Valuation of Maize Storage Facility

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Submitted in partial fulfillment of the academic requirement for the Degree of Master of Science in Mathematical Finance at Strathmore University

Strathmore Institute of Mathematical Sciences
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Nairobi, Kenya

June 2024

Declaration and Approval

Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Abstract

Grain supply in Kenya depends strongly on weather, i.e. the seasons of the year. This implies that grain prices such as maize depend on the season of the year and overhead costs like storage and transportation. The analysis of historical spot prices of maize and weather shows a dependency between day-ahead prices and seasons of the year, especially around the harvesting period (October to December). Typically, the maize supply increases during this period. This study advocates for the adoption of a stochastic model that intertwines maize spot prices with the seasons, focusing on the assessment and risk mitigation of maize storage facilities. This is achieved through a spot-based valuation framework coupled with a financial hedging strategy, implemented via futures contracts. The key contributions of this study encompass proposing a comprehensive model that captures the dynamics of the futures curve and spot prices while accommodating essential characteristics of the commodities market, such as seasonality and sporadic spikes in the spot market. Additionally, the study addresses the evaluation of associated model risk.

Employing both "*intrinsic*" and "*extrinsic*" valuation techniques, this study notably utilizes the "*rolling intrinsic*" valuation method. This approach considers both spot and future prices of maize in the valuation process, revolving around a trading strategy where a trader secures spot and futures positions by solving an optimization problem based on market information on the first day. The trader retains the flexibility to adjust positions over time in response to new market information, with the storage value calculated as the cumulative sum of the initial day's value and subsequent added values.

To solve the problem, this study formulates the Bellman equation and employs a recursive solution through Monte Carlo simulation with ordinary least square regression.

Keywords: Spot price; futures; valuation, hedging; "*intrinsic*" and "*extrinsic*" valuation; optimization; Bellman equation; Monte Carlo; ordinary least square regression.

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Acknowledgment

I deeply appreciate Dr. Samuel C. Maina, the project supervisor, whose support and advice has led to the completion of this project. Secondly, I appreciate Prof. Livingstone S. Luboobi who has given me enough pressure to see me through the process. I wish also to appreciate Strathmore University for providing conducive environment and resources during the whole of this project work. Finally, I thank God Almighty for providing me with the strength and endurance to complete this journey.

Dedication

This thesis is dedicated to my family, friends and classmates for their support during the entire period I was working on this project.

Abbreviations

ARMA	Autoregressive–Moving-Average
BBO	Best Bid-Offer
CBK	Central Bank of Kenya
CBOT	Chicago Board of Trade
CI	Confidence Interval
DPP	Dynamic Programing Problem
EPV	Expected Present Value
EV	Extrinsic Value
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
HJB	Hamilton-Jacobi-Bellman
IV	Intrinsic Value
KALRO	Kenya Agricultural & Livestock Research Organization
KEBS	Kenya Bureau of Standards
KFA	Kenya Farmers' Association
KNBS	Kenya National Bureau of Statistics
LSMC	Least Square Monte Carlo
MC	Monte Carlo
MLE	Maximum Likelihood Estimation
MPC	Monetary Policy Committee
NCPB	National Cereals & Produce Board
SDE	Stochastic Differential Equation
USDA	United States Department of Agriculture
WRS	Warehouse Receipt System

Chapter 1

Introduction

This thesis focuses on the valuation and hedging of storage facilities for commodities such as maize. The approach involves a spot-based valuation framework combined with a financial hedging strategy utilizing futures contracts. The contribution of this study to the existing literature is twofold. Firstly, it suggests the adoption of a model that integrates the dynamics of the futures curve and spot prices, simultaneously considering key characteristics of the commodities market such as seasonality and the occurrence of jumps or spikes in the spot market, as discussed by Hénaff et al. (2018). Secondly, this study delves into the evaluation of the associated model risk.

1.1 Motivation

Maize is an important agricultural commodity in Kenya. It is a staple food for the populace. It is also used as animal feed and key industrial raw material. According to Simiyu (2014), demand for maize will increase by roughly 6% over the next two decades resulting from an increased demand for food. He further argues that agricultural production, including that of maize, within this period will either stagnate or increase at a slower rate than the demand.

Price of maize is dependent on several factors: (a) seasons of the year, (b) ease of transportation and (c) availability of storage facilities.

Several studies in the field of crop science have looked at the ideal storage for agricultural produce including maize to minimize post-harvest losses (Farnworth et al., 2021, Kimani et al., 2018, Likhayo et al., 2016). Their studies have been backed up by the Kenya Agricultural & Livestock Research Organization (KALRO).

