STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
BACHELOR OF SCIENCE IN ACTURIAL SCIENCE
BSA 3108: ACTUARIAL MODELLING 2

Date: 21st March 2018 **Time:** 2Hours

INSTRUCTIONS

Answer question one and any other two questions

Question 1
Part A

(i) Define how the following forms of censoring arise in a survival investigation:

- right censoring
- type I censoring
- random censoring [3]

An experience analysis is conducted where the event of interest is the lapse of a term assurance policy.

(ii) Explain whether each form of censoring listed in part (i) occurs in each of the following situations. If it is not possible to state whether a form of censoring occurs, explain why this is the case.

(a) A policyholder dies.

(b) A subset of the policies is migrated to a new administration system and no data are provided from the new system to the experience analysis team.

(c) A policy reaches its maturity date. [4]

Part B

Company A and Company B are two small insurance companies which have recently merged to form Company C. Company C is reviewing its premium rates for a whole of life product and so is conducting an analysis of mortality rates experienced.

Company A recorded the number of policies in force every 1 January using a definition of age next birthday whereas Company B recorded the number of policies in force every 1 April using an age definition of age last birthday. Both companies recorded deaths as they happened using an age definition of age last birthday.

These are the data for the most recent years.

Company A

Age next birthday	*Number of policies 1 Jan. 2012*	*Number of policies 1 Jan. 2013*	*Number of policies 1 Jan. 2014*
51	8,192	6,421	8,118
52	7,684	8,298	7,187
53	9,421	8,016	9,026

Company B

Age last birthday	*Number of policies 1 April 2012*	*Number of policies 1 April 2013*	*Number of policies 1 April 2014*
51	4,496	3,817	4,872
52	5,281	5,218	3,812
53	4,992	5,076	5,076

In the calendar year 2013 Company A recorded 28 deaths of those aged 52 last birthday and Company B recorded 17 deaths of those aged 52 last birthday.

(i) Estimate the force of mortality for the combined company for age 52 last birthday, stating all assumptions that you make. [6]

(ii) Explain the exact age to which your estimate applies. [1]

Part C

A study was made of a group of people seeking jobs. 700 people who were just starting to look for work were followed for a period of eight months in a series of interviews after exactly one month, two months, etc. If the job seeker found a job during a month, the job was assumed to have started at the end of the month. Unfortunately, the study was unable to maintain contact with all the job seekers.

The data from the study are shown in the table below:

Months since start of study	*Found employment*	*Contact lost*
1	100	50
2	70	0
3	50	20
4	40	20
5	20	30
6	20	60
7	12	38
8	6	0

(i) (a) Describe two types of censoring present in the investigation.

(b) Describe an example of a person to whom each type applies. [3]

(ii) Calculate the Kaplan-Meier estimate of the function for "remaining without employment". [6]

A Weibull distribution with a rate $h(t)$ given by the formula $h(t) = \lambda^{\beta}\beta\, t^{\beta-1}$ was fitted to these data. The estimated value of λ was 0.18 and the estimated value of β was 0.3.

(iii) Test the goodness-of-fit of the data to this Weibull distribution. [6]

Question 2
Part A

(i) Explain what is meant by a proportional hazards model. [3]

(ii) Outline three reasons why the Cox proportional hazards model is widely used in empirical work. [3]

Part B

(i) Describe an example of a situation when graduation by parametric formula would be used. [1]

(ii) State two advantages and two disadvantages of graduation by parametric formula. [4]

(iii) (a) Explain why the χ^2 test is different when considering the goodness of fit of graduated data compared with when considering the similarity of two sets of data.

(b) Describe how this is dealt with when the graduation has been carried out by parametric formula. [4]

Part C

A school offers a one year course in a foreign language as an evening class. This is divided into three terms of 13 weeks each with one lesson per week. At the end of each lesson all the students sit a test and any that pass are awarded a qualification, and no longer attend the course.

Last year 33 students started the course. Of these 13 dropped out before completing the year, and 16 passed the test before the end of the year. The last lesson attended by the students who did not stay for the whole 39 lessons is shown in the table below along with their reason for leaving.

Number of students	*Last lesson attended*	*Reason for leaving*
5	1	Dropped out
1	6	Dropped out
2	7	Passed test
2	13	Dropped out
5	14	Passed test
6	27	Passed test
4	28	Dropped out
1	30	Dropped out
3	36	Passed test

Calculate the Nelson-Aalen estimate of the survival function. [5]

Question 3

(i) Define a Markov Jump Process. [1]

A company provides phones on contracts under which it is responsible for repairing or replacing any phones which break down.

When a customer reports a fault with a phone, it is immediately taken to the company's repair shop and it is assessed whether it can be fixed (meaning fixable at reasonable cost). Based on previous experience, it is estimated that the probability of a phone being fixable is 0.75. If a phone is not fixable it is discarded and the customer is provided with a new phone.

