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Intertemporal Equity Asset Pricing with Stochastic Volatility at the NSE and the JSE

Chelimo John Kigen

Reg. Number: 063946

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School of Finance and Applied Economics
Strathmore University

Nairobi, Kenya

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Declaration
I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

John Kigen Chelimo

Signed:...

Date: 19TH JANUARY 2015

This Research Project has been submitted for examination with my approval as the Supervisor.

Ferdinand O. Othieno

Signed:...

Date: 19/JAN/2015

School of Finance and Applied Economics.

Strathmore University

Abstract

This paper explores the implementation of an intertemporal asset pricing model with stochastic volatility. This model is applied to equity asset pricing at the Nairobi Securities Exchange (NSE) and the Johannesburg Stock Exchange (JSE). The return on the aggregate stock market is modelled using a vector auto regression (VAR) model and the volatility of all shocks to the VAR is modelled using GARCH and EGARCH models. It is shown that the reduced form of the ICAPM with stochastic volatility is inadequate in the context of equity asset pricing at the NSE and JSE. However, the variables indicate the existence of a significant relationship between asset returns and realized market variance and PE ratios to motivate further research.

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1 Introduction

1.1. Background to the study

The extent to which asset prices in the future can be predicted on the basis of currently available information is a matter of great significance to practical investors as well as academic model builders. (Bailey, 2005). For investors, the objective is to exploit their knowledge to obtain the best rates of return from their portfolios of assets. (Bailey, 2005).

The CAPM of Sharpe (1964) and Lintner (1965) is widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios. The attraction of the CAPM is that it offers powerful and intuitively pleasing predictions about how to measure risk and the relation between expected return and risk. (Fama & French, 2004).

The CAPM builds on the model of portfolio choice developed by Markowitz (1959). In Markowitz's model, an investor selects a portfolio at time *t-1* that produces a stochastic return at time *t*. The model assumes that investors are risk averse, and when choosing among portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that the portfolios 1) minimize the variance of portfolio return, given expected return, and 2) maximize expected return, given variance. (Fama & French, 2004).

Fama & French (1992) produce two negative conclusions about the empirical adequacy of the CAPM of Sharpe (1964) and Lintner (1965); (i) when one allows for variation in CAPM market βs that is unrelated to size, the univariate relation between β and average return for their sample period is weak; (ii) β does not suffice to explain average return. Size (market capitalization) captures differences in average stock returns that are missed by β. (Fama & French, 1996)

Researchers have identified many patterns in average stock returns for example DeBondt and Thaler (1985) find a reversal in long-term returns; stocks with low long-term past returns tend to have higher future returns. In contrast, Jegadeesh and Titman (1993) find that short-term returns tend to continue; stocks with higher returns in the previous twelve months tend to have higher future returns. Others show that a firm's average stock return is related to its size (ME, stock price times number of shares), book-to-market equity (BE/ME, the ratio of the book value of common equity to its market value), earnings/price (E/P), cash flow/price (C/P), and past sales growth. (Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer, and Vishny (1994).) Because these patterns in average stock returns are not explained in the CAPM they are typically called anomalies. (Fama & French, 1996).

Fama & French (1996) argue that many of the CAPM average-return anomalies are related, and they are captured by the three-factor model in Fama and French (1993). The model says that the expected return on a portfolio in excess of the risk free rate $[E(R_i) - R_f]$ is explained by the sensitivity of its return to three factors; (i) the excess return on a broad market portfolio $(R_m - R_f)$; (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big); and (iii) the difference between the return on a portfolio of low-book-to-market stocks (HML, high minus low). (Fama & French, 1996)

The average-return anomalies of the CAPM suggest that, if asset pricing is rational, a multifactor version of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's, (1976) Arbitrage Pricing Theory (APT) can provide a better description of average returns. (Fama & French, 1996). The excess market return of the CAPM is a relevant risk in many multifactor alternatives, like the ICAPM and Connor's (1984) equilibrium version of the APT. Thus evidence of a positive relation between $\beta$and expected return does not favour the CAPM over these alternatives. (Fama & French, 1996)

The Arbitrage Pricing Theory (APT) presented in Ross (1976) was proposed as the testable alternative, and perhaps the natural successor to the CAPM. An important intuition in modern portfolio theory is that it is the co-variability of an asset's returns with the return on other assets, rather than its total variability, that is important from the perspective of the risk averse investor who holds a well-diversified portfolio of many assets. Ross's seminal contribution was his insight that this intuition can be transformed into a theory of asset pricing with implications similar to the CAPM. (Shanken, 1982).

The APT assumes that returns conform to a K-factor linear model($K < N$):

$$R_i = E_i + \beta_{i1}\delta_1 + \cdots + \beta_{iK}\delta_K + \epsilon_i, \qquad i = 1, N$$

R_i is the random return on asset i, and E_i its expected return. The $\delta_k$are mean zero common factors and the $\epsilon_i$are mean zero asset-specific disturbances assumed to be uncorrelated with the $\delta_K$and with each other. In the language of factor analysis, the β_{iK} are the factor loadings. N is the number of assets under consideration. (Shanken, 1982).

The fundamental insight of intertemporal asset pricing is that long-term investors should care just as much about the returns they earn on their invested wealth as about the level of that wealth. In the case of time-varying investment opportunities, conservative investors will seek to hold "inter-temporal" hedges, assets that perform well when investment opportunities deteriorate. Such assets should deliver lower average returns in equilibrium if they are priced from conservative long-term investors' first-order conditions. (Campbell J. , Giglio, Polk, & Turley, 2012)

Campbell, Giglio, Polk, & Turley (2012) build on the Intertemporal CAPM (ICAPM) of Merton (1973) and expand this by extending the closed-form ICAPM to allow for stochastic volatility. While a great deal of literature has followed from the work of Merton (1973), most of them fail to consider the time variation in the volatility of stock returns. Campbell, Giglio, Polk, & Turley (2012) build a model that explains the risk premia in the stock market using three priced risk factors that correspond to three important attributes of aggregate market returns: revisions in expected future cash flows, discount rates, and volatility. An attractive feature of their model is that the prices of these three risk factors depend on only one free parameter, the long-horizon investor's coefficient of risk aversion.

This research tests the intertemporal CAPM with stochastic volatility presented in Campbell, Giglio, Polk, & Turley (2012) to evaluate the applicability of the model for practical equity asset pricing at the Nairobi Securities Exchange (NSE) and the Johannesburg Securities Exchange (JSE).

1.2. Problem statement

Asset pricing theory has had particular challenges in generalising the insights of static asset pricing theory to incorporate multi-period considerations. The intertemporal CAPM presented in Merton (1973) provides a basic framework for analysis of multi-period capital asset pricing. In their study, Campbell, Giglio, Polk, & Turley (2012) find that their three-beta model explains over 62% of the cross-sectional variation in average returns of 25 portfolios sorted on size and book-to-market ratios. The model is not rejected at the 5% level while the CAPM is strongly rejected. This research seeks to test this model's performance with regard to equity asset pricing at the Nairobi Securities Exchange and the Johannesburg Securities Exchange.

1.3. Research objective

The objective of this research is to find out if an Intertemporal Capital Asset Pricing Model (Intertemporal CAPM) with Stochastic Volatility is a good model for equity asset pricing at the Nairobi Securities Exchange and the Johannesburg Securities Exchange.

1.4. Hypothesis

The research hypotheses are as follows;

H_{01}; An Intertemporal CAPM with Stochastic Volatility is an appropriate model for equity asset pricing.

H_{A1}; An Intertemporal CAPM with Stochastic Volatility is not an appropriate model for equity asset pricing.

1.5. Importance of the Research

This research will contribute to the asset pricing debate by providing additional empirical evidence regarding the Intertemporal CAPM with Stochastic Volatility. This research aims to provide a guide to practical implementation of the Intertemporal CAPM in a frontier economy such as Kenya and an emerging market economy in this case, South Africa. This will therefore assist in the estimation of the cost of equity for firms by incorporating multi-period considerations into the asset pricing model. This research also aims to provide empirical support for intertemporal models of equity asset pricing.

2 Literature review

2.1 Overview

The organisation of this literature review is as follows: Section 2 reviews the classical capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) and consideres the critique levelled against the CAPM in Roll (1977) which highlights the inadequacy of the CAPM for practical equity asset pricing. Section 3 then considers two alternative asset pricing models; the three-factor model presented in Fama & French (1993) and the Arbitrage Pricing Theory (APT) presented in Ross (1976). Section 4 then considers the Intertemporal CAPM (ICAPM) as presented in Merton (1973) and evaluates the body of literature that has followed from the paper. Finally, the review of literature is concluded with an evaluation of the Intertemporal CAPM with stochastic volatility as presented in Campbell, Giglio, Polk, & Turley (2012), how this model fits into the asset pricing debate and how it differs from other intertemporal forms of the CAPM.

2.2 The Classical Capital Asset Pricing Model (CAPM)

The CAPM builds on the model of portfolio choice developed by Markowitz (1959). Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model to identify a portfolio that must be mean-variance efficient. The first assumption is complete agreement and the second assumption is that there is borrowing and lending at a risk-free rate, which is the same for all investors and does not depend on the amount borrowed or lent. (Fama & French, 2004).