Maize supply can be well described by a deterministic dependency on maize production (volumes) and weather (seasons) whereas the demand for maize is fairly constant over the year. Consumers generally pay a fixed price for maize for their individual consumption. On the other hand, the producers and suppliers (millers) take the volume risks of deviations from the projected load profile of the consumers and the production constraints. In dry seasons, there is usually a higher demand for maize than its supply. This situation forces the suppliers (millers) to buy extra maize either offshore or from strategic food reserve silos to meet the demand of consumers. Similarly, the supply of maize is usually higher than demand during harvesting season. The suppliers (millers) have to incur storage costs for the excess supply of maize at this time. According to Cucu et al. (2016), a similar scenario occurs in the gas market and there exists energy quanto contracts that provide a risk management tool to mitigate the correlated gas price temperature risks inherited in consumer full supply contracts. For pricing purposes, they choose a model that treats gas and temperature as coupled risk factors. Wang and Secomandi (2009) dealt with the transport problem of natural gas. Their model can be applicable to any commodity including maize.

These forces of demand and supply significantly affect the prices of maize. When demand is higher than supply, there are spikes in the prices of maize and when the demand is relatively equal to the supply, the prices are also relatively fair. However, prices are sticky upwards meaning that it takes time before prices stabilize once there is a price jump.

This raises the need to protect consumers and farmers against exploitation by aggregators, middlemen, or suppliers (millers). To this effect, a Bill was recently introduced in the Kenyan Parliament to address the storage and trading of agricultural commodities and has so far been enacted into an Act of Parliament. The Bill was christened 'The Warehouse Receipt Bill, 2018'. This Bill aimed to establish a legal framework governing the development and regulation of a warehouse receipt system specifically designed for agricultural commodities. It also outlined the creation of the Warehouse Receipt System Council and addressed related matters (Kenya, 2018). As defined by Kenya (2018), the Warehouse Receipt System (WRS) encompasses the entire process, including depositing commodities in a licensed warehouse, issuing a warehouse receipt detailing the quantity and quality of the deposited commodity, managing the transfer of these receipts as title documents, and regulating warehouses and associated processes.

These warehouses can be either publicly or privately owned, and they were to be operated by licensed warehouse operators. These operators are responsible for ensuring that agricultural commodities stored in the warehouse adhere to standards set by the Kenya Bureau of Standards (KEBS). Additionally, the commodities must be weighed using equipment certified in accordance with the *Weights and Measures Act*. The grading of these commodities is to be carried out by agricultural commodities graders certified by a recognized authority (Kenya, 2018). The warehouse operator is obligated to issue a warehouse receipt for any agricultural commodity deposited in their facility, and these receipts may exist in either hard copy or electronic form. These receipts form the basis of the contract between the depositor and the warehouse operator and are delivered to the central registry under the leadership of a chief registrar. This mitigates the risk of forgery. This security feature on the warehouse receipts allows the depositor to negotiate with potential buyers and then transfer ownership of the stored commodities by way of endorsement. The warehouse charges the depositor a fee for storing the commodities, providing insurance during the storage period, and delivering the commodities to the depositors or buyers upon surrender of a legitimate warehouse receipt. The largest warehouse operator in Kenya is the National Cereals & Produce Board (NCPB). It is run by the government and it is essential for strategic food reserve for the populace. To complement the efforts by the government, most of the maize millers and farmers associations e.g. the Kenya Farmers' Association (KFA) also have their own silos/ warehouses to buffer their supplies in the event there is no adequate supply of maize.

More formally, the WRS Bill for agricultural commodities sought to be used as a market intervention policy. Upon approval (the Bill has so far been passed and accented by the President into an Act of Parliament), the proposed legislation was meant to establish a system in which licensed warehouse operators would issue receipts to depositors or suppliers upon the delivery of commodities. This sets the stage for the creation of a national commodity exchange, facilitating the trading of agricultural commodities. This development is crucial as it is expected to enhance profitability, increase liquidity, and contribute to price stability within the agricultural trade. The issued receipts serve as a document of title for the commodities stored in the facility and can be presented to a lender as collateral for a loan or transferred to another party through a sale. This sets the stage for the creation of a national commodity exchange, facilitating the trading of agricultural commodities. This development is crucial as it is expected to enhance profitability, increase liquidity, and contribute to price stability within the agricultural trade. The issued receipts serve as a document of title for the commodities stored in the facility and can be presented to a lender as collateral for a loan or transferred to another party through a sale. The Warehouse Receipt System (WRS) introduced by the Bill, now an Act of Parliament (Law), will enable agricultural producers to access credit by borrowing against the receipts issued for goods stored in regulated warehouses. This system empowers producers to postpone the sale of their produce immediately after harvest to a time when market prices are more favorable. It is important to note that the WRS comes with a storage fee, allowing for the sustainable maintenance of the system and its associated benefits.

