



Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

END OF SEMESTER EXAMINATION

MAT1101: MATHEMATICS I

DATE: 19th OCTOBER 2023

Time: 2.5 **Hours**

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE [30 MARKS]

(a) Differentiate the following:

i. $y = e^{2x} \ln 5x$ [2 Marks]

ii. $y = \frac{\cos 2x}{x^2}$ [2 Marks]

(b) Evaluate $\lim_{x \rightarrow 0} x \cot x$ [2 Marks]

(c) Determine:

(i) $\int 3x(1-x^2)^{1/2} dx$ [2 Marks]

(ii) $\int x \cos 3x dx$ [3 Marks]

(d) Evaluate $\int_0^{\pi/4} \frac{\sin 2x}{1 + \cos^2 x} dx$. [4 Marks]

(e) Given that $y = \sin(2x+3)$, find $\frac{dy}{dx}$ from first principles. [5 Marks]

(f) Determine the particular solution of $\frac{dy}{dx} = 3e^{2x-3y}$ given that $y = 0$ when $x = 0$.

[5 Marks]

(g) Use implicit differentiation to determine the equation of the normal to the curve $x^3 + 2y^2 - 4xy = 2$ at the point $(0,1)$.

[5 Marks]

QUESTION TWO [15 MARKS]

(a) In finding the electric field E caused by a surface electric charge on a disk, the equation

$$E = k \int_0^R \frac{r}{(x^2 + r^2)^{3/2}} dr \text{ is used. Evaluate the integral.}$$

[5 Marks]

(b) The charge q on a capacitor in a circuit containing a capacitance C , a resistance R and a source of voltage E , is given by $q = CE\left(1 - e^{-t/RC}\right)$. Show that this equation satisfies the equation

$$R \frac{dq}{dt} + \frac{q}{C} = E.$$

[4 Marks]

(c) The power developed in a resistor R by a battery of e.m.f E and internal resistance r is

$$\text{given by } P = \frac{E^2 R}{(R + r)^2}.$$

i. Find $\frac{dP}{dR}$;

[4 Marks]

ii. Show that at turning point $R = \frac{r}{2E - 1}$.

[2 Marks]

QUESTION THREE [15 MARKS]

(a) A curve is defined parametrically by $y = \cos^3 \theta$, $x = \sin^3 \theta$. Find $\frac{d^2y}{dx^2}$ at the point $\theta = \frac{\pi}{3}$.

[4 Marks]

(b) Determine the turning points of the function $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 5$, hence distinguish them.

[5 Marks]

(c) Evaluate $\int_6^7 \frac{18 + 21x - x^2}{(x-5)(x-2)^2} dx$.

[6 Marks]

QUESTION FOUR [15 MARKS]

- (a) The power P (in W) developed in a certain resistor as a function of the current i (in A) is $P = 6.0i^2$. Find the average power with respect to the current as the current changes from $2.0 A$ to $5.0 A$. **[4 Marks]**
- (b) 500 tons of farm effluent is released into a river. The amount of effluent (E) present is given by the differential equation $\frac{dE}{dt} = 0.1E$, where t is in hours.
- Deduce an expression for E in terms of t . **[4 Marks]**
 - Calculate the amount of effluent present after 5 hours. **[1 Mark]**
 - Calculate the time taken for the amount of effluent to reduce to 10.50 tons. **[2 Marks]**
- (c) Given $y = \sqrt{3 + \sqrt{3 + x}}$, find $\frac{dy}{dx}$. **[4 Marks]**

QUESTION FIVE [15 MARKS]

- (a) Given that $y^2 \sin x - 3y^2 = \sec x$, determine the value of $\frac{dy}{dx}$ when $x = \pi$. **[4 Marks]**
- (b) Obtain the first four non-zero terms in the Maclaurin series expansion of $\cos \lambda x$, where λ is a constant. Hence show that $\int_0^1 \cos \lambda x dx = 1 - \frac{\lambda^2}{6} + \frac{\lambda^4}{120} - \frac{\lambda^6}{5040}$. **[5 Marks]**
- (c) Expand $\ln x$ as a power series of ascending powers of $(x - a)$ up to the term containing $(x - a)^3$ and use it to calculate $\ln 1.01$, correct to five decimal places. **[6 Marks]**

END