for BTC over a short time horizon. Given the high spot price of BTC, the relatively low call option price of USD 199.73 suggests either a short time to expiration, limiting the option's value, market expectation of limited extreme upside in the short term and possibly a low implied volatility assumption in the option pricing model.

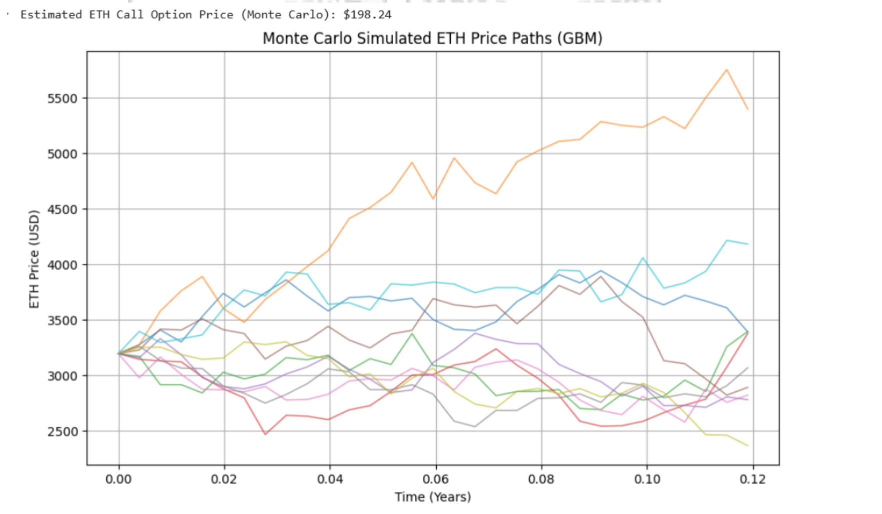


Figure 4.16: Monte Carlo Simulated ETH Price Paths (GBM)

The estimated ETH call option price of USD 199.24 reflects the market's expectation of future volatility. Given the current price of USD 3191.90, this option price indicates a moderate risk premium, suggesting the market does not expect extreme price changes in the short term. Compared to BTC's call option price of USD199.73, the ETH option price is similar, showing a comparable volatility and risk outlook for ETH and BTC in the short term. The ETH market appears to have a balanced outlook with both upside potential and downside risks. The majority of paths hover around the current price level, showing market stability. The call option price aligns with this stability, indicating that while there is some expected volatility, extreme movements are less likely.

The diversity in price paths as shown in both assets, underscores the need for robust stress testing and scenario analysis in the regulatory framework to prepare for both extreme losses and gains. For policy consideration, introducing margin requirements or volatility controls in derivative markets could mitigate risks associated with sharp price movements, while also monitoring liquidity and setting position limits to manage speculative risk effectively.

4.10 GARCH Modeling

Given the persistent volatility clustering observed in BTC and ETH markets, it became necessary to introduce Generalized Autoregressive Conditional Heteroskedasticity (G(ARCH)) models to provide additional insights into short-term volatility dynamics.

Unlike SDE based models, G(ARCH) models explicitly account for time-dependent volatility and asymmetric market reactions to shocks. Their inclusion in this discussion serves to complement the findings from the stochastic models by further examining the persistence and conditional heteroskedasticity in VA volatility.

4.10.1 BTC Model Selection

The following shows the BTC model selection results;

Model	AIC	BIC	Log-Likelihood
GJR-GARCH (1,1)	11136.182488	11164.527629	-5563.091244
EGARCH (1,1)	11154.154864	11176.830177	-5573.077832
GARCH (1,1)	11154.731949	11177.480863	-5573.365975
ARCH	11310.862466	11327.869551	-5652.431233

Table 4.5: BTC G(ARCH) Model Selection Results

From the BTC dataset, the best model is GJR-GARCH (1,1) due to its lowest AIC and BIC, indicating the best balance of goodness-of-fit and model complexity. The highest log-likelihood suggests that it is the most probable model given the data.

4.10.2 ETH Model Selection

The table below shows the ETH model selection results.