This study proposes a dynamic valuation of the storage cost and a hedging strategy to be applied by the suppliers/ producers so as to smoothen the prices of the commodities they are producing and/or selling.

1.2 Storage Facilities

Leasing a storage unit can be likened to paying for the right, without the obligation, to deposit or withdraw storable commodities from that unit, as highlighted by Hénaff et al. (2018). For a lessee, the objective is twofold: (1) optimizing the exercise of the right by managing deposits or withdrawals of the trade commodity from the unit, and (2) engaging in trading activities involving the commodity on both the spot and futures markets.

These decisions are made within operational constraints, such as maximum and minimum volume in storage and limited deposit and withdrawal rates. Approaches for determining the optimal operating policy and the value of storage fall into two main categories: static (*intrinsic*) and dynamic (*extrinsic*) methods.

In the *intrinsic* approach, the lessee projects an optimal schedule of deposits and withdrawals, using the term structure of future prices. The corresponding discounted cash flow is then computed, providing a lower bound to the storage value. An example of such a contract is a swing contract, also known as a $take - and - pay$ $option$. According to Warin (2012), swing options are akin to American-style options with daily and annual quantity exercise constraints. The option holder must decide daily how much of the quantities, within the constraints, to consume, employing a strategy to maximize expected profit. Typically, this involves commodities like gas and oil. In this method, a predetermined trading strategy is established, and to determine optimal futures positions, a linear optimization problem is solved, considering constraints imposed by the storage unit.

Conversely, the *extrinsic* approach involves the lessee determining the expected present value (EPV) of future cash flows, akin to financial options. This approach considers the dynamics of the futures curve, relying on a precise model of the commodity's, in this case, maize, term structure. Notably, it involves modeling the correlation between future prices and the cash price of the commodity (maize).

1.3 Literature Review

Hénaff et al. (2018) comment that prior to 2005, storage units were valued using a discounted cash flow method, also known as the *intrinsic* method. This method involved projecting an optimal schedule of deposits and withdrawals, using the term structure of future prices, and calculating the corresponding present value. This approach provided a lower bound to the storage value. The storage manager would observe the futures curve at the start of the storage contract and buy (or sell) futures contracts, determining the complete deposit-withdrawal schedule upfront. However, as the seasonal (summer-winter) spread started to shrink over time, the viability of this futures-based methodology was questioned. The static strategies based on futures contracts were found to be inadequate for monetizing storage facilities, failing to recover operating costs. This led to the adoption of dynamic strategies that capitalized on the real options embedded in commodity storage facilities — the *extrinsic* method of valuation.

The *extrinsic* method, valuing storage units with higher deposit-withdrawal rates at a higher value than the *intrinsic* method, considers the time value of embedded calendar spread options. Hénaff et al. (2018)'s study aimed to investigate this paradigm shift and quantify the inherent model uncertainty in the *extrinsic* method. In contrast to the *intrinsic* method, where the operator determines a trading strategy upfront, the *extrinsic* method involves solving a constrained stochastic control problem, often described using the notations of Warin (2012). This is typically addressed using Least Square Monte Carlo (LSMC) simulations, combined with the Longstaff and Schwartz (2001) algorithm to approximate the Hamilton–Jacobi–Bellman (HJB) equation. Boogert and De Jong (2008) provided a detailed explanation of this algorithm and its implementation.

Subsequent research by Gray and Khandelwal (2004) extended the static (*intrinsic*) methodology to *rolling intrinsic* valuation, allowing for iterative optimization of positions as the futures curve evolves over each Monte Carlo (MC) path. This helped lock in additional storage value. However, the *rolling intrinsic* approach faced challenges with narrowing spreads over time, leading to the adoption of extrinsic methods.

Various models have been proposed to capture the dynamics of the term structure of commodities, including one-factor and two-factor mean-reverting models. This study adopts Hénaff et al. (2018)'s joint, multi-factor model for the futures curve and spot process, accounting for seasonality in futures prices, the correlation between spot and prompt prices, and the presence of spikes in spot prices. This model aims to address limitations in existing research by providing a more comprehensive and realistic representation of storage valuation.

Chapter 2

Methodology

2.1 Problem Formulation

Valuation and Hedging of a Storage Unit

Leasing a storage unit involves paying for the right, but not the obligation, to deposit or withdraw a storable commodity, in our case - maize, from the unit. The primary objective is to optimize the exercise of this right by managing the deposit and/or withdrawal of the commodity from the unit. Simultaneously, the lessee engages in trading activities on the commodity's spot or futures market. The lessee makes these decisions within various operational constraints. Two main methods can be employed to determe the optimal operating policy and the value of storage: *intrinsic* and *extrinsic* methods.