In the view of Fama & French (1996), the evidence that β does not suffice to explain expected return is compelling. And the average-return anomalies of the CAPM are serious enough to infer that the model is not a useful approximation. Their bet, is that the payoffs in empirical asset pricing are in showing that the failures of the CAPM can be explained by multifactor ICAPM or APT alternatives – or that they are consistent with irrational-asset-pricing stories. (Fama & French, 1996).

Fama & French (1996) tests of the CAPM against a multifactor alternative illustrate that a positive β premium does not in itself resuscitate the CAPM, or justify using it in applications. (Fama & French, 1996).

2.2.1 The Roll Critique

The CAPM is testable in principle; but arguments are given in Roll (1977) that: (a) No correct and unambiguous test of the theory has appeared in the literature, and (b) there is practically no possibility that such a test can be achieved in the future.

The critiques presented in Roll (1977) are that; the only testable hypothesis associated with the CAPM is whether the market portfolio is mean-variance efficient. All other implications of the model follow from the market portfolio's efficiency and are not independently testable. His critique postulates that the theory is not testable unless the exact composition of the true market portfolio is known and used in the tests. All individual assets must therefore be included in the sample. Use of a proxy for the market portfolio is subject to two difficulties; first that the proxy itself might be mean-variance efficient even when the true market portfolio is not. Alternatively, the chosen proxy may turn out to be inefficient but this alone implies nothing about the true market portfolio's efficiency.

Roll (1977) then considers the case where the market portfolio is knowable. In this case, he proposes that a test of the proxy's mean-variance efficiency is difficult computationally because the full sample covariance matrix of individual returns must be inverted because the sampling distribution of the efficient set is generally unknown. Further, testing for the proxy's efficiency by using the return/beta linearity relation also poses empirical difficulties. Also, deviations from the return/beta linearity relation are frequently linked with some other phenomenon. Finally, Roll (1977) critiques the beta itself as a risk measure on two grounds; First, that it will always be significantly related to observed average individual returns if the market index is on the positively sloped section of the ex-post efficient frontier, regardless of investors' attitudes towards risk; and second, that it depends, non-monotonically, on the particular market proxy used.

2.3 Other Asset Pricing Models

2.3.1 Fama & French (1993)

Fama & French (1993) identify five common risk factors in the returns on stocks and bonds. There are three stock-market factors: an overall market factor and factors related to firm size and book-to-market equity. There are two bond-market factors, related to maturity and default risks. Stock returns have shared variation due to the stock-market factors, and they are linked to bond returns through shared variation in the bond-market factors. Except for low-grade corporates, the bond market factors capture the common variation in bond returns. (Fama & French, 1993)

The three-factor model in Fama & French (1993) provides a better description of average returns than the CAPM, and it captures most of the average-return anomalies missed by the CAPM. Because of its strong theoretical standing, the excess market return is one of the three risk-factors in the model, and Fama & French (1996) tests confirm that it is important. It captures strong common time-series variation in returns, and the market premium is needed to explain the large differences between the average returns on stocks and bills. Moreover as in the CAPM, the market premium in the multifactor model is just the average return on M in excess of the risk-free rate. Tests on long sample periods say that this premium is reliably positive. (Fama & French, 1996).

Specifically, Fama & French, (1993) give the expected excess return on portfolio i as,

$$E(R_i - R_f) = b_i[E(R_m) - R_f] + s_i[E(SMB)] + h_i[E(HML)]$$

Where $(ER_m - R_f)$,$E(SMB)$, and $E(HML)$are expected premiums, and the factor sensitivities or loadings, b_i, s_i, h_i, are the slopes in the time-series regression,

$$R_i - R_f = \alpha_i + b_i(R_m - R_f) + s_i(SMB) + h_i(HML) + \varepsilon_i$$

Fama and French (1995) show that book-to-market equity and slopes on HML proxy for relative distress. Weak firms with persistently low earnings tend to have high BE/ME and positive slopes on HML; Strong firms with persistently high earnings have low BE/ME and negative slopes on HML. (Fama & French, 1996)

The above three-factor model seems to capture much of the cross-sectional variation in average stock returns. Fama and French (1993) show that the model is a good description of returns on portfolios formed on sized and BE/ME. Fama and French (1994) use the model to explain industry returns. In Fama & French (1996), it is shown that the three-factor model captures the returns to portfolios formed on E/P, C/P and sales growth. Fama & French (1996) find that the low E/P, low C/P, and high sales growth are typical of strong firms that have negative slopes on the HML.

Since the average HML return is strongly positive (about 6 percent per year), these negative loadings which are similar to the HML slopes for low-BE/ME stocks, imply lower expected returns in the three factor model. Conversely, like high-BE/ME stocks, stocks with high E/P, high C/P, or low sales growth tend to load positively on HML (they are relatively distressed), and they have higher average returns. (Fama & French, 1996). The three-factor model also captures the reversal of long-term returns documented by DeBondt and Thaler (1985). Stocks with low long-term past returns (losers) tend to have positive future average returns. Conversely, long-term winners tend to be strong stocks that have negative slopes on HML and low future returns. (Fama & French, 1996). Thus a market factor and the proxies for the risk factors related to size and book-to-market equity seem to do a good job explaining the cross-section of average stock returns. (Fama & French, 1993)

The three factor model however, cannot explain the continuation of short-term returns documented by Jegadeesh and Titman (1993). Like long-term losers, stocks that have low short-term past returns tend to load positively on HML; like long-term winners, short-term past winners load negatively on HML. As it does for long-term returns, this pattern in the HML slopes predicts reversal rather than continuation for future returns. The continuation of short-term returns is thus left unexplained by the model. (Fama & French, 1996)

The available evidence suggests that the three-factor model is a parsimonious description of returns and average returns. The model captures much of the variation in the cross-section of average stock returns, and it absorbs most of the anomalies that have plagued the CAPM. Fama & French (1996) argue that the empirical successes of the three-factor model suggest that it is an equilibrium pricing model, a three-factor version of Merton's (1973) Intertemporal CAPM (ICAPM) or Ross's (1976) Arbitrage Pricing Theory (APT). In this view, SMB and HML mimic combinations of two underlying risk factors or state variables of special hedging concern to investors. (Fama & French, 1996)

2.3.1.1 CAPM versus Three-factor models

Fama & French (1996) show that the GRS test always rejects the CAPM at the 0.99 level (p-values less than 0.01). The CAPM fails because univariate market βs show little relation to variables like BE/ME, E/P, C/P, and sales rank, that are strongly related to average return. Fama & French (1996) also show that except for portfolios formed on short-term past return, where all models fail, the CAPM is dominated by the three-factor model. The average absolute pricing errors (intercepts) of the CAPM are large (25 to 30 basis points per month), and they are three to five times those of the three-factor model (5 to 10 basis points per month). (Fama & French, 1996)

2.3.1.2 Scepticism for the Fama & French (1993) model

Tests of a three-factor ICAPM or APT ask whether loadings on three portfolios can describe the average returns on other portfolios. Such tests in effect ask whether the explanatory portfolios span the three-factor portfolios that can be formed from the returns to be explained. (Fama & French, 1996). The explanatory portfolios are found to span the sets of three-factor multifactor-minimum-variance (MMV) portfolios that can be formed from sorts on size, BE/ME, E/P, C/P, sales rank, and long-term past returns. These explanatory portfolios cannot however span the three-factor-MMV portfolios that can be constructed from sorts on short-term past returns.

Fama & French (1996) recognize the contention in the interpretation of their results with three approaches being considered. The first line of argument states that asset pricing is rational and conforms to a three-factor ICAPM or APT that does not reduce to the CAPM. The second argument agrees that a three-factor model describes returns but argues that it is investor irrationality that prevents the three-factor model from collapsing to the CAPM with the irrational pricing causing the high premium for relative distress (the average HML return). The third line of argument says that the CAPM holds but is spuriously rejected because (i) there is survivor bias in the returns used to test the model, (ii) CAPM anomalies are the result of data snooping, or (iii) the tests use poor proxies for the market portfolio. (Fama & French, 1996).

2.3.2 The Arbitrage Theory of Capital Asset Pricing

The Arbitrage Pricing Theory (APT) presented in Ross (1976) was proposed as the testable alternative, and perhaps the natural successor to the CAPM. The APT is a particularly appropriate alternative to the CAPM because it agrees perfectly with what appears to be the intuition behind the CAPM. Indeed, the APT is based on a linear generating process as a first principle, and requires no utility assumptions beyond monotonicity and concavity. Nor is it restricted to a single period; it will hold in both the multi-period and single period cases. Though consistent with every conceivable prescription for portfolio diversification, no particular portfolio plays a role in the APT. Unlike the CAPM, there is no requirement that the market portfolio be mean-variance efficient. (Roll & Ross, 1980). Rather, it is the co-variability of an asset's returns with those random factors which systematically influence the returns on most assets, that is reflected in the expected return relation. This ability of the APT to accommodate several sources of "systematic risk" has been considered by many an advantage in comparison with the CAPM. (Shanken, 1982)

The theory begins with the traditional neo-classical assumptions of perfectly competitive and frictionless markets. Just as the CAPM is derived from the assumption that random asset returns follow a multivariate normal distribution, the APT also begins with an assumption on the return generating process. Individuals are assumed to believe (homogeneously) that the random returns on the set of assets being considered are governed by a k-factor generating model. (Roll & Ross, 1980)

There are two major differences between the APT and the original Sharpe (1964) "diagonal" model, a single factor generating model which Roll & Ross (1980) believe to be the intuitive grey eminence behind the CAPM. First, and most simply, the APT allows more than just one generating factor. Second, the APT demonstrates that since any market equilibrium must be consistent with no arbitrage profits, every equilibrium will be characterized by a linear relationship between each asset's expected return and its return's response amplitudes, or loadings, on the common factors. (Roll & Ross, 1980). With minor caveats, given the factor generating model, the absence of riskless arbitrage profits – an easy enough condition to accept a priori – leads immediately to the APT. (Roll & Ross, 1980)

Nevertheless, there seems to be enough evidence in past empirical work to conclude that there may exist multiple factors in the returns generating processes of assets. The APT provides a solid theoretical framework for ascertaining whether those factors, if they exist, are "priced" i.e., are associated with risk premia. (Roll & Ross, 1980)

2.4 Intertemporal CAPM

Following the development of the Sharpe-Lintner-Mossin mean-variance equilibrium model of exchange, Merton (1973) developed the Inter-temporal CAPM which allows the current demands to be affected by the possibility of uncertain changes in future investment opportunities unlike the original static CAPM. The model developed is consistent with both the expected utility maxim and the limited liability of assets. (Merton, 1973). It was shown that the equilibrium relationships among expected returns specified by the classical capital asset pricing model will obtain only under very special additional assumptions. (Merton, 1973).

