Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
END OF SEMESTER EXAMINATION

MAT 1202 Applied Mathematics II

Instructions **TIME: 11:00-13:30** **Date**: 13th March, 2025

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) The probability distribution of a discrete random variable X is given by

$$P(X=x)=\begin{cases}\dfrac{1}{12}, & x=1,2,3,\cdots,12\\ 0, & \text{otherwise}\end{cases}$$

Determine $P(X+2<3X-4\leq 2X+7)$. [3 Marks]

(b) Find the value of the following multiple integral [3 Marks]

$$\int_0^{\pi}\int_0^{\cos\theta} r\sin\theta\, dr d\theta.$$

(c) The current i in an electric circuit containing resistance R and inductance L in series with a constant voltage source E is given by the differential equation $E-L\left(\frac{di}{dt}\right)=Ri$.
Solve the equation and find i in terms of time t given that when $t=0, i=0$. [4 Marks]

(d) Show that the solution of the differential equation [4 Marks]

$$e^x\frac{dy}{dx}+y^2=xy^2, \quad x>0, y>0,$$

subject to $y=e$ at $x=1$, is $y=\frac{1}{x}e^x$.

(e) $X\sim Geo(p)$ and it is known that $P(X=2)=0.21$ and $p<0.5$. Find $P(X=1)$. [4 Marks]

(f) Calculate $\frac{\partial \omega}{\partial u}$ and $\frac{\partial \omega}{\partial v}$ using the following functions. [4 Marks]

$$\begin{aligned}
\omega &= f(x,y,z) = 3x^2 - 2xy + 4z^2 \\
x &= x(u,v) = e^u \sin v \\
y &= y(u,v) = e^u \cos v \\
z &= z(u,v) = e^u
\end{aligned}$$

(g) A Christmass draw aims to sell 5000 tickets, 50 of which will win a prize. A syndicate buys 200 tickets. Let X represent the number of these tickets that win a prize.
Calculate $P(X \leq 3)$. [4 Marks]

(h) Find the critical point(s) of the following function [4 Marks]

$$f(x,y) = \sqrt{4y^2 - 9x^2 + 24y + 36x + 36}$$

QUESTION TWO (15 MARKS)

(a) Emma plays a game in which she throws two dice. If she gets two sixes, she wins 20p, if she gets one six, she wins 10p, otherwise she wins nothing. She has to pay 5p to enter. Write out the probability distribution of X, the amount Emma gains in one turn. [4 Marks]

(b) The probability that a telephone box is occupied is 0.2. Find, to two significant figures, the probability that a person wishing to make a telephone call will find a telephone box which is not occupied only at the sixth box tried. [5 Marks]

(c) In the mass production of bolts, it is found that 5% are defective. Bolts are selected at random and put into packets of ten. A packet is selected at random. Find the probability that it contains (i) three defective bolts, (ii) less than three defective bolts. [4 Marks]

Two packets are selected at random.

(iii) Find the probability that there are no defective bolts in either packet. [2 Marks]

QUESTION THREE (15 MARKS)

(a) An equation used in thermodynamics is the Benedict-Webb-Rubine equation of state for the expansion of a gas. The equation is:

$$p = \frac{RT}{V} + \left(B_0RT - A_0 - \frac{C_0}{T^2}\right)\frac{1}{V^2} + (bRT - a)\frac{1}{V^3} + \frac{A\alpha}{V^6} + \frac{C\left(1 + \frac{\gamma}{V^2}\right)}{T^2}\left(\frac{1}{V^3}\right)e^{-\frac{\gamma}{V^2}} - C_0$$

Show that $\dfrac{\partial^2 p}{\partial T^2} = \dfrac{6}{V^2T^4}\left\{\dfrac{C}{V}\left(1 + \frac{\gamma}{V^2}\right)e^{-\frac{\gamma}{V^2}} - C_0\right\}$ [4 Marks]

(b) Safe Shades produces two kinds of sunglasses; one kind sells for \$17, and the other for \$21. The total revenue in thousands of dollars from the sale of x thousand sunglasses at \$17 each and y thousand at \$21 each is given by

$$R(x, y) = 17x + 21y.$$

The company determines that the total cost, in thousands of dollars, of producing x thousand of the \$17 sunglasses and y thousand of the \$21 sun- glasses is given by

$$C(x, y) = 4x^2 - 4xy + 2y^2 - 11x + 25y - 3$$

Find the number of each type of sunglasses that must be produced and sold in order to maximize profit. [7 Marks]

(c) The gradient of a curve satisfies

$$\frac{dy}{dx} = \frac{1}{3y^2(x-1)}, \quad x > 1.$$

Given the curve passes through the point $P(2, -1)$ and the point $Q(q, 1)$, determine the exact value of q. [4 Marks]

QUESTION FOUR (15 MARKS)

(a) A sixth form class consists of 6 boys and 4 girls. Three students are selected at random from this class and the variable X represents the number of girls selected. Generate a probability distribution of the variable X. [6 Marks]

(b) Identical independent trials of an experiment are carried out. The probability of a successful outcome is p. On average, five trials are required until a successful outcome occurs.

(i) Find the value of p. [3 Marks]

(ii) Find the probability that the first successful outcome occurs on the fifth trial.

(c) A game consists of throwing tennis balls into a bucket from a given distance. The probability that William will get the tennis ball in the bucket is 0.4. A turn consists of three attempts.

(i) Construct the probability distribution for X, the number of tennis balls that land in the bucket in a turn. **Hint**: use a tree diagram to display all the probabilities. [4 Marks]

(ii) William wins a prize if, at the end of his turn, there are two or more tennis balls in the bucket. What is the probability that William does not win a prize? [2 Marks]

QUESTION FIVE (15 MARKS)

(a) Show that if $y = a$ at $t = 0$, the solution of the differential equation [5 Marks]

$$\frac{dy}{dt} = \omega(a^2 - y^2)^{\frac{1}{2}},$$

where a and ω are positive constants, can be written as $y = a\cos\omega t$.

(b) The potential difference (p.d), V, between the plates of a capacitor C charged by a steady voltage E through a resistor R is given by the equation

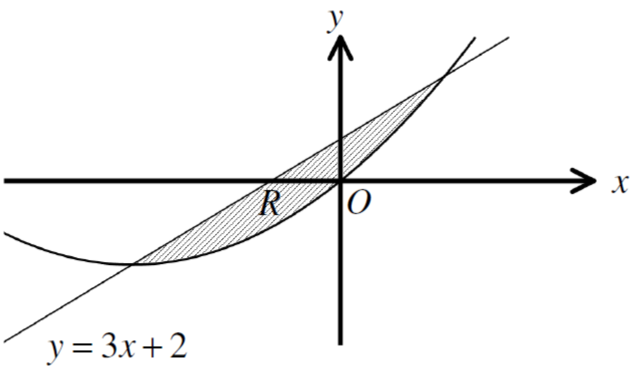
$$CR\frac{dV}{dt} + V = E$$

(i) Solve the equation for V given that at $t = 0$, $V = 0$ [4 Marks]

(ii) Calculate V, correct to 3 significant figures, when $E = 25V$, $C = 20 \times 10^{-6}F$, $R = 200 \times 10^{3}\Omega$ and $t = 3.0s$ [2 Marks]

(c) The finite region **R** in the x-y plane is bounded by a curve and a straight line with the following equations.

$$y = x^2 + 4x \quad \text{and} \quad y = 3x + 2$$



Use double integration to find [4 Marks]

$$\int_R 2\ dxdy.$$

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