Let us consider a storage facility with constraints on the volume of stored commodity, V_{min} and V_{max}. By definition, $V_{min} \leq V \leq V_{max}$ should hold. We assume a discrete set of dates $t_i = i\Delta t$ for $i = 0, 1, ..., n-2, n-1$ with $\Delta t = T/n$. At each date t_i and starting from a volume V_{t_i}, one has three decisions to make: deposit the commodity at a rate of p_{dep}, withdraw the commodity at a rate of p_{with}, or take no action. Let d_i be the decision at time t_i. Clearly, $d_i = dep$ (with respect to *deposit*), $d_i = with$ (with respect to *withdrawal*), and $d_i = no$ (with respect to *no action taken*). The user can follow the strategy $(d_i)_{i=0,1,...,n-2,n-1}$ and the resulting volume (of the commodity) in storage $(V_{t_i})_i$ is given by:

$$V_0 = v \tag{2.1}$$

$$V_{t_{i+1}}(d) = \begin{cases} min(V_{t_i}(d) + p_{dep}\Delta t, V_{max}) & ;if \quad d_i = dep \\ max(V_{t_i}(d) - p_{with}\Delta t, V_{min}) & ;if \quad d_i = with \quad i = 0, 1, ..., n-3, n-2 \\ V_{t_i}(d) & ;if \quad d_i = no, \end{cases} \tag{2.2}$$

We further denote the spot price of maize by S_t and come up with the following cash flow (positive when withdrawing maize and negative when depositing):

$$\phi_{d_i}(S_{t_i}) \coloneqq S_{t_i}(V_{t_i}(d) - V_{t_{i+1}}(d)).$$

According to Hénaff et al. (2018), we assume that the maximum deposit (p_{dep}) and maximum withdrawal (p_{with}) rates are constant for simplicity. This assumption is made with the knowledge that these rates are a function of quantities of maize stored in the facility. We can summarize the decisions an operator can make in the table below:

Deposit: $d_i = dep$	$V_{t_{i+1}}(d) = min(V_{t_i}(d) + p_{dep}\Delta t, V_{max})$	$\phi_{dep} = S_{t_i}(V_{t_i}(d) - V_{t_{i+1}}(d))$
Withdrawal: $d_i = with$	$V_{t_{i+1}}(d) = max(V_{t_i}(d) - p_{with}\Delta t, V_{min})$	$\phi_{with} = S_{t_i}(V_{t_i}(d) - V_{t_{i+1}}(d))$
No Action: $d_i = no$	$V_{t_{i+1}}(d) = V_{t_i}(d)$	$\phi_{no} = 0$

Table 2.1: Possible decisions.

The trading strategy under the *intrinsic* method is predetermined, and the optimal futures positions are determined by solving a linear optimization problem. The constraints in this optimization problem are imposed by the storage unit (Eydeland and Wolyniec, 2003).

Let us consider N futures contracts available for trading. Let these contracts expire at times T_j, $j = 1, 2, ..., N-1, N$. Let $F(t, T_j)$ be the price of a futures contract at time t. The lessee's (or storage operator's) objective under the *intrinsic* strategy is to find the number of futures contracts $\alpha_j(t) \cong \alpha_j(t_o)$ s/he can buy (or sell) at the onset of the lease. This optimization problem can be expressed as follows:

$$\begin{aligned} IV(t) \; &\coloneqq \max_{(\alpha_j(t))_{j=1,2,\ldots,N}} -\sum_{j=1}^{N} \alpha_j(t) F(t, T_j) \\ -p_{with} &\leq \alpha_j(t) \leq p_{dep}, \; for \; j = 1, 2, ..., N-1, N \qquad (2.3) \\ V_{min} &\leq V(t) + \sum_{j=1}^{N} \alpha_j(t) \leq V_{max}, \; for \; j = 1, 2, ..., N-1, N; \end{aligned}$$

We implement the *intrinsic* (or static) trading strategy at the beginning of the lease to obtain the cash flow corresponding to the optimal solution. It is assumed that the subsequent evolution of the futures curve does not impact the cash flow.

Next, we look at the *extrinsic* method as described by Warin (2012). In this method, the storage operator's objective is to find a strategy d that maximizes the expected cumulative cash flows. The optimal value, denoted by J^*, is the solution to the following problem:

$$\begin{aligned} J^*(t_0, x_0, v_0; d) &= \max_{(d_i)_{t=0,1,\ldots,n-2,n-1}} J(t_0, x_0, v_0; d) &(2.4)\\ &= \max_{(d_i)_{i=0,1,\ldots,n-2,n-1}} \mathbb{E}\left[\sum_{i=0}^{n-1} \phi_{d_i}(S_{t_i})\right] &(2.5)\\ &= J(t_0, x_0, v_0; d^*). &(2.6)\end{aligned}$$

From Table 2.1 on page 8, it is recalled that $V_{t_{i+1}}(d)$ only depends on $V_{t_i}(d)$ and d_i. To emphasize this, if $V_{t_i} = v$, then $V_{t_{i+1}}(d)$ can be expressed as $\hat{V}_{d_i}(v)$.