Since the seminal work of Merton (1973) on the intertemporal capital asset pricing model (ICAPM), a large empirical literature has explored the relevance of intertemporal considerations for the pricing of financial assets in general, and the cross-sectional pricing of stocks in particular. (Campbell, Giglio, Polk, & Turley, 2012).

A major weakness of the most of the empirical research is that they fail to consider the time-variation in the volatility of stock returns. In general, investment opportunities may deteriorate either because expected stock returns decline or because the volatility of stock returns increases, and it is an empirical question which of these types of intertemporal risk have a greater effect on asset returns. (Campbell J. , Giglio, Polk, & Turley, 2012)

2.5 Intertemporal CAPM with Stochastic Volatility

The three-factor model presented in Campbell, Giglio, Polk, & Turley (2012) incorporates the volatility process directly in the ICAPM, allowing heteroskedasticity to affect and to be predicted by all state variables, and showing how the price of volatility risk is pinned down by the time-series structure of the model along with the investor's coefficient of risk aversion.

Campbell, Giglio, Polk, & Turley (2012) find that growth stocks have low average returns because they outperform not only when the expected stock return declines, but also when stock market volatility increases.

In their study, Campbell, Giglio, Polk, & Turley (2012) find that their three-beta model explains over 62% of the cross-sectional variation in average returns of 25 portfolios sorted on size and book-to-market ratios. The model is not rejected at the 5% level while the CAPM is strongly rejected.

Campbell, Giglio, Polk, & Turley (2012) present their model by first laying out the approximate closed-form ICAPM and then showing how to extend it to incorporate stochastic volatility. The following is a discussion of the model.

A representative agent with Epstein-Zin preferences is first assumed. The value function, V_t, is expressed as;

$$V_t = [(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[V_{t+1}^{1-\gamma}])^{1/\theta}]^{\frac{\theta}{1-\gamma}}$$

Where C_t is consumption and the preference parameters are the discount factor δ, risk aversion γ, and the elasticity of intertemporal substitution ψ. For convenience θ is defined as follows; $\theta = (1-\gamma)/(1-1/\psi)$.

The corresponding stochastic discount factor (SDF) is then written as

$$M_{t+1} = \left(\delta\left(\frac{C_t}{C_{t+1}}\right)^{1/\psi}\right)^{\theta}\left(\frac{W_t - C_t}{W_{t+1}}\right)^{1-\theta},$$

Where W_t is the market value of the consumption stream owned by the agent, including current consumption C_t. The log return on wealth is $r_{t+1} = \ln(W_{t+1}/(W_t - C_t))$,the log value of wealth tomorrow divided by reinvested wealth today. The log SDF is therefore

$$m_{t+1} = \theta ln\delta - \frac{\theta}{\psi}\Delta C_{t+1} + (\theta - 1)r_{t+1}.$$

Campbell, Giglio, Polk, & Turley (2012) obtain an ICAPM pricing relation that relates the risk premium on any asset to the asset's covariance with the wealth return and with shocks to future consumption claim values:

$$E_t r_{t+1} - r_{f,t} + \frac{1}{2} Var_t r_{t+1} = \gamma Cov_t[r_{i,t+1}, r_{t+1}] - \frac{\theta}{\psi} Cov_t[r_{i,t+1}, h_{t+1}].$$

The relationship between the intertemporal hedging component h_{t+1} and z_{t+1} is then approximated, where $z_{t+1} = \ln((W_t - C_t)/C_t)$.

By solving for the value of h_{t+1}, and substituting back into the intertemporal model, Campbell, Giglio, Polk, & Turley (2012) end up with an extension of the ICAPM as written by Campbell (1993), with no reference to consumption or the elasticity of intertemporal substitution as below;

$$\begin{aligned}
E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} Var_t r_{i,t+1} \\
&= \gamma Cov_t[r_{i,t+1}, r_{t+1}] + (\gamma - 1) Cov_t[r_{i,t+1} N_{DR,t+1}] \\
&- \frac{1}{2} Cov_t[r_{i,t+1}, N_{RISK,t+1}] \\
&= \gamma Cov_t[r_{i,t+1}, N_{CF,t+1}] + Cov_t[r_{i,t+1} N_{DR,t+1}] \\
&- \frac{1}{2} Cov_t[r_{i,t+1}, N_{RISK,t+1}]
\end{aligned}$$

Where N_{DR} is news about discount rates and is used for revisions in expected future returns. Revisions in expectations of future risk (the variance of future log returns plus the log stochastic discount factor) is written as N_{RISK}.

The first equality expresses the risk premium as risk aversion γ times covariance with the current market return plus $(\gamma - 1)$ times covariance with news about future market returns, minus one half covariance with risk.

The second equality rewrites the model, following Campbell and Vuolteenaho (2004), by breaking the market return into cash-flow news and discount rate news. Cash flow news N_{CF} is defined by $N_{CF} = r_{t+1} - E_t r_{t+1} + N_{DR}$. The price of risk for cash-flow news is γ times greater than the price of risk for discount-rate news.

By supposing that the economy is described by a first-order VAR

$$x_{t+1} = \bar{x} + \Gamma(x_t - \bar{x}) + \sigma_t u_{t+1},$$

Campbell, Giglio, Polk, & Turley (2012) then proceed to expand the variables N_{DR}, and N_{RISK} and substitute them back into the Intertemporal CAPM developed in Campbell (1993). This allows them to obtain an empirically-testable intertemporal CAPM with stochastic volatility:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} Var_t r_{i,t+1}$$
$$= \gamma Cov_t[r_{i,t+1}, N_{CF,t+1}] + Cov_t[r_{i,t+1} N_{DR,t+1}]$$
$$- \frac{1}{2} \omega Cov_t[r_{i,t+1}, N_{V,t+1}]$$

where the conditional variance $Var_t[(m_{t+1} + r_{t+1})/\sigma_t] = \omega_t$ is a constant that does not depend on the state variables.

3 Methodology

3.1 Overview

This empirical analysis will estimate the returns process using a vector auto regression (VAR) system to describe the conditional means and GARCH and EGARCH models to describe the conditional variances.

3.2 Population

This research restricts its population of study to the equity securities listed on the Nairobi Securities Exchange and the Johannesburg Stock Exchange.

3.3 Sample and Sampling Method

3.3.1 Using stocks or portfolios in Tests of factor Models

The finance literature takes two approaches to specifying base assets in tests of cross-sectional factor models. One approach is to aggregate stocks into portfolios. Another approach is to use individual stocks. The motivation for creating portfolios is originally stated in Blume (1970); betas are estimated with error and this estimation error is diversified away by aggregating stocks into portfolios. Numerous authors, including Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and Fama and French (1993), use this motivation to choose portfolios as base assets in factor model tests. The literature suggests that more precise estimates of factor loadings should translate into more precise estimates, and lower standard errors, of factor risk premia. (Ang, Liu, & Schwarz, 2010)

Ang, Liu, & Schwarz (2010) show analytically and confirm that this motivation is wrong. The sampling uncertainty of factor loadings is markedly reduced by grouping stocks into portfolios, but this does not translate into lower standard errors for factor risk premia estimates. An important determinant of the standard error of risk premia is the cross-sectional distribution of risk factor loadings. Intuitively, the more dispersed the cross section of betas, the more information the cross section contains to estimate risk premia. Aggregating stocks into portfolios loses information by reducing the cross-sectional dispersion of betas. While creating portfolios reduces the sampling variability of the estimates of factor loadings, the standard errors of factor risk premia actually increase. It is the decreasing dispersion of the cross-section of beta when stocks are grouped into portfolios that leads to potentially large efficiency losses in using portfolios versus individual stocks. (Ang, Liu, & Schwarz, 2010).

The most important message of Ang, Liu, & Schwarz (2010) is that using individual stocks permits more efficient tests of whether factors are priced. When just two-pass cross-sectional regression coefficients are estimated there should be no reason to create portfolios and the asset pricing tests should be run on individual stocks instead. Thus the use of portfolios in cross-sectional regressions should be carefully motivated. (Ang, Liu, & Schwarz, 2010).