Model	AIC	BIC	Log-Likelihood
EGARCH (1,1)	12086.190900	12108.867013	-6039.095450
GJR-GARCH (1,1)	12098.719548	12127.064689	-6044.359774
GARCH (1,1)	12099.209464	12121.885577	-6045.604732
ARCH	12377.253453	12394.206537	-6185.626726

Table 4.6: ETH G(ARCH) Model Selection Results

ETH is best captured by EGARCH (1,1), suggesting that ETH experiences stronger asymmetric volatility reactions and more extreme price fluctuations than BTC. The ARCH model in both cases has the highest AIC/BIC, confirming that a simple conditional variance model is not sufficient for capturing VA market volatility.

Further analysis of the parameter estimates yielded the following;

GARCH (1,1) Model Estimates

Table 1: GARCH (1,1) Model Parameters and P-Values

	BTC		ETH	
Parameter	Value	P-Value	Value	P-Value
Mean (μ)	0.1888	0.0057	0.1789	0.0329
Omega (ω)	0.8370	0.0048	0.6046	0.0107
Alpha (α)	0.1077	0.0281	0.9909	0.0275
Beta (β)	0.8301	0.0000	0.8765	0.0000

EGARCH (1,1) Model Estimates

Table 2: EGARCH (1,1) Model Parameters and P-Values

	BTC		ETH	
Parameter	Value	P-Value	Value	P-Value
Mean (μ)	0.0879	0.0511	0.0999	0.1036
Omega (ω)	0.0612	0.0026	0.0663	0.0506
Alpha ($\alpha[1]$)	0.1636	0.0000	0.1646	0.0003
Beta ($\beta[1]$)	0.9898	0.0000	0.9846	0.0000
Nu (ν)	2.7540	0.0000	3.3569	0.0000

GJR-GARCH (1,1) Model Estimates

Table 3: GJR-GARCH (1,1) Model Parameters and P-Values

	BTC		ETH	
Parameter	Value	P-Value	Value	P-Value
Mean (μ)	0.1020	0.0279	0.1327	0.0356
Omega (ω)	0.1387	0.5593	0.2703	0.4773
Alpha ($\alpha[1]$)	0.0702	0.0000	0.0920	0.0047
Gamma ($\gamma[1]$)	−0.0233	0.4314	−0.0221	0.4046
Beta ($\beta[1]$)	0.9414	0.0000	0.9190	0.0000
Nu (ν)	2.9719	0.0000	3.3654	0.0000

Figure 4.17: Parameter estimates for BTC and ETH

In comparing our results with previous research such as the studies by (Katsiampa, 2017), Chu et al. (2017) and Chan (2024), which found that GARCH (1,1) performs well for capturing general volatility dynamics, but models like EGARCH (1,1) or GJR-GARCH (1,1) better capture the asymmetric response of volatility to market shocks, our findings align with their conclusion. Our GARCH (1,1) results effectively captures overall volatility persistence as shown by the high beta values in both assets, aligning with the study by(Zhang, 2018). Further to that, our EGARCH (1,1) results show that past shocks significantly impact volatility in both assets as indicated by the significant alpha values and the heavy-tailed distributions shown by both assets, implies a higher probability of extreme returns.

The heavy-tailed distributions shown by both assets, align with findings from previous research by Zhang (2018), (De Sousa, 2021) and Chan (2024) whereby they found the price returns of BTC and ETH together with other VAs and NFTs are characterized by heavy tails.

The ability of EGARCH (1,1) to capture asymmetry, allowing different impacts for positive and negative shocks shows that it is better suited for modeling the asymmetric effects of volatility shocks, aligning with Chu et al. (2017). Zhang et al., (2018) in their study, showed that Bitcoin exhibits long memory in volatility, meaning shocks have long-lasting effects and our results confirm this conclusion as shown in the high and significant beta in GARCH (1,1) which indicates that past volatility has a long-lasting influence on future volatility.

Looking at the GJR-GARCH (1, 1) model, our gamma parameter values which measure the asymmetric impact of positive verses negative shocks on volatility for BTC is -0.23282 with a p-value of 0.4314.and that for ETH is -0.022101 and p-value is 0.4046. The results show that for both assets, negative shocks do not necessarily increase volatility more than positive shocks. The p-values are greater than 0.05, meaning the gamma values are not statistically significant, thus, we cannot conclude that negative shocks have a stronger impact on volatility than positive shocks in our dataset. The findings confirm that EGARCH (1,1) is better suited for capturing asymmetry.