At time t, for $X_t = x$ and with current volume level v, the optimal value for the storage will be denoted by $J^*(t, x, v)$. The Dynamic Programming Principle (DPP) implies

$$J^*(t_i, x, v) = \max_{(d_i)_{i=0,1,\ldots,n-1}\in\{dep,no,with\}} \left\{\phi_{d_i} + \mathbb{E}\left[J^*(t_{i+1}, X_{t_{i+1}}, \hat{V}_{d_i}(v))|X_{t_i} = x, V_{t_i} = v\right]\right\}. \tag{2.7}$$

The problem of finding the optimal strategy d^* and its corresponding value J^* can be solved numerically, and one approach is to use Monte Carlo (MC) simulations. However, in this case, the preference is to solve the conditional expectation using a regression technique. The backward algorithm employed provides an estimate of the optimal strategy d^*, which is a function of time, the quantity of maize under storage, and the forward curve. The steps of the algorithm involve determining the optimal decision rule backward in time.

Once the optimal strategy d^* has been determined, the storage value is then computed using a forward algorithm. This algorithm can be divided into two main steps: (1) simulate paths for the forward curve and the spot price, and (2) along each simulated paths, apply the optimal decision rule d^* determined earlier and calculate the corresponding cash flow. The Expected Present Value (EPV) of the cash flow over these paths serves as an estimate of J^*, the optimal Expected Value (EV) of the storage unit.

This process integrates both backward and forward steps to estimate the optimal strategy and its corresponding value. The regression technique is used to handle the conditional expectation, providing a practical and efficient way to solve the optimization problem in a numerical setting.

While the *intrinsic* method provides a price that can be obtained with certainty through a trading strategy in the futures market, the same certainty does not apply to a manager following the optimal strategy d^* on a single path. In this case, there is no guarantee that the realized cumulative cash flows on a given path will match the expected value J^*. Discrepancies between realized and expected cumulative cash flows are expected.

Therefore, it becomes crucial for the lessee to address the variance of the cumulative cash flows, which is a random variable. One effective way to achieve this is by implementing a financial hedging strategy based on futures contracts. Financial hedging aims to minimize the impact of market fluctuations and uncertainties on the cash flows, providing a level of protection against adverse movements in the futures market.

By strategically using futures contracts, the storage manager can offset or mitigate the potential risks associated with the optimal strategy d^* on a single path. This hedging strategy helps to reduce the variance in cumulative cash flows and enhances the manager's ability to achieve more reliable and consistent financial outcomes in the face of market uncertainties.

Financial Hedging Strategy

The optimal operating strategy, which includes both physical and financial operations, will be complemented by additional financial trades. This combination is designed so that the expectation of the cumulative wealth generated by the integrated strategy remains J^*, but its variance is reduced. The additional financial hedging strategy functions analogously to control variates in the variance reduction of Monte Carlo (MC) simulations. The primary objective is to maintain the expected value while decreasing its variance.

In the context of MC simulations, to reduce the variance of an estimator for a random variable Y, a mean-zero control variate is introduced. This control variate is deliberately chosen to be highly negatively correlated with Y. By adding this control variate to the estimator, the resulting combined estimator preserves the expected value of Y but achieves a reduction in variance.

Similarly, in the financial context described, the additional financial hedging strategy serves as a control variate. By integrating it with the optimal operating strategy, the overall cumulative wealth is expected to have the same mean as the original strategy (i.e., J^*), but its variance is diminished. This approach helps enhance the reliability and stability of financial outcomes by mitigating the impact of uncertainties and fluctuations associated with the integrated strategy.

This study chooses futures contracts as a hedging instrument due to their high liquidity and strong positive correlation with the spot price. Despite the fact that the futures contract price $F(t,T)$ does not converge to the spot price as the time to maturity $(T-t)$ approaches zero, the correlation between the prompt contract and the spot price is very high. In fact, the prompt contract and spot price move in the same direction. The fundamental idea behind hedging is to supplement physical spot trading with a strategy that involves buying and selling a quantity $\Delta(t_i, T_j)$ of futures contracts $F(., T_j)$ at a trading date t_i, where $1 \leq j \leq m$. It is important to note that these quantities, $\Delta(t_i, T_j)$, will be dependent on the spot S and futures $\{F(., T_j)\}_{1\leq j\leq m}$ prices, as well as the current volume level V_t.