This research follows the methodology presented in Campbell, Giglio, Polk, & Turley (2012) but where possible, individual stock returns will be used rather than creating portfolios. This is primarily motivated by the limited number of equity securities at the Nairobi Securities Exchange and the Johannesburg Securities Exchange compared to the same population for the US market. The size of the sample will only be limited by the availability of data. The support provided by the findings of Ang, Liu, & Schwarz (2010) give sufficient comfort that the use of individual stocks will not handicap the model and may even support a more efficient test.

3.4 Data Collection Methods and Procedures

In this research, we follow closely the methodology presented in Campbell, Giglio, Polk, & Turley (2012) as the primary basis for implementing the ICAPM with stochastic volatility. The full VAR specification of the vector x_{t+1} includes three state variables with all the data being monthly. The first variable in the VAR is the log real return on the market, r_M, the difference between the log return on the Nairobi Securities Exchange-20 (NSE-20) index and the log return on the Consumer Price Index. For the Johannesburg Securities Exchange, the log real return on the market is computed as the difference between the log return on the JSE All share index and the log return on the Consumer Price Index.

The second variable is expected market variance (EVAR). This variable is meant to capture the volatility of market returns, σ_t , conditional on information available at time t, so that innovations to this variable can be mapped to the N_V, term, the news about volatility. In constructing EVAR, a series of within-month realized variance of daily returns for each time t, $RVAR_t$ is constructed.

The third variable is the price-earnings ratio (PE), obtained as the price of the Index divided by a one-year trailing moving average of aggregate earnings of companies in the Index. The ratio is log transformed. This variable must predict low stock returns over the long run if smoothed earnings growth is close to unpredictable.

This research departs from the use of the additional three variables of term yield (TY), the small stock value spread (VS), and the default spread (DEF) as presented in (Campbell J. , Giglio, Polk, & Turley, 2012). This is largely because of paucity of data to facilitate a deeper analysis.

3.5 Data Analysis

3.5.1 Vector Auto regression (VAR)

Following Campbell (1993), Campbell, Giglio, Polk, & Turley (2012) estimate a first-order VAR, where x_{t+1} is a 6×1 vector of state variables ordered as follows;

$$x_{t+1} = [r_{M,t+1}\, EVAR_{t+1}\, PE_{t+1}\, TY_{t+1}\, DEF_{t+1}\, VS_{t+1}]$$

So that the real market return $r_{M,t+1}$ is the first element and $EVAR$ is the second element. $\bar{x}$ is a 6×1 vector of the means of the variables, and Γ is a 6×6 matrix of constant parameters. Finally, $\sigma_t\, u_{t+1}$ is a 6×1 vector of innovation, with the conditional variance-covariance matrix of u_{t+1} a constant Σ, so that the parameter σ_t^2 scales the entire variance covariance matrix of the vector of innovations.

The first stage regression forecasting realized market return variance generates the variable $EVAR$.

Campbell, Giglio, Polk, & Turley (2012) define the following asset pricing equation;

$$E[R_i - R_f] = \gamma\sigma_M^2\beta_{i,CF_M} + \sigma_M^2\beta_{i,DR_M} - \frac{1}{2}\omega\sigma_M^2\beta_{i,V_M}$$

Where;

$$\beta_{i,CF_M} = \frac{Cov(r_{i,t}, N_{CF_t})}{Var(r_{M,t} - E_{t-1}, r_{M,t})}$$

$$\beta_{i,DR_M} = \frac{Cov(r_{i,t}, N_{DR_t})}{Var(r_{M,t} - E_{t-1}, r_{M,t})}$$

$$\beta_{i,V_M} = \frac{Cov(r_{i,t}, N_{V_t})}{Var(r_{M,t} - E_{t-1}, r_{M,t})}.$$

3.5.2 GARCH and EGARCH

Campbell, Giglio, Polk, & Turley (2012) estimate two standard GARCH-type models, specifically designed to capture the long-run component of volatility. The first one is the two-component EGARCH model proposed by Adrian and Rosenberg (2008). This model assumes the existence of two separate components of volatility, one which is more persistent than the other, and therefore will tend to capture the long-run dynamics of the volatility process. The other model estimated is the FIGARCH model of Baillie, Bollerslev,and Mikkelsen (1996), in which the process for volatility is modelled as a fractionally-integrated process, and whose slow, hyperbolic rate of decay of lagged, squared innovations potentially captures long-run movements in volatility better. Both GARCH models are estimated using the full sample of daily returns before generating the appropriate forecast of $LH\ RV$. To these two models, the set of variables from the Vector auto regression (VAR) are added and their forecasting ability is then compared.

For the tests of the model in Equity Asset Pricing at the Nairobi Securities Exchange and the Johannesburg Securities Exchange, our approach differs from the above in two ways. First, this research will use the GARCH model to capture the long term dynamics of the volatility process. The EGARCH model is also specified for a comparison of the difference in volatility behaviour. Second, the FIGARCH model of Baillie, Bollerslev, & Mikkelsen (1996) is not used in conditional volatility estimation.

3.5.3 Analysis of results

To evaluate whether the ICAPM with stochastic volatility is a good model, we will primarily consider the R^2 of the cross-sectional variation in average returns. This is the same approach considered in Fama & French (1996) and Campbell J. , Giglio, Polk, & Turley (2012). We will conclude that the model is good in the case where the results for equity asset pricing for the Nairobi Securities Exchange and the Johannesburg Securities Exchange are at least as good as the findings presented for a developed market given in Campbell J. , Giglio, Polk, & Turley (2012).

4 Results

The asset pricing relation is considered to follow a joint VAR (1) and GARCH (1, 1) process. A VAR (1) model is fitted to describe the dynamics of the conditional expected returns of the variables in the system while a GARCH (1, 1) specification describes the dynamics of the conditional covariances. The VAR-GARCH models provide estimates of the forecasting ability of the state variables. The full VAR specification of the vector x_{t+1} includes three state variables all of which are the same as those presented in Campbell J. , Giglio, Polk, & Turley (2012). The variables are discussed in detail in Section 3. The model is implemented at the Nairobi Securities Exchange (NSE) and the Johannesburg Stock Exchange (JSE). The analysis uses monthly data for both the NSE and the JSE. The data covers the sample period February 2010 to June 2014 for NSE data and October 1995 to June 2014 for the JSE data.

4.1 VAR Estimation

4.1.1 Lag Length Selection

In carrying out the Vector Auto Regression, it is important to determine the correct order of lag to use for the analysis. In determining the correct lag specification, the Information Criteria approach is used. For the lag length selection, the NSE data sample spans 44 observations and has RM, PE and RVAR as endogenous variables. The results of the analysis for NSE data are as shown in the table below;

Lag	LogL	LR	FPE	AIC	SC	HQ
0	355.1474	NA	2.24e-11	-16.00670	-15.88505	-15.96159
1	406.7667	93.85323	3.24e-12*	-17.94394	-17.45734*	-17.76349*
2	415.5523	14.77590	3.29e-12	-17.93420	-17.08265	-17.61840
3	420.6953	7.948180	3.99e-12	-17.75888	-16.54238	-17.30774
4	428.9749	11.66666	4.24e-12	-17.72613	-16.14469	-17.13966
5	443.4943	18.47928*	3.47e-12	-17.97701	-16.03062	-17.25520
6	449.4019	6.713205	4.32e-12	-17.83645	-15.52511	-16.97930
7	462.9119	13.51003	3.94e-12	-18.04145	-15.36517	-17.04896
8	467.9659	4.364808	5.56e-12	-17.86209	-14.82086	-16.73425
9	483.7693	11.49333	5.16e-12	-18.17133	-14.76515	-16.90815
10	494.8435	6.543858	6.57e-12	-18.26561*	-14.49448	-16.86710

Table 4.1 Lag length selection results for VAR estimation for NSE data covering the period from February 2010 to June 2014.

For the NSE Data, lag 1 is selected with this choice being supported by the Final Prediction Error (FPE), the Schwarz Information Criterion (SC), and the Hannan-Quinn Information Criterion (HQ). This decision discards the indicated lag order by the sequential modified LR test statistic (LR) and the AIC Criterion. The LR Test statistic indicates that Lag 5 would be the most appropriate while the AIC criterion indicates that lag 10 would be most appropriate.

The JSE data sample spans 216 observations and has RM, EVAR01 and PERATIO as endogenous variables. The tests of lag length used covered 10 lags of the variables.

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1302.397	NA	1.20e-09	-12.03145	-11.98458	-12.01252
1	1548.175	482.4529*	1.33e-10*	-14.22384*	-14.03633*	-14.14809*
2	1554.419	12.08408	1.37e-10	-14.19833	-13.87018	-14.06575
3	1563.234	16.81256	1.37e-10	-14.19661	-13.72782	-14.00722
4	1567.889	8.750065	1.43e-10	-14.15638	-13.54695	-13.91017
5	1569.852	3.635818	1.52e-10	-14.09123	-13.34116	-13.78820
6	1570.947	1.995811	1.64e-10	-14.01802	-13.12732	-13.65818
7	1579.240	14.89752	1.65e-10	-14.01148	-12.98015	-13.59482
8	1582.475	5.721947	1.75e-10	-13.95811	-12.78613	-13.48463
9	1585.489	5.246372	1.85e-10	-13.90268	-12.59007	-13.37238
10	1594.458	15.36363	1.85e-10	-13.90239	-12.44915	-13.31528

Table 4.2 Lag length selection results for VAR estimation for JSE data covering the period from October 1995 to June 2014

The JSE data panel is more conclusive as all the criteria indicate the use of a vector auto regression with lag 1 as being most appropriate. This also weighs in the decision to use lag 1 for the NSE data.