Under our (G)ARCH model selection, we found that BTC volatility is best modeled using GJR-GARCH (1,1), reflecting a strong leverage effect where negative price shocks lead to higher future volatility which is also a conclusion that has been made by various studies. Conrad et al. (2018) and Bouri et al. (2017) found that BTC's volatility is highly persistent, exhibits leverage effects meaning that it reacts more strongly to negative economic news, especially macroeconomic shocks, showing that negative shocks increase volatility more than positive shocks of the same magnitude. However, our study shows that the gamma was not significant, suggesting no strong asymmetric volatility effect. The difference could be attributed to different time periods or model specifications.

In looking at the residuals of BTC and ETH as shown below, they both exhibit volatility clustering, with occasional large spikes, suggesting that some heteroscedasticity effects remain, even after modeling.

The plot of the residuals is as follows;

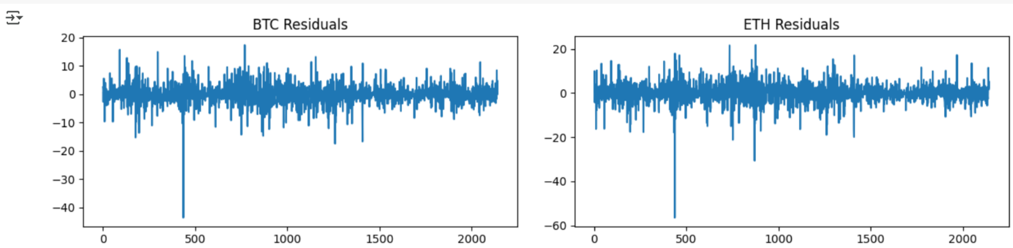


Figure 4.18: Residual plots for BTC and ETH

The BTC Ljung-Box test which assesses autocorrelation, has its results as 15.622491 with a p-value of 0.110961 for BTC and that for ETH as 36.108986 with a p-value of 0.000081. BTC's p-value is greater than 0.05, showing no significant auto-correlation while that of ETH shows significant auto-correlation indicating that persistent nonlinear dependencies. The ARCH-LM test which checks for the presence of heteroscedasticity in residuals has the test static of BTC as 31.1419953 with a p-value of 0.00055586512 and that for ETH is 60.957156 with a p-value of 2.39×10^{-9}. The p-values for both assets are less than 0.05 showing that the (G)ARCH models did not fully remove heteroscedasticity.

The histogram of the residuals as shown below, illustrates fat tails, meaning that large deviations occur more frequently than expected under normality. The results go contrary to the assumption of normality for independent and identically distributions (i.i.d s). Based on the volatility clustering, Ljung-Box test, ARCH-LM test and the histogram, we can conclude that residuals are not independent and identically distributed (i.i.d).

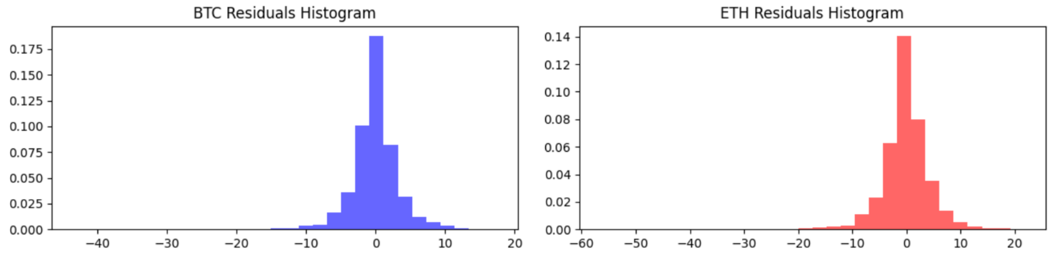
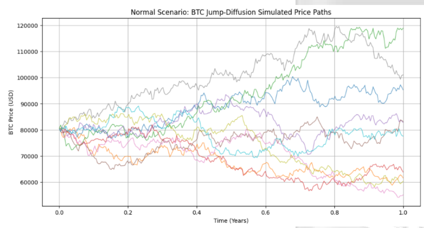


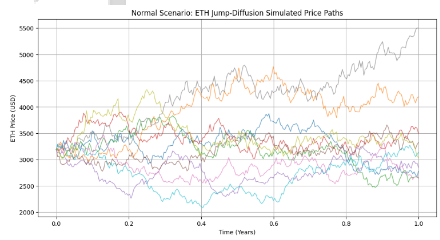
Figure 4.19: Histograms of BTC and ETH residuals.