If the storage manager follows this hedging strategy along with spot physical trading, then the cumulative cash flows of the combined strategy are given by:

$$Wealth_{spot+futures} = \sum_{i=0}^{n-1} \phi_{d_i^*}(S_{t_i}) + \sum_{i=0}^{n-1}\sum_{j=1}^{m} \Delta(t_i, T_j)(F(t_{i+1}, T_j) - F(t_i, T_j)). \quad (2.8)$$

We use $\Delta(t, T_j) = 0$ for $t \geq T_j$ because the futures contract $F(., T_j)$ stops trading after expiry date T_j.

Given that the futures price process is a martingale under risk-neutral probability, i.e.,

$$\mathbb{E}_{t_i}\left[F(t_{i+1}, T_j)\right] = F(t_i, T_j), \quad (2.9)$$

then the expectation of such a hedging strategy is null;

$$\mathbb{E}\left[\sum_{i=0}^{n-1}\sum_{j=1}^{m}\Delta(t_i,T_j)(F(t_{i+1},T_j)-F(t_i,T_j))\right]=0.$$

Following the optimal spot strategy in consideration of the future hedging portfolio gives the same cash flows in expectation but lower variance.

$$\mathbb{E}\left[Wealth_{spot+futures}\right]=\mathbb{E}\left[Wealth_{spot}\right]=\mathbb{E}\left[\sum_{i=0}^{n-1}\phi_{d_i^*}(S_{t_i})\right]=J^*.$$

Note that the specification of such a hedging strategy depends on the nature of the relation between the spot price and the futures curve.

Two heuristic delta strategies, $\Delta_1(t_i,T_j)$ and $\Delta_2(t_i,T_j)$, are considered. The first, Δ_1, is set equal to the conditional expectation of the volume to be exercised during the delivery period of the futures contract, given the information at t_i:

$$\Delta_1(t_i,T_j)=\mathbb{E}_{t_i}\left[\sum_{T_{j-1}\le t_l\le<T_j}(V_l(d^*)-V_{l+1}(d^*))\right]. \tag{2.10}$$

The second, Δ_2, modifies the heuristic delta using Warin (2012)'s concept of tangent process, assuming that the prompt (P) converges towards the spot (S):

$$\Delta_2(t_i,T_j)=\mathbb{E}_{t_i}\left[\sum_{T_{j-1}\le t_l<T_j}(V_l(d^*)-V_{l+1}(d^*))\frac{F(t_l,T_j)}{F(t_i,T_j)}\right]. \tag{2.11}$$

It is important to note that these hedging strategies, Δ_1 and Δ_2, may not be perfect, and residual risk may still remain within the model due to their heuristic nature. The choice of these strategies aims to significantly reduce the uncertainty of cash flows associated with the spot trading strategy.

2.2 The Term Structure

The valuation of storage units has predominantly been approached using numerical methods, with less emphasis on modeling the underlying price processes, which play a crucial role in valuation and hedging decisions. This study explores two modeling approaches: (1) modeling the spot price with the classical mean-reverting models and (2) modeling of the dynamics of the entire term structure. Both approaches have their merits and limitations, and the choice between them may depend on the specific characteristics of the commodities being studied, market conditions, and the level of detail required in the analysis.

Spot Price Process

This approach involves employing classical mean-reverting models to capture the dynamics of the spot price. Mean-reverting models are characterized by a tendency of prices to move towards a long-term average or equilibrium level over time. The choice of mean-reverting models may include various mathematical formulations that describe the mean-reverting behavior of the spot price.

We employ the use of a one-factor mean-reverting process for the spot price dynamics. The simplest form of this model, as per Boogert and De Jong (2008), is expressed as follows:

$$\frac{\boldsymbol{d}S_t}{S_t} = k[\mu(t) - log(S_t)]dt + \sigma dB_t, \tag{2.12}$$

where B_t is the standard Brownian Motion, $\mu(t)$ is a time-dependent parameter, calibrated to the initial futures curve $(F(0,T))_{T\geq 0}$, provided by the market, k is the mean reversion parameter and σ is the volatility, both of which are positive constant.

This model suggests that the spot price undergoes mean-reverting behavior, with the rate of reversion determined by k, the difference between the current spot price and the parameter $\mu(t)$, and a stochastic component σdB_t. The model reflects the idea that the spot price tends to move towards a certain mean level over time, and the volatility term introduces randomness into the price dynamics. The calibration of $\mu(t)$ to the initial futures curve aligns the model with market observations.