4.1.2 Tests of Stationarity

For both data from the NSE and the JSE, the unit root tests indicate that the Vector Autoregressions satisfy the stability condition.

The roots of the characteristics polynomials lie between 0.093834 and 0.184384 for the NSE sample.

Endogenous variables: RM PE RVAR Lag specification: 1 1	
Root	Modulus
0.902558	0.902558
0.184384	0.184384
0.093834	0.093834
No root lies outside the unit circle. VAR satisfies the stability condition.	

Table 4.3 Unit root test for NSE data for the period February 2010 to June 2014

The roots of the characteristics polynomials lie between -0.056292 and 0.878477 for the JSE sample as seen from the table 4.4 below. The roots are not unity and this indicates that the VAR specification is stationary.

Endogenous variables: RM EVAR01 PERATIO Lag specification: 1 1	
Root	Modulus
0.878477	0.878477
0.490982	0.490982
-0.056292	0.056292
No root lies outside the unit circle. VAR satisfies the stability condition.	

Table 4.4 Unit root test for JSE data for the period October 1995 to June 2014

4.1.3 Causality Tests

For the NSE sample, none of the variables Granger-causes the other at the 5% significance level but at the 10% significance level, it is found that both RM and RVAR Granger-cause the variable PE.

For the JSE sample, it is also found that RM Granger-causes EVAR01 and PERATIO Granger-causes RM at the 5% signficance level. This result does not change at the 10% signficance level.

4.1.4 Estimated VAR results

	RM	RVAR	PE
RM(-1)	**0.177119**	**-2.77E-05**	**5.090843**
	(0.14365)	(0.00012)	(2.77752)
	[1.23303]	[-0.23369]	[1.83288]
RVAR(-1)	**-19.49841**	**0.100883**	**1418.404**
	(166.579)	(0.13770)	(3220.96)
	[-0.11705]	[0.73262]	[0.44037]
PE(-1)	**-7.66E-05**	**1.97E-07**	**0.902775**
	(0.00252)	(2.1E-06)	(0.04877)
	[-0.03038]	[0.09472]	[18.5124]
C	**0.007150**	**3.70E-05**	**1.139029**
	(0.03391)	(2.8E-05)	(0.65574)
	[0.21083]	[1.32124]	[1.73700]

Table 4.5 Estimation results of a VAR (1) model for the market Returns, PE ratio, and realized market variance for the NSE-20 share index from February 2010 to June 2014

The Vector Auto regression for the NSE data indicates that neither of the variables RM and RVAR is statistically different from zero at the 5% significance level.

However, the null hypothesis is rejected for the PE variable which has a p-value of 0.00252. It can be concluded that the variable PE is important in estimating the mean return of the market.

	RM	EVAR01	PERATIO
RM(-1)	**-0.016681**	**-0.000719**	**1.431616**
	(0.06787)	(0.00021)	(1.66631)
	[-0.24576]	[-3.38203]	[0.85916]
EVAR01(-1)	**-30.81300**	**0.445879**	**-307.7077**
	(19.2184)	(0.06018)	(471.807)
	[-1.60330]	[7.40936]	[-0.65219]
PERATIO(-1)	**-0.003992**	**-4.22E-06**	**0.883969**
	(0.00130)	(4.1E-06)	(0.03199)
	[-3.06328]	[-1.03526]	[27.6294]
C	**0.076625**	**0.000159**	**1.775978**
	(0.02085)	(6.5E-05)	(0.51186)
	[3.67512]	[2.43729]	[3.46968]

Table 4.6 Estimation results of a VAR (1) model for the market returns, PE ratio, and realized market variance for the JSE all-share index from October 1995 to June 2014

In considering the JSE data, similar results are found where neither the RM nor the EVAR variables is statistically different from zero at the 5% significance level. The null hypothesis is however rejected for the PE variable at the 5% confidence level. Of key note however is the fact that it is possible to reject the null hypothesis for the variable RM at the 10% significance level.

4.2 GARCH Estimation

4.2.1 Coefficients

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
PEKE	0.036185	0.003620	9.995685	0.0000
EVARKE	-184.1049	91.72431	-2.007154	0.0447
Variance Equation				
C	0.002188	0.000438	4.991721	0.0000
RESID(-1)^2	0.019568	0.065982	0.296564	0.7668
GARCH(-1)	-1.068350	0.149459	-7.148135	0.0000

Table 4.7 Estimation results for a GARCH model for the Return, P/E ratio and the realised market variance of the NSE-20 share index for the period from February 2010 to June 2014

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVAR	-56.76111	13.93423	-4.073500	0.0000
PE	0.017144	0.001626	10.54104	0.0000
Variance Equation				
C	9.92E-05	8.40E-05	1.181395	0.2374
RESID(-1)^2	0.096430	0.049852	1.934334	0.0531
GARCH(-1)	0.856558	0.078831	10.86581	0.0000

Table 4.8 Estimation results for a GARCH model for the Return, P/E ratio and the realised market variance of the JSE All-share index for the period from October 1995 to June 2014

4.2.2 Conditional Variance

The conditional variance graphs for the NSE data is shown in Figure 1 . There is a period of very high volatility in the last quarter of 2011 and the first quarter of 2012 which coincides largely with the macroeconomic shocks experienced in the same period. The country experienced a sharp depreciation in the local currency unit against major global currencies and high inflation rates resulted in a significant increase in volatility. The second spike in volatility is seen to arise towards the end of 2013 which was a period when uncertainty in the global markets increased significantly especially with the Federal Reserve of the United States announcing plans to gradually taper Quantitative Easing (QE).

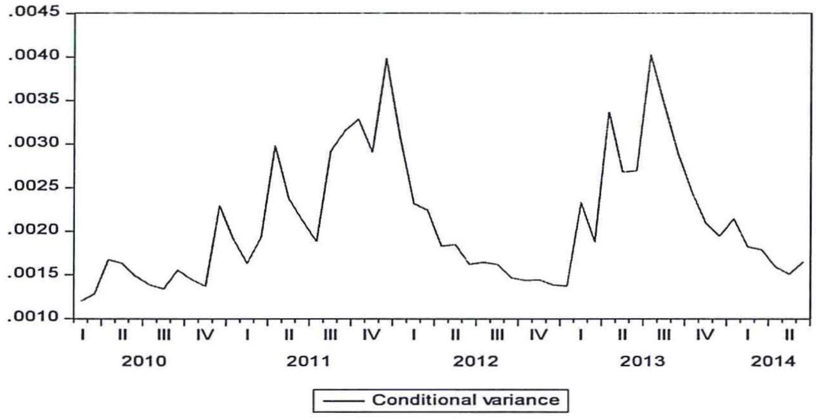

Figure 1 Graph of conditional variance for GARCH estimation for NSE data for the period February 2010 to June 2014

The conditional variance graph for the NSE data is shown in Figure 2. There is a significant increase in variance in 1998 coinciding with the United States stock market dot-com bubble. There is also a significant spike in the period 2008 and 2009 during the global financial crisis.

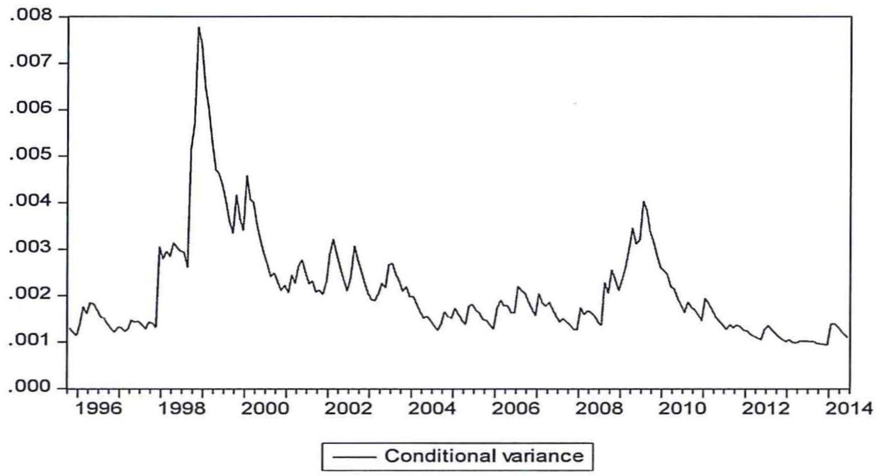

Figure 2 Graph of conditional variance for GARCH estimation for JSE data for the period October 1995 to June 2014

4.3 EGARCH Estimation

In the context of the EGARCH model, similar results as in the GARCH (1,1) are obtained. The adjusted R-squared in the case of the NSE data is found to be 39.23%. This is indicated in Panel A of Table 5.. The PE ratio remains statistically significant at the 5% level. The EGARCH model for JSE data indicates that both the EVAR and PE ratio are statistically significant at the 5% level. The adjusted R-squared is estimated at 30.7% which is lower than the values obtained using the GARCH (1,1) model.