4.11 Regulatory Insights from Model Results

To get regulatory insights, we simulated extreme market conditions for the two assets and compared it with their normal scenarios and the following are the results;



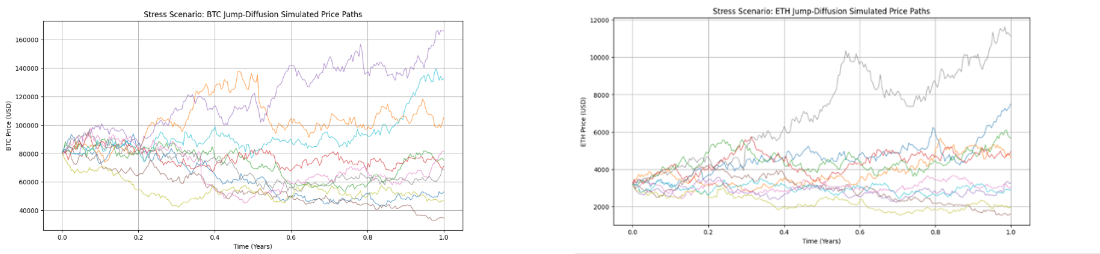
(a) BTC Jump-Diffusion Simulated Price Paths



(b) ETH Jump-Diffusion Simulated Price Paths

Figure 4.20: Normal Scenario: BTC and ETH Jump-Diffusion Simulated Price Paths

Under the normal scenario, the simulated BTC price paths show moderate fluctuations with occasional jumps, reflecting a typical market where BTC moves with its expected drift and experiences infrequent, minor shocks. The paths for ETH, show slightly higher volatility and more noticeable fluctuations compared to BTC, consistent with ETH's generally higher volatility observed historically.



(a) Stress Scenario: BTC Jump-Diffusion Simulated Price Paths

(b) Stress Scenario: ETH Jump-Diffusion Simulated Price Paths

Figure 4.21: Simulated Price Paths for BTC and ETH under Stress Scenario

Under the stress scenario, the simulated BTC paths show much higher volatility and larger negative jumps, indicating how BTC could behave during extreme market conditions, such as regulatory shocks or liquidity crises. Similarly, ETH's stress scenario demonstrates significantly increased volatility, more frequent jumps, and greater price dispersion, suggesting that ETH is even more susceptible to extreme market movements under stress.

4.11.1 Recommendations that can Inform Regulatory Strategies for VA Oversight in Kenya

The stress testing simulations provide a way to stress test the VA market and regulators can use such results to gauge how extreme events may impact market stability. With evidence of extreme volatility under stress, risk-based frameworks could include requirements for liquidity buffers, capital adequacy, and dynamic hedging strategies. This information can help inform oversight strategies in Kenya's emerging VA market.

For further regulatory insights, the research computed the VaR and CVaR (Expected Shortfall). The VaR measures the potential loss in value of an asset or portfolio over a given time period with a specified confidence level while the CVaR estimates the average loss beyond the VaR threshold, considering the worst-case losses.

In computing the VaR and CVaR to look at the general risk management practices, the research applied 95% VaR confidence level as it provides a good balance between capturing significant risks and not being overly conservative. For the regulatory stress tests 99% confidence level was applied as it is helpful to ensure institutions are adequately prepared for severe market events. The 99% confidence level, indicates a much lower probability of exceeding the calculated loss, meaning it is used for more stringent scenarios like regulatory stress tests where regulators want to assess a firm's resilience under extreme market conditions.