The one-factor model introduced earlier has certain limitations in capturing the dynamics of the spot price and, consequently, the value of maize storage. Boogert and De Jong (2008) identified drawbacks in terms of the calibration of the time-varying function $\mu(t)$, which was considered unstable and led to unrealistic sensitivity of spot dynamics and storage value concerning the initial futures curve. Additionally, this model assumes a perfect correlation between future contracts, limiting the ability to formulate trading strategies involving spreads between futures contracts. Moreover, it (equation (2.12)) does not account for price spikes, which are an essential source of storage value.

In an effort to address these limitations, Parsons (2013) proposed a two-factor mean-reverting model with the following dynamics:

$$\frac{\boldsymbol{d}S_t}{S_t} = a[\mu(t) + log(L_t) - log(S_t)]dt + \sigma_{S,t}dB_t, \tag{2.13}$$

$$\frac{\boldsymbol{d}L_t}{L_t} = b[log(\mathcal{L}) - log(L_t)]dt + \sigma_{L,t}dZ_t, \tag{2.14}$$

where $S - t$ is the spot price of maize at time t, L_t is a stochastic process representing the long-run mean of the spot price, $\mu(t)$ is a time-dependent parameter, a and b are constants representing mean-reversion parameters, and $\sigma_{S,t}$ and $\sigma_{L,t}$ are volatilities.

In this model, the spot price follows a mean-reverting process with a long-run mean that is stochastic and reverts to a deterministic value $\mathcal{L}$. However, this model still suffers from the instability of the deterministic function μ and does not incorporate the possibility of jumps (or spikes) in the spot price.

To address these issues, Safarov and Atkinson (2017) introduced a time in-homogeneous exponential Lévy process, considering seasonality, mean-reversion, and price spikes with seasonality jump intensities. This provides a more comprehensive framework for modeling the spot price dynamics and associated storage value.

Term Structure Models

In contrast to focusing solely on the spot price, this approach considers the dynamics of the entire term structure, which includes futures prices at different maturities. The term structure refers to the relationship between spot prices and futures prices across various delivery dates. Modeling the entire term structure provides a more comprehensive view of how prices evolve over time and can capture the interplay between different maturities.

Clewlow and Strickland (1999) proposed a one-factor model for the futures curve. Clewlow et al. (1999) extended it to a multi-factor setting. A two-factor version of their model can be expressed as

$$\frac{\boldsymbol{d}F(t,T)}{F(t,T)} = e^{-\lambda(T-t)}\sigma_{ST}dB_t^S + \sigma_{LT}dB_t^L,$$

where λ, σ_{ST} (short term volatility) and σ_{LT} (long term volatility) are positive constants, and B^S and B^L are two correlated Wiener processes, with $\boldsymbol{d}\langle B^S, B^L\rangle_t = \rho dt$.

This model is an adaptation of the Gabillon (1991) model proposed for spot prices. It fits the initial futures curve exactly and has the advantage of capturing the dependence of volatility on the maturity parameter (term-structure type). However, it does not incorporate the essential seasonality feature.

An alternative approach is to model the entire term structure with an $n-$factor log-normal dynamics for the futures curve (Warin, 2012):

$$\frac{\boldsymbol{d}F(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t) e^{-a_i(T-t)} dZ_t^i, \tag{2.15}$$

where n is the number of factors, $\sigma_i(t)$ and a_i are parameters and dZ_t^i represents Brownian motions.

The spot process is assumed to be the limit of the futures contract price as time to maturity goes to zero ($S_t = \lim_{T \downarrow t} F(t,T)$) under this model. However, this is a misrepresentation of the reality; the spot price corresponds to the commodity (maize) delivered the next day whereas futures contracts are settled over an entire calendar month by rated delivery.

The modeling choice should consider the trade-offs between accurately fitting the initial futures curve, capturing term structure dynamics, and accounting for the specific features of commodity delivery.

2.3 Model Motivation

Stylized Facts

We motivate the model used by; (1) presenting some stylized facts about maize prices, and then (2) introduce the model formulation and its estimation. We first underscore essential stylized facts about maize markets that may influence the value of the storage unit. These properties revolve around the dynamics of maize demand and supply. As pointed out earlier, maize supply can be well described by a deterministic dependency on maize production (volumes) and weather (seasons) whereas the demand for maize is fairly constant over the year. This interplay between demand and supply gives rise to a seasonal price pattern, while unpredictable fluctuations in weather conditions, and consequently variations in output, can lead to abrupt shifts in maize prices. These observations serve as the primary drivers behind the value attributed to the storage unit, as ownership of such a facility allows one to capitalize on both seasonality and price fluctuations.