4.3.1 Coefficients

LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5) *RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVARKE	-6.243222	92.68614	-0.067359	0.9463
PEKE	0.036257	0.003860	9.393851	0.0000
Variance Equation				
C(3)	-1.940748	3.87E-11	-5.02E+10	0.0000
C(4)	-1.093201	0.049899	-21.90843	0.0000
C(5)	-0.235768	0.126405	-1.865181	0.0622
C(6)	0.573830	0.007066	81.20521	0.0000

Table 4.9 Estimation results for an EGARCH model for the Return, P/E ratio and the realised market variance of the NSE-20 share index for the period from February 2010 to June 2014

LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5) *RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVAR	-56.03491	14.95309	-3.747379	0.0002
PE	0.016724	0.001570	10.65453	0.0000
Variance Equation				
C(3)	-0.533605	0.377432	-1.413776	0.1574
C(4)	0.201231	0.100845	1.995454	0.0460
C(5)	-0.099993	0.056794	-1.760609	0.0783
C(6)	0.940540	0.051164	18.38293	0.0000

Table 4.10 Estimation results for an EGARCH model for the Return, P/E ratio and the realised market variance of the JSE All-share index for the period from October 1995 to June 2014

4.3.2 Conditional Variance

The conditional variance graph for NSE data under EGARCH is as shown in Figure 3. The trend of the graph is largely similar to that of the NSE graph under GARCH shown in Figure 1.

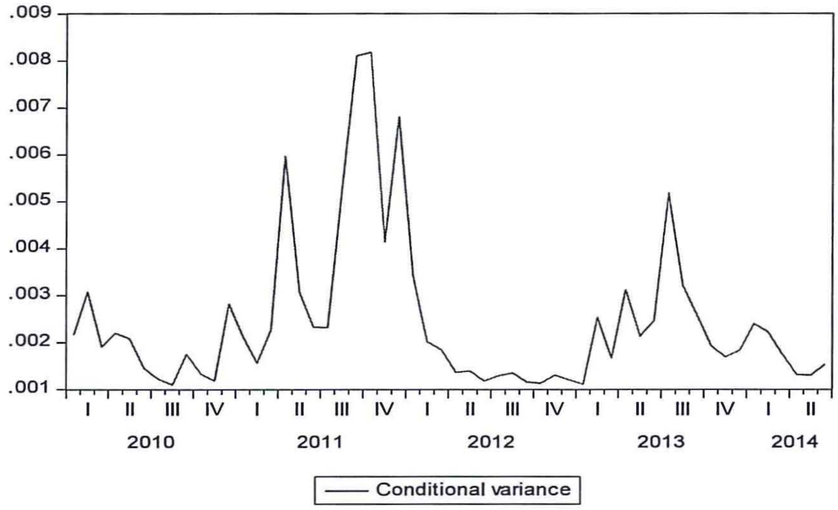

Figure 3 Graph of conditional variance for EGARCH estimation for NSE data for the period February 2010 to June 2014

The conditional variance graph for EGARCH estimation for JSE data in Figure 4 shows a similar pattern to the estimation under GARCH shown in Figure 2. The effect of the 2008 financial crisis is however less significant.

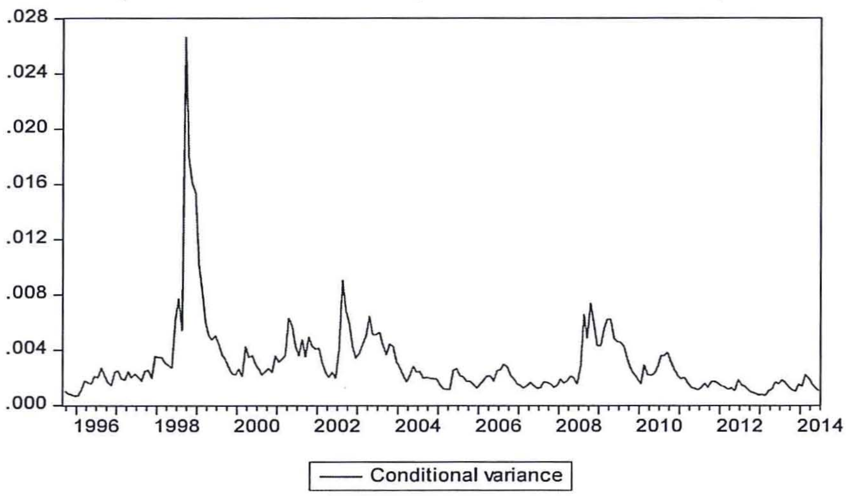

Figure 4 Graph of conditional variance for EGARCH estimation for JSE data for the period October 1995 to June 2014

4.3.3 Forecast Power

GARCH and EGARCH models are used to model the conditional variance of our asset pricing model. For both the NSE and JSE data, it is found that the two variables of PE and EVAR are statistically significant at the 5% significance level. In the case of the NSE data, the adjusted R-squared is found to be 41.21% which is low when compared against the R-squared of 62% obtained by Campbell J. , Giglio, Polk, & Turley (2012). The performance of the Model in the context of the JSE Data is less robust with the adjusted R-squared being 31.19%. In as much as the variables do not explain a significant part of the variation in asset returns, it is clear from the p-values that the variables of EVAR and PE are important in incorporating changes in conditional volatility within the pricing model.

The sum of the ARCH and GARCH coefficients ($\alpha + \beta$) for NSE data is negative indicating that the volatility shocks are not persistent. However, the volatility shocks are seen to be quite persistent in the JSE data with the sum of the ARCH and GARCH coefficents being 0.9530.

5 Conclusion

This research set out to test the adequacy of an intertemporal asset pricing model that incorporates stochastic volatility in estimating asset returns.

Given the results of the empirical analysis, we can conclude that the model as specified in this analysis is not a robust model for equity asset pricing. The model is only able to explain between 30% and 40% of the conditional variation of market returns. However, the coefficients of the variables in the GARCH estimation are statistically significant. This is likely to indicate the presence of a significant relationship which can be the subject of additional empirical evaluation. The findings of this research are quite different from those obtained by Campbell J. , Giglio, Polk, & Turley (2012) in their analysis. This difference stems from two key factors. First, this analysis faces a restriction in the availability of data which rules out the use of three additional variables considered in the former study. This restriction also necessitated the use of monthly data rather than quarterly data. Secondly, the time covered by this analysis is relatively smaller and the analysis may be extended to cover longer time horizons.

Appendix

Table 1

Vector Auto regression Estimates.

Panel A. NSE Data

Estimation results of the VAR (1) model for the market Returns, PE ratio, and realized market variance for the NSE-20 share index from February 2010 to June 2014. Sample (adjusted) covers the period 2010M02 to 2014M06. (n = 53). Standard errors in ( ) & t-statistics in [ ]

	RM	RVAR	PE
RM(-1)	**0.177119**	**-2.77E-05**	**5.090843**
	(0.14365)	(0.00012)	(2.77752)
	[1.23303]	[-0.23369]	[1.83288]
RVAR(-1)	**-19.49841**	**0.100883**	**1418.404**
	(166.579)	(0.13770)	(3220.96)
	[-0.11705]	[0.73262]	[0.44037]
PE(-1)	**-7.66E-05**	**1.97E-07**	**0.902775**
	(0.00252)	(2.1E-06)	(0.04877)
	[-0.03038]	[0.09472]	[18.5124]
C	**0.007150**	**3.70E-05**	**1.139029**
	(0.03391)	(2.8E-05)	(0.65574)
	[0.21083]	[1.32124]	[1.73700]
R-squared	0.033237	0.012717	0.892018
Adj. R-squared	-0.025953	-0.047729	0.885407
Sum sq. resids	0.119097	8.14E-08	44.52777
S.E. equation	0.049301	4.08E-05	0.953273
F-statistic	0.561527	0.210380	134.9265
Log likelihood	86.39604	462.5973	-70.58800
Akaike AIC	-3.109284	-17.30556	2.814642
Schwarz SC	-2.960583	-17.15686	2.963343
Mean dependent	0.006721	4.41E-05	13.18651
S.D. dependent	0.048673	3.98E-05	2.816036
Determinant resid covariance (dof adj.)		2.10E-12	
Determinant resid covariance		1.66E-12	
Log likelihood		493.1739	
Akaike information criterion		-18.15750	
Schwarz criterion		-17.71140	

Table 2

Estimation results of a VAR (1) model for the market Returns, PE ratio, and realized market variance for the JSE All share index from October 1995 to June 2014.Sample (adjusted) covers the period 1995M10 to 2014M06. (n = 225 after adjustments). Standard errors in ( ) & t-statistics in [ ]

	RM	EVAR01	PERATIO
RM(-1)	**-0.016681**	**-0.000719**	**1.431616**
	(0.06787)	(0.00021)	(1.66631)
	[-0.24576]	[-3.38203]	[0.85916]
EVAR01(-1)	**-30.81300**	**0.445879**	**-307.7077**
	(19.2184)	(0.06018)	(471.807)
	[-1.60330]	[7.40936]	[-0.65219]
PERATIO(-1)	**-0.003992**	**-4.22E-06**	**0.883969**
	(0.00130)	(4.1E-06)	(0.03199)
	[-3.06328]	[-1.03526]	[27.6294]
C	**0.076625**	**0.000159**	**1.775978**
	(0.02085)	(6.5E-05)	(0.51186)
	[3.67512]	[2.43729]	[3.46968]
R-squared	0.043525	0.300862	0.792476
Adj. R-squared	0.030541	0.291372	0.789659
Sum sq. resids	0.695454	6.82E-06	419.1414
S.E. equation	0.056097	0.000176	1.377159
F-statistic	3.352213	31.70120	281.3122
Log likelihood	330.9090	1628.331	-389.2483
Akaike AIC	-2.905858	-14.43850	3.495541
Schwarz SC	-2.845127	-14.37777	3.556271
Mean dependent	0.011957	0.000158	14.94386
S.D. dependent	0.056974	0.000209	3.002770
Determinant resid covariance (dof adj.)		1.20E-10	
Determinant resid covariance		1.13E-10	
Log likelihood		1618.518	
Akaike information criterion		-14.28016	
Schwarz criterion		-14.09797	

Table 3

Roots of Characteristic Polynomial; Unit root tests

Panel A; NSE Data

Endogenous variables: RM PE RVAR
Exogenous variables: C
Lag specification: 1 1

Root	Modulus
0.902558	0.902558
0.184384	0.184384
0.093834	0.093834

No root lies outside the unit circle.
VAR satisfies the stability condition.