If a repaired phone breaks again the company, in line with its customer charter, will not attempt to repair it again, and so discards the phone and replaces it with a new one.

The status of a phone is to be modelled as a Markov Jump Process with state space {Never Broken (NB), Repaired (R), Discarded (D)}.

The company considers the rate at which phones break down to vary according to whether a phone has previously been repaired as follows:

Status	*Probability of break down in small interval of time, dt*
Never Broken	$0.1dt + o(dt)$
Repaired	$0.2dt + o(dt)$

(ii) Draw a transition diagram for the possible transitions between the states, including the associated transition rates. [2]

Let $P_{NB}(t)$, $P_R(t)$ and $P_D(t)$ be the probabilities that a phone is in each state after time t since it was provided as a new phone.

(iii) Determine Kolmogorov's forward equations in component form for $P_{NB}(t)$, $P_R(t)$ and $P_D(t)$. [2]

(iv) Solve the equations in part (iii) to obtain $P_{NB}(t)$ and $P_R(t)$. [4]

(v) Calculate the probability that a phone has not been discarded by time t. [1]

Part B:

A health insurance company has collected data on sickness rates during the calendar year 2013 among a sample of its policyholders aged 40–64 years inclusive. It compares these to the rates among its policyholders of the same age in 2012. It finds that at ages 40–50 years inclusive, and at ages 56–61 years inclusive, the sickness rates in 2013 are higher than those in 2012. At other ages, the sickness rates in 2013 were lower than those in 2012.

(i) Carry out two tests of the null hypothesis that the underlying sickness rates in 2013 are the same as those in 2012. [6]

(ii) Comment on the implications of the results of your test for the company's sickness insurance business. [2]

Question 4

Part A:

Doctors at a health centre are carrying out an investigation to see if obesity affects the likelihood of dying from heart disease. They propose to use a model with four states:

1. Obese
2. Not obese
3. Dead due to heart disease, and
4. Dead due to other causes

(i) Write down, defining all the terms you use, the likelihood for the transition intensities. [3]

(ii) Derive the maximum likelihood estimator of the force of mortality from heart disease for Obese people. [3]

The investigation has followed several thousand people aged 50–59 years for five years and has the following data:

Waiting time in state Obese (in person-years)	14,392
Waiting time in state Not obese (in person-years)	18,109
Number of deaths due to heart disease for those persons who are Obese	178
Number of deaths due to heart disease for those persons who are Not obese	190
Number of deaths due to other causes for those persons who are Obese	89
Number of deaths due to other causes for those persons who are Not obese	53

The doctors want to promote healthy living and therefore wish to claim that Obese people have a much higher chance, statistically, of dying from heart disease than do people who are Not obese.

(iii) Test whether this claim is true at the 90% confidence level. [5]

Part B:

(i) State the principle of correspondence as it applies to death rates. [1]

A nightclub opens at 10.00 p.m. and closes at 2.00 a.m. It admits only people aged over 21 years on the production of an identity card giving date of birth.

The table below shows the number of people entering in various intervals between 10.00 p.m. and 2.00 a.m. on 30 June 2013. No-one was admitted after 1.00 a.m., and you may assume that all those who enter the premises stay until 2.00 a.m.

Year of birth	*10.00–11.30 p.m.*	*11.30–12.00 p.m.*	*12.00 p.m.–1.00 a.m.*
1989	100	300	200
1990	200	400	350
1991	150	400	300
1992	100	250	200

During the period of opening, 40 people aged 22 last birthday required medical attention for heat exhaustion.

(ii) Calculate the rate per person-hour at which those attending the night club aged 22 last birthday required medical attention for heat exhaustion, stating any assumptions you make. [6]

Question 5

(i) Describe why an insurance company might want to compare the results of a mortality investigation with previous experience. [2]

A large life insurance company has undertaken an investigation of the mortality of its policyholders. Currently it assumes that mortality at age x, μ_x, is equal to a standard table. The company wishes to use the results from the investigation to see whether the standard table is still appropriate. Below are shown some data from the investigation.

Age x	*Number of policies in force*	*Actual death claims*	*Expected death claims from standard table*
70	1,000	13	23.74
71	1,200	28	31.80
72	1,100	31	32.50
73	1,100	34	36.20
74	1,000	39	36.63
75	1,000	41	40.73
76	950	41	42.99
77	900	40	45.20
78	850	46	47.34
79	800	48	49.35

(ii) Perform an overall test of the hypothesis that the underlying mortality of the company's policyholders is, over this range of ages, represented by the standard table. [6]

(iii) Evaluate the suitability of the standard table for use in the company's financial modelling by performing two additional tests for different possible inconsistencies between the actual death rates and those represented by the standard table. [6]

The company discovers that at age 70 years, one individual owns 25 of the policies in the investigation, the remaining policies each being owned by different individuals.

(iv) Assess the impact of this on the variance of the number of claims at age 70 years. [4]

END