From the 95% interval level applied in the research, the following are the results of the risk measures;

Asset	**VaR**	**CVaR**
BTC	-0.5160	-0.5823
ETH	-0.6103	-0.6913

Table 4.7: VaR and CVaR for BTC and ETH

The BTC VaR of -0.5160 shows that there is a 5% chance that BTC could lose at least 51.60% of its value under extreme conditions while the ETH VaR of -0.6103 shows that ETH has a higher risk than BTC, with a potential loss of 61.03% in the worst 5% of cases. From this, ETH appears to be more volatile and riskier under stress than BTC.

The BTC CVaR of 0.5823 shows that if BTC exceeds the 95% VaR threshold, the expected average loss is 58.23%, while the ETH CVaR of -0.6913 shows that ETH's expected worst-case loss is even higher at 69.13%. The CVaR confirms that ETH is more prone to extreme losses compared to BTC under stress scenarios.

The results show that ETH exhibits a higher tail risk than BTC, meaning more capital buffers are needed for ETH trading. The analysis indicate that risk-averse investors may need hedging strategies to mitigate these losses.

From the 99% interval level applied in the research, the following are the results of the risk measures;

Asset	**VaR**	**CVaR**
BTC	$9,401.53	$7,669.60
ETH	$872.76	$732.92

Table 4.8: Risk Measures at 99% Confidence Level

The BTC VaR of USD 9,401.53 means that with 99% confidence, BTC is not expected to lose more than USD 9,401.53 in a given time period under normal market conditions while the CVaR of USD 7,669.60 shows that if losses exceed the 99% threshold, the expected average loss beyond that point is USD 7,669.60.

The ETH VaR of USD 872.76 with 99% confidence shows that ETH is not expected to lose more than USD 872.76 over the given time period while the CVaR of USD 732.92 shows that if losses surpass this level, the expected average loss beyond that threshold is USD 732.92.

In summary, BTC carries higher risk exposure in extreme market conditions, and the CVaR is lower than VaR, suggesting that extreme losses (beyond the 99% threshold) tend to stabilize rather than escalate drastically. The BTC's higher tail risk suggests that liquidity buffers or capital adequacy requirements should be higher for BTC-related financial products than for ETH.

Chapter 5
Discussion, Conclusion, Recommendations and Future Work

5.1 Discussion

This study focused on evaluating the effectiveness of stochastic models including GBM, Heston, Jump-Diffusion,and O-U in capturing the volatility and price dynamics of BTC and ETH. These models were selected for their ability to incorporate stochastic volatility, mean reversion, and price jumps, which are essential for modeling the complex nature of VA markets. It also fitted the G(ARCH) models to help further understand the characteristics of VAs.

In comparing the models, we have used in this research, we can conclude that GARCH models work well in capturing volatility clustering, and they align with findings from Stŕale & Tjernström (2014). The GBM model is simple and widely used in asset pricing and financial modeling, serving as the foundation for the Black-Scholes Model. It captures general price trends with a constant volatility assumption and works well for long-term price movement estimation. It is however, not suitable for VAs as they exhibit volatility clustering and sudden shocks which GBM cannot model. Additionally, GBM assumes prices drift continuously without reverting to any mean, which is unrealistic for some assets. Heston models improve upon GBM by introducing stochastic volatility, making them better suited for modeling VA price dynamics.

Jump-Diffusion models are the best for handling regulatory shocks and news events, which are common in the VA sector. The O-U is relevant, especially in analyzing mean-reverting VAs. In conclusion, stochastic models like Heston and Jump-Diffusion outperform GBM in modeling VA price behavior due to their ability to capture volatility clustering and jumps. GARCH models are effective but lack jump components, making them slightly less adaptable to major shocks.

The results of the (G)ARCH models shows that our choice of using other stochastic volatility models, including jump-diffusion models, Heston and O-U, is better in capturing extreme price movements of VAs. The persistent heteroscedasticity implies that risk estimation methods such as VaR, Expected Shortfall must account for residual volatility effects in valuation models. Understanding these residual patterns is crucial for regulatory frameworks in Kenya, as risk-based supervision must account for hidden dependencies in VA price movements.