Panel B; JSE Data

Endogenous variables: RM EVAR01 PERATIO
Exogenous variables: C
Lag specification: 1 1

Root	Modulus
0.878477	0.878477
0.490982	0.490982
-0.056292	0.056292

No root lies outside the unit circle.
VAR satisfies the stability condition.

Table 4; Lag length selection; VAR Lag order selection criteria

In the NSE Data sample (shown in panel A), the endogenous variables are RM, PE and RVAR with the exogenous variable being C. 44 observations are included. In the JSE Data sample (shown in panel B), the endogenous variables are RM, EVAR01, and PERATIO. 216 observations are included.

* indicates lag order selected by the criterion. LR: sequential modified LR test statistic (each test at 5% level). FPE: Final prediction error. AIC: Akaike information criterion. SC: Schwarz information criterion. HQ: Hannan-Quinn information criterion

Panel A; NSE Data

Lag	LogL	LR	FPE	AIC	SC	HQ
0	355.1474	NA	2.24e-11	-16.00670	-15.88505	-15.96159
1	406.7667	93.85323	3.24e-12*	-17.94394	-17.45734*	-17.76349*
2	415.5523	14.77590	3.29e-12	-17.93420	-17.08265	-17.61840
3	420.6953	7.948180	3.99e-12	-17.75888	-16.54238	-17.30774
4	428.9749	11.66666	4.24e-12	-17.72613	-16.14469	-17.13966
5	443.4943	18.47928*	3.47e-12	-17.97701	-16.03062	-17.25520
6	449.4019	6.713205	4.32e-12	-17.83645	-15.52511	-16.97930
7	462.9119	13.51003	3.94e-12	-18.04145	-15.36517	-17.04896
8	467.9659	4.364808	5.56e-12	-17.86209	-14.82086	-16.73425
9	483.7693	11.49333	5.16e-12	-18.17133	-14.76515	-16.90815
10	494.8435	6.543858	6.57e-12	-18.26561*	-14.49448	-16.86710

Panel B; JSE Data

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1302.397	NA	1.20e-09	-12.03145	-11.98458	-12.01252
1	1548.175	482.4529*	1.33e-10*	-14.22384*	-14.03633*	-14.14809*
2	1554.419	12.08408	1.37e-10	-14.19833	-13.87018	-14.06575
3	1563.234	16.81256	1.37e-10	-14.19661	-13.72782	-14.00722
4	1567.889	8.750065	1.43e-10	-14.15638	-13.54695	-13.91017
5	1569.852	3.635818	1.52e-10	-14.09123	-13.34116	-13.78820
6	1570.947	1.995811	1.64e-10	-14.01802	-13.12732	-13.65818
7	1579.240	14.89752	1.65e-10	-14.01148	-12.98015	-13.59482
8	1582.475	5.721947	1.75e-10	-13.95811	-12.78613	-13.48463
9	1585.489	5.246372	1.85e-10	-13.90268	-12.59007	-13.37238
10	1594.458	15.36363	1.85e-10	-13.90239	-12.44915	-13.31528

Table 5

Variance decomposition tests for the Estimation of a VAR (1) where the dependent variable is the return on the market (RM). The estimation covers the period February 2010 to June 2014 for NSE data.

Variance Decomposition of RM:				
Period	S.E.	RM	PE	RVAR
1	0.049301	100.0000	0.000000	0.000000
2	0.050084	99.97461	0.001098	0.024295
3	0.050108	99.97232	0.001454	0.026224
4	0.050108	99.97204	0.001634	0.026330
5	0.050108	99.97190	0.001762	0.026338
6	0.050108	99.97180	0.001862	0.026339
7	0.050108	99.97172	0.001944	0.026340
8	0.050108	99.97165	0.002010	0.026341
9	0.050108	99.97160	0.002064	0.026341
10	0.050109	99.97155	0.002108	0.026341
Variance Decomposition of PE:				
Period	S.E.	RM	PE	RVAR
1	0.953273	40.65345	59.34655	0.000000
2	1.410692	50.11701	49.72094	0.162050
3	1.712865	53.55677	46.23750	0.205738
4	1.926939	55.19038	44.58664	0.222982
5	2.085547	56.10615	43.66181	0.232043
6	2.206403	56.67778	43.08462	0.237600
7	2.300175	57.06099	42.69771	0.241309
8	2.373828	57.33073	42.42535	0.243917
9	2.432179	57.52728	42.22691	0.245817
10	2.478697	57.67412	42.07864	0.247237
Variance Decomposition of RVAR:				
Period	S.E.	RM	PE	RVAR
1	4.08E-05	1.585496	1.904758	96.50975
2	4.10E-05	1.752176	1.912360	96.33546
3	4.10E-05	1.756046	1.914608	96.32935
4	4.10E-05	1.756398	1.915795	96.32781
5	4.10E-05	1.757503	1.916674	96.32582
6	4.10E-05	1.758602	1.917379	96.32402
7	4.10E-05	1.759532	1.917951	96.32252
8	4.10E-05	1.760296	1.918416	96.32129
9	4.10E-05	1.760919	1.918796	96.32029
10	4.10E-05	1.761427	1.919105	96.31947
Cholesky Ordering: RM PE RVAR				

Table 6

Variance decomposition tests for the Estimation of a VAR (1) where the dependent variable is the return on the market (RM). The estimation covers the period October 1995 to June 2014 for JSE data.

Variance Decomposition of RM:				
Period	S.E.	RM	EVAR01	PERATIO
1	0.056097	100.0000	0.000000	0.000000
2	0.056541	98.60617	0.724405	0.669427
3	0.056705	98.05988	0.810903	1.129217
4	0.056838	97.69543	0.816072	1.488500
5	0.056951	97.42210	0.812860	1.765037
6	0.057044	97.21034	0.812157	1.977507
7	0.057119	97.04576	0.813533	2.140703
8	0.057179	96.91819	0.815693	2.266116
9	0.057226	96.81954	0.817897	2.362564
10	0.057262	96.74337	0.819842	2.436787
Variance Decomposition of EVAR01:				
Period	S.E.	RM	EVAR01	PERATIO
1	0.000176	8.244679	91.75532	0.000000
2	0.000202	16.84217	83.09926	0.058578
3	0.000208	18.11884	81.80238	0.078787
4	0.000210	18.44903	81.45962	0.091349
5	0.000210	18.54294	81.35725	0.099803
6	0.000210	18.57283	81.32138	0.105793
7	0.000210	18.58357	81.30626	0.110176
8	0.000210	18.58798	81.29857	0.113453
9	0.000210	18.59006	81.29400	0.115936
10	0.000210	18.59117	81.29099	0.117831
Variance Decomposition of PERATIO:				
Period	S.E.	RM	EVAR01	PERATIO
1	1.377159	28.74045	0.458356	70.80120
2	1.877240	31.36396	0.757671	67.87837
3	2.194825	32.50254	1.013384	66.48407
4	2.415439	33.17293	1.199539	65.62753
5	2.574418	33.60046	1.331428	65.06811
6	2.691373	33.88639	1.424665	64.68895
7	2.778554	34.08393	1.491080	64.42499
8	2.844129	34.22377	1.538907	64.23732
9	2.893771	34.32465	1.573745	64.10161
10	2.931528	34.39850	1.599392	64.00211
Cholesky Ordering: RM EVAR01 PERATIO				

Table 7

Estimation results for a GARCH model for the Return, P/E ratio and the realised market variance of the NSE-20 share index for the period from February 2010 to June 2014

Dependent Variable: RMKE
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 53 after adjustments
Convergence achieved after 15 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PEKE	0.036185	0.003620	9.995685	0.0000
EVARKE	-184.1049	91.72431	-2.007154	0.0447
		Variance Equation		
C	0.002188	0.000438	4.991721	0.0000
RESID(-1)^2	0.019568	0.065982	0.296564	0.7668
GARCH(-1)	-1.068350	0.149459	-7.148135	0.0000

R-squared	0.423393	Mean dependent var	4.08E-18
Adjusted R-squared	0.412087	S.D. dependent var	0.047857
S.E. of regression	0.036695	Akaike info criterion	-3.749519
Sum squared resid	0.068672	Schwarz criterion	-3.563642
Log likelihood	104.3622	Hannan-Quinn criter.	-3.678039
Durbin-Watson stat	1.935127		

Table 8

Estimation results for a GARCH model for the Return, P/E ratio and the realised market variance of the JSE All-share index for the period from October 1995 to June 2014

