



Strathmore
UNIVERSITY

**STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BBS ACTUARIAL SCIENCE, FINANCIAL ECONOMICS & FINANCIAL ENGINEERING
SPECIAL EXAMINATION
BSM 2111: STATISTICAL INFERENCE**

DATE: 4th May, 2022

TIME 2 hour

Instructions: Answer question ONE and any other TWO questions

Question 1 (30marks)

(a) Define the following terms.

- i. Statistic **(2 Marks)**
- ii. Margin of error **(2 Marks)**
- iii. Confidence level **(2 Marks)**

(b) A man arrives at a bus stop at a random time to catch the next bus. Buses run every 30 minutes without fail, hence the next bus will come any time during the next 30 minutes with evenly distributed probability (a uniform distribution).

- i. Find the probability that a bus will come within the next 10 minutes? **(2 Marks)**
- ii. Find expected value and the standard deviation of the amount of time a person must wait for the bus **(5 Marks)**

(c) Consider a population with the following density: $f(x | \theta) = 2\theta^2 x^{-3}$, $x > \theta$, $\theta > 0$, and a random sample of size six: (15, 8, 10, 5, 17, 20). Compute the estimate of θ by the method of moments. **(4 Marks)**

(d) Suppose that y_1, y_2, \dots, y_n are a random sample with a common density depending on a parameter θ . Suppose further that θ_1 and θ_2 are both unbiased estimators of θ based on y_1, y_2, \dots, y_n , and that

$$Eff(\theta_1, \theta_2) = \frac{Var(\theta_2)}{Var(\theta_1)} = \frac{2n^3 + 5n + 1}{2n^3 + n^2 - 1}$$

Which estimator is preferred? **(2 Marks)**

(e) In order to estimate the proportion of students who graduate within six years, the administration at a private university examined the records of 600 randomly selected students who entered the university six years ago, and found that 312 had graduated. Assuming that the sample is sufficiently large, construct a 90% confidence interval for the six-year graduation rate **(5 Marks)**

(f) A bank wishes to estimate, to within KSh 2500, the mean average monthly balance in its checking accounts, with 95% confidence. Assuming $\sigma = \text{Ksh } 2500$, estimate the minimum size sample required. **(3 Marks)**

(g) A two-sided test of $H_0: \mu=0$ yields a P-value of 0.04. Will the 90% confidence interval for μ include 0? Explain your reasoning. **(3 Marks)**

Question 2 (20 marks)

(a) State without proof the central limit theorem **(2 Marks)**

(b) A businessman makes a sale on 60% of his calls on regular customers. He makes 6 sales calls each day.

i. Construct the probability distribution of X, the number of sales made each day. **(2 Marks)**

ii. Find the probability that, on a randomly selected day, the salesman will make a sale. **(4 Marks)**

iii. Assuming that the salesman makes 30 sales calls per week, find the mean and standard deviation of the number of sales made per week **(4 Marks)**

(c) Suppose that the mean cost across the country of a week supply of a product is Ksh250 with a standard deviation of Ksh 20.

i. Find the probability that the cost of a randomly selected week supply is between Ksh 300 and Ksh 350. **(4 Marks)**

ii. Find the probability that the mean cost of 20 randomly selected weeks' supply is between Ksh 270 and Ksh 350. **(4 Marks)**

Question 3 (20 marks)

- (a) State and explain three (3) characteristics of a good estimator. **(6 Marks)**
- (b) Suppose a random sample x_1, x_2, \dots, x_n of size n was collected from a Uniform distribution with unknown mean θ .
- Use the method of maximum likelihood (ML) to estimate θ . **(6 Marks)**
 - Determine if the ML estimator found in i above an unbiased estimator. **(4 Marks)**
- (c) Suppose a random sample x_1, x_2, \dots, x_n are iid with the Poisson population on the interval $(0, \theta)$ Where θ ($\theta > 0$) is the unknown parameter.
- Derive the method of moments estimator for θ **(4 Marks)**

Question 4 (20 marks)

- (a) Differentiate between type I error and type II error as used in hypothesis testing **(4 Marks)**
- (b) The recipe for a bakery item is designed to result in a product that contains 8 grams of fat per serving. The quality control department samples the product periodically to ensure that the production process is working as designed.
- State the relevant null and alternative hypotheses. **(1 Mark)**
 - Suppose that it is known that the population is normally distributed with standard deviation $\sigma = 0.15$ gram and suppose that the test of hypotheses will be performed with a sample of size 5. Construct the rejection region for the test for the choice at $\alpha = 0.10$. Explain the decision procedure and interpret it. **(6 Marks)**
- (c) In the previous year the proportion of deposits in checking accounts at a certain bank that were made electronically was 35%. The bank wishes to determine if the proportion is higher this year. It examined 30,000 deposit records and found that 9,000 were electronic. Determine, at the 5% level of significance, whether the data provide sufficient evidence to conclude that more than 35% of all deposits to checking accounts are now being made electronically. Use the p-value approach. **(9 Marks)**

Question 5 (20 marks)

(a) A beverage company markets a new product with two experimental packaging designs. Design A is sent to 10 stores; their average sales the first month is 60 units with sample standard deviation 15 units. Design B is sent to 5 stores; their average sales the first month is 48 units with sample standard deviation 12 units. Construct a point estimate and a 95% confidence interval for the difference in average monthly sales between the two package designs. Assume that the two populations have the equal variances. Interpret your answer **(10 Marks)**

(b) A business owner had been working to improve employee relations in his company. He predicted that he met his goal of increasing employee satisfaction from 65% to 80%. Employees from 3 departments were asked if they were satisfied with the working conditions of the company. The results are shown in the following table:

Level of satisfaction	Department		
	Finance	Sales	Technology
Satisfied	12	25	23
dissatisfied	6	8	7

Is there association between the level of satisfaction and the department in the company at 5% level of significance? **(10 Marks)**