According to the research by Zhang(2018), out of the eight VAs they studied including,Bitcoin (BTC), Dash (DASH),Ethereum(ETH), Litecoin (LTC), NEM (XEM), Stellar(XLM), Monero (XMR) and Ripple (XRP), they concluded that VAs exhibit there are heavy tails, absence of autocorrelations, volatility clustering, leverage effect and long-range dependence for the returns of crypto-currencies. Their conclusion, aligns to some extent with our findings where BTC exhibited no significant auto-correlation while that of ETH showed significant auto-correlation indicating persistent nonlinear dependencies.

This study examined the relationship between liquidity measures, market depth and price volatility in BTC and ETH markets. The findings suggest that BTC exhibits stronger liquidity ETH reducing its exposure to significant price impact risks. Conversely, ETH's higher volatility and more pronounced variations in its VMC ratio suggest that, at times, liquidity conditions fluctuate more significantly in ETH markets. In periods of reduced trading volume, liquidity risks increase, potentially exacerbating volatility spikes.

ETH exhibited higher VMC peaks in earlier periods, suggesting that, at certain times, ETH's trading volume was disproportionately large relative to its market capitalization. This could be attributed to ETH's central role in DeFi and NFT booms, and other ETH-centric speculative activities, which led to increased turnover and short-term liquidity surges. On the other hand, BTC maintained a lower VMC ratio, indicating that despite its much larger market capitalization, its trading volume was proportionally smaller. This suggests that BTC, while having deeper absolute liquidity (higher dollar volume), sees less relative turnover compared to ETH.

The relationship between liquidity and volatility aligns with established financial literature. As (Chordia,2001) suggests, liquidity shocks can lead to heightened price volatility, particularly in markets with fluctuating trading volume. The findings in this study reinforce this notion, as ETH's episodic liquidity spikes correlate with higher price fluctuations. Furthermore(Kyle, 1985) proposed that lower market depth increases price sensitivity to large trades, a phenomenon observed in ETH during volume downturns.

The results from our Monte Carlo simulations based on the GBM model provided estimated call option prices for BTC as USD 199.73 and ETH as USD 198.24. The results show that Heston and Jump-Diffusion models offer improvements by incorporating stochastic volatility and market jumps, making them more suitable for pricing BTC and ETH derivatives. The O-U process is useful in futures pricing due to its mean-reverting nature, aligning well with temporary market dislocations.

For the market expectations and risk reflected in derivative prices, we see that despite BTC and ETH's high spot prices, the relatively low call option prices suggest that a short time to expiration limits the options' extrinsic value and there is an impact of market liquidity and risk sentiment on derivative valuation. The near similarity between BTC and ETH option prices implies a comparable volatility outlook, despite differences in liquidity and market structure.

5.2 Conclusion

This study evaluated the effectiveness of stochastic models, such as jump-diffusion and stochastic volatility models, in capturing the unique volatility patterns of BTC and ETH. The findings indicate that models like the Heston and Jump-Diffusion outperform the standard GBM in modeling VA price behavior due to their ability to account for volatility clustering and abrupt price jumps. This aligns with prior research, such as Andersen et al. (2001), which highlights the limitations of GBM in capturing financial market volatility.

Additionally, while GARCH models (Bollerslev, 1986) effectively capture conditional heteroscedasticity, they lack jump components, making them less adaptable to major shocks in the VA market. Stŕale & Tjernström (2014) similarly identified key drivers of BTC price volatility using the GARCH (1,1) model, including market sentiment, regulatory actions, and technological disruptions. However, as Carr & Wu (2003) suggest, models incorporating both stochastic volatility and jump components provide a more robust framework for derivative pricing and risk management.

These results underscore the importance of incorporating stochastic volatility and jump processes when modeling BTC and ETH price dynamics. Given the complex and evolving nature of VA markets, regulatory frameworks should account for the limitations of traditional models and leverage more sophisticated stochastic approaches for risk assessment and policy development. Further, it highlights that stochastic models provide a crucial foundation for accurately valuing VA derivatives, supporting informed decision-making for traders, risk managers, and regulators alike.