Dependent Variable: RM
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 225 after adjustments
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVAR	-56.76111	13.93423	-4.073500	0.0000
PE	0.017144	0.001626	10.54104	0.0000
Variance Equation				
C	9.92E-05	8.40E-05	1.181395	0.2374
RESID(-1)^2	0.096430	0.049852	1.934334	0.0531
GARCH(-1)	0.856558	0.078831	10.86581	0.0000

R-squared	0.311875	Mean dependent var	7.40E-19
Adjusted R-squared	0.308789	S.D. dependent var	0.055720
S.E. of regression	0.046325	Akaike info criterion	-3.369502
Sum squared resid	0.478559	Schwarz criterion	-3.293588
Log likelihood	384.0689	Hannan-Quinn criter.	-3.338863
Durbin-Watson stat	2.006168		

Table 9

Estimation results for an EGARCH model for the Return, P/E ratio and the realised market variance of the NSE-20 share index for the period from February 2010 to June 2014

Dependent Variable: RMKE
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 53 after adjustments
Convergence achieved after 49 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVARKE	-6.243222	92.68614	-0.067359	0.9463
PEKE	0.036257	0.003860	9.393851	0.0000
Variance Equation				
C(3)	-1.940748	3.87E-11	-5.02E+10	0.0000
C(4)	-1.093201	0.049899	-21.90843	0.0000
C(5)	-0.235768	0.126405	-1.865181	0.0622
C(6)	0.573830	0.007066	81.20521	0.0000

R-squared	0.403969	Mean dependent var	4.08E-18
Adjusted R-squared	0.392283	S.D. dependent var	0.047857
S.E. of regression	0.037308	Akaike info criterion	-3.752474
Sum squared resid	0.070986	Schwarz criterion	-3.529422
Log likelihood	105.4406	Hannan-Quinn criter.	-3.666699
Durbin-Watson stat	1.915063		

Table 10

Estimation results for an EGARCH model for the Return, P/E ratio and the realised market variance of the JSE All-share index for the period from October 1995 to June 2014

Dependent Variable: RM
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 225 after adjustments
Convergence achieved after 25 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EVAR	-56.03491	14.95309	-3.747379	0.0002
PE	0.016724	0.001570	10.65453	0.0000
		Variance Equation		
C(3)	-0.533605	0.377432	-1.413776	0.1574
C(4)	0.201231	0.100845	1.995454	0.0460
C(5)	-0.099993	0.056794	-1.760609	0.0783
C(6)	0.940540	0.051164	18.38293	0.0000

R-squared	0.310099	Mean dependent var	7.40E-19
Adjusted R-squared	0.307005	S.D. dependent var	0.055720
S.E. of regression	0.046385	Akaike info criterion	-3.375351
Sum squared resid	0.479795	Schwarz criterion	-3.284255
Log likelihood	385.7270	Hannan-Quinn criter.	-3.338584
Durbin-Watson stat	2.006467		

Table 11

Panel A

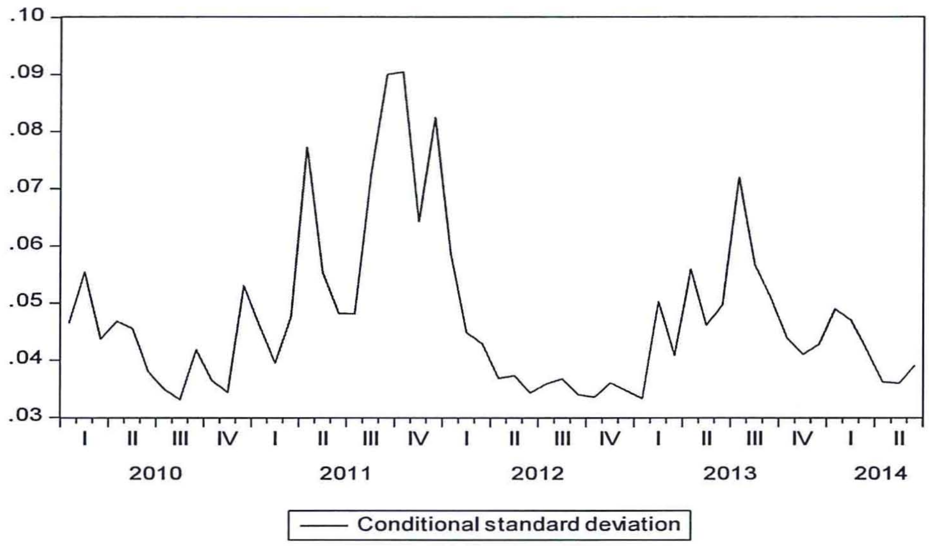

Figure 5 Graph of conditional standard deviation for GARCH estimation for NSE data for the period February 2010 to June 2014

Panel B

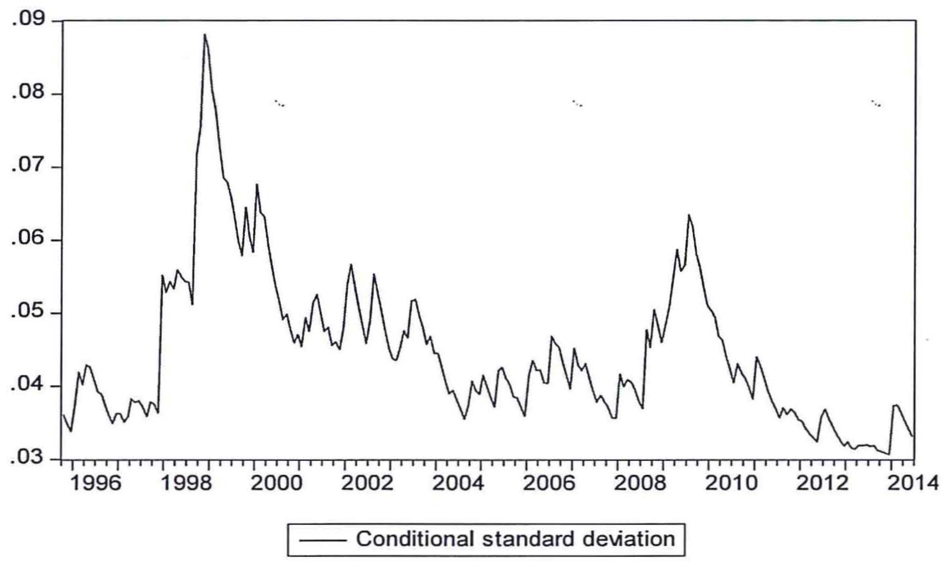

Figure 6 Graph of conditional standard deviation for GARCH estimation for JSE data for the period October 1995 to June 2014

Table 7; EGARCH GRAPHS

Panel A

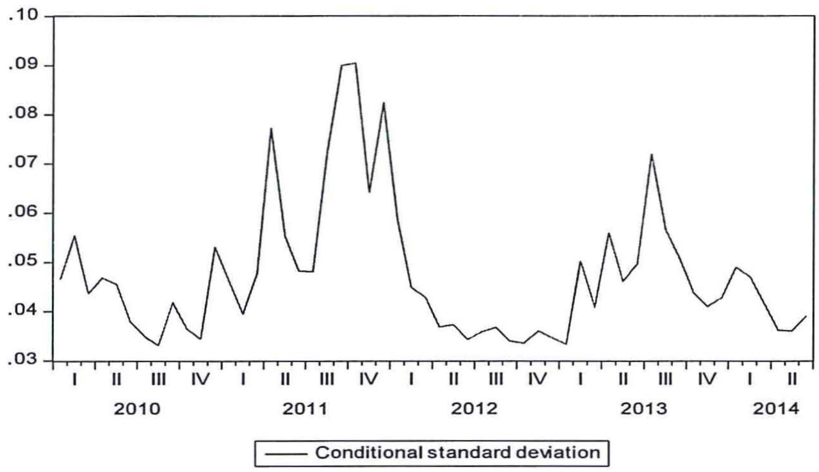

Figure 7 Graph of conditional standard deviation for EGARCH estimation for NSE data for the period February 2010 to June 2014

Panel B

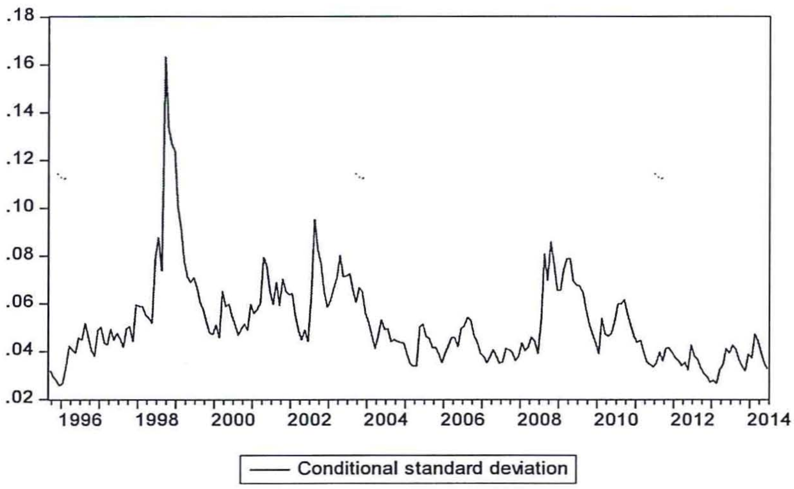

Figure 8 Graph of conditional standard deviation for EGARCH estimation for JSE data for the period October 1995 to June 2014

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