The analysis highlights that liquidity conditions play a crucial role in shaping price volatility in BTC and ETH markets. BTC's stronger liquidity profile provides more stability, while ETH's market structure, influenced by DeFi activity and speculative trading, results in higher liquidity variability and volatility. Regulators and market participants should consider volume-based signals when assessing market stability and designing risk mitigation strategies. Implementing adaptive liquidity measures, such as market-wide circuit breakers or volume-adjusted trade execution algorithms, may enhance market efficiency and reduce excessive volatility in virtual asset markets.

The study further demonstrates that stochastic modeling provides essential insights into risk dynamics in VA markets, offering a solid foundation for regulatory frameworks in Kenya. By leveraging advanced volatility models, stress testing techniques, and liquidity assessments, regulators can implement a proactive, risk-based approach to market oversight. The findings suggest that a multi-layered regulatory strategy including incorporating market surveillance, capital buffers, and targeted interventions such as margin requirements and circuit breakers can enhance market stability and reduce systemic risk. Ultimately, integrating insights from stochastic modeling into Kenya's regulatory framework will help navigate the complexities of VA markets, fostering a more stable, transparent, and resilient financial system.

5.3 Recommendations

To enhance market resilience and investor protection, Kenya's regulatory approach should consider several key policy interventions. First, introducing margin requirements for leveraged trading would help mitigate the impact of excessive speculation, which can trigger liquidity crises. Additionally, implementing volatility-based circuit breakers, such as trading halts during extreme price fluctuations, could help stabilize the market and prevent panic-driven sell-offs. Continuous monitoring of liquidity indicators, including bid-ask spreads and order book depth, is crucial for assessing market stability and potential disruptions. Furthermore, a risk-based licensing framework for virtual asset exchanges and custodians should be introduced, ensuring compliance with capital adequacy and security requirements to safeguard investor funds.

5.4 Future Work

Future research could expand on this study by exploring hybrid stochastic models that integrate jump components with stochastic volatility, such as the Bates model, which extends the Heston model by incorporating price jumps. This would allow for a more comprehensive understanding of extreme market movements in VAs. Additionally, further investigation into stress-testing scenarios specific to the Kenyan VA market such as regulatory interventions, capital control measures, and liquidity crises would provide valuable insights into market resilience. The application of machine learning techniques to refine volatility forecasting and improve option pricing accuracy could also be a valuable area of exploration, particularly in adapting models dynamically to real-time market conditions. Furthermore, studying the impact of tokenized derivatives and structured crypto products on market stability and investor behavior could enhance regulatory decision-making.

Lastly, a deeper analysis of VASP activities, including liquidity provision, risk exposure, and compliance with Anti-Money Laundering (AML) regulations, would offer critical insights for developing risk-based regulatory frameworks tailored to Kenya's evolving VA landscape.

Overall, this research lays a foundation for further exploration of stochastic modeling as a tool for regulatory and risk management advancements in Kenya's evolving VA sector. It offers a basis for regulators, especially since it clearly brings out the nature of VAs, which regulators can use to come up with policies around the VA sector.

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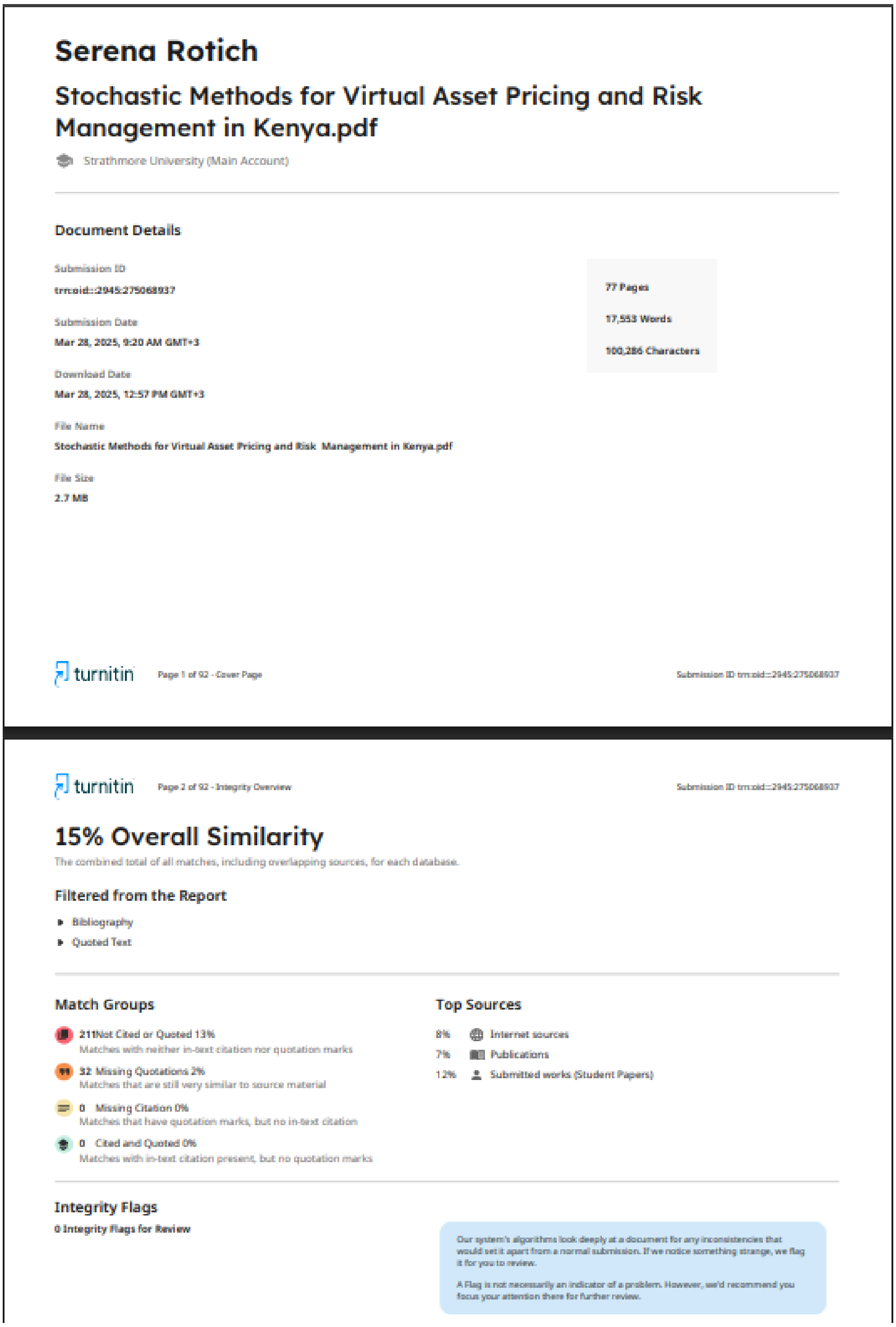
Appendices

Appendix A: Similarity Report



Serena Rotich

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Appendix B: Ethical Clearance Confirmation

Strathmore
UNIVERSITY

17th January 2025

Ms Rotich Serena,
serena.rotich@strathmore.edu

Dear Ms Rotich,

<u>RE: Stochastic Modeling for Virtual Asset Valuation and Risk Management in Kenya</u>

This is to inform you that SU-ISERC has reviewed and **approved** your above **SU-masters** proposal. Your application reference number is **SU-ISERC2545/24.** The approval period is from **17th January 2025 to 16th January 2026.**

This approval is subject to compliance with the following requirements:

i. Only approved documents including (informed consents, study instruments, MTA) will be used.
ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-ISERC.
iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-ISERC within 72 hours of notification.
iv. Any changes anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-ISERC within 72 hours.
v. Clearance for the export of biological specimens must be obtained from relevant institutions.
vi. Submission of a request for renewal of approval at least 60 days prior to the expiry of the approval period. Attach a comprehensive progress report to support the renewal.
vii. Submission of an executive summary report within 90 days of completion of the study to SU-ISERC.

Before commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology, and Innovation (NACOSTI) https://research-portal.nacosti.go.ke/ and obtain other clearances needed.

Yours sincerely,

Mr Ambrose Rachier,
Chairperson; SU-ISERC

Ole Sangale Rd, Madaraka Estate. PO Box 59857-00200, Nairobi, Kenya. Tel +254 (0)703 034000
Email admissions@strathmore.edu www.strathmore.edu