Etienne and Mattos (2016) studied the term structure of agricultural commodity prices, focusing on corn (maize) as a case study. They employed a dynamic latent factor model to approximate the commodity futures price curve using three latent factors: level, slope, and curvature. These factors, though unobserved, were found to be linked to observable economic fundamentals. Their analysis unveiled the significant influence of real economic activity and the relative scarcity of the commodity on shaping the corn futures price curve. Moreover, Granger causality tests conducted by Etienne and Mattos (2016) revealed that all three unobserved factors of the futures price curve contained predictive information regarding real economic activity and the relative scarcity of the commodity. These findings align with the theory of storage, indicating a forward-looking element inherent in the term structure of prices of commodity.

The point of delivery significantly influences the price of corn (maize). Kimathi (2018)'s study on the valuation of locational spread options defines these options based on price differences between the same commodity in different locations. In commodity markets, spread options can be categorized into location spreads (price differences between the same commodity at different locations), calendar spreads (price differences at different points in time), processing spreads (price differences between inputs and outputs of a production process), and quality spreads (price differences between different grades of the same commodity).

Kimathi (2018)'s study specifically investigated the price difference of tomatoes between the Kenyan cities of Mombasa and Nairobi, confirming that the point of delivery does influence commodity prices. In the case of maize, this study uses the average monthly spot prices of corn obtained from the Kenya National Bureau of Statistics (KNBS). Additionally, the average annual prices of maize over time are used as a proxy for futures, which act as hedge instruments.

As highlighted in the previous sections, the notation used in the study includes: S_t is the spot price of maize at date t, $(F(t, T_i))_i$ is the futures contract prices at t for a set of maturities $\{T_i\}$ (monthly spaced), and P_t is the prompt contract, representing the futures contract with the closest maturity to the current time t. Maize (corn) price is quoted in Kenyan shilling (KES) per kilogram (Kg).

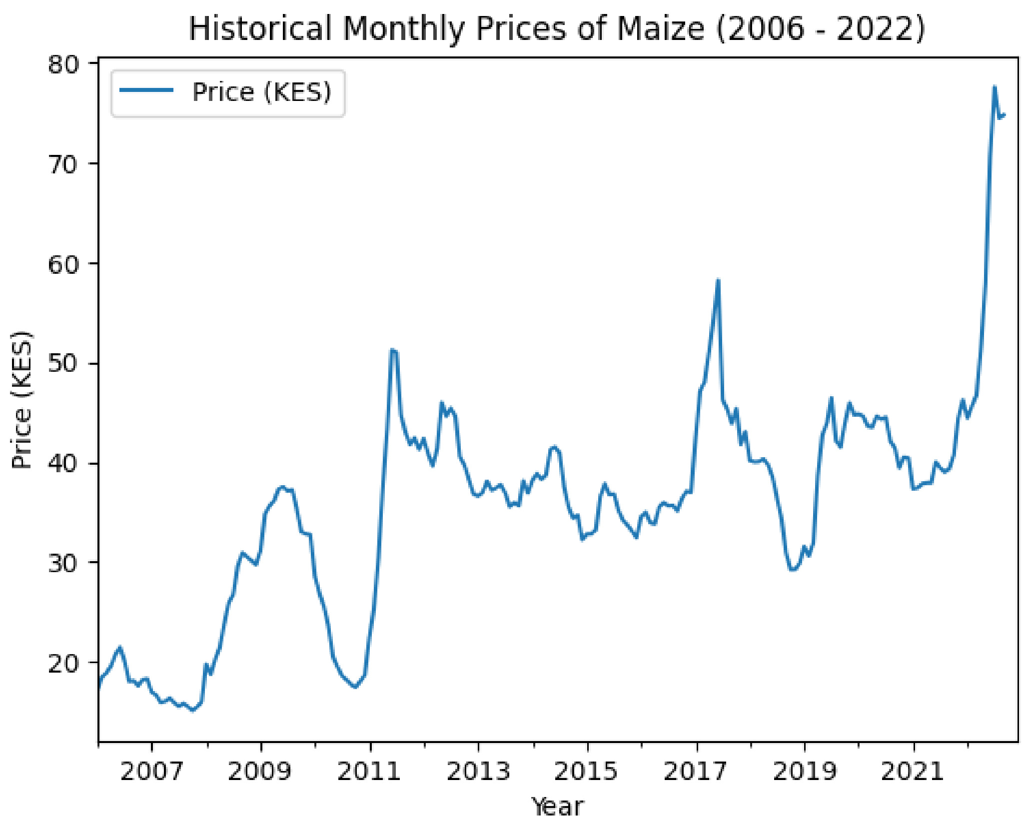

Figure 2.1: Historical monthly spot prices of maize in KES per Kg

We observe from the above plot in figure 2.1 that the price of maize (corn) has been consistently rising from lows of KES 17.36 per Kg (average price for the period between January 2006 and December 2007) to the highs of KES 47.55 per Kg (average price for the period between October 2020 and September 2022). The least price was KES 15.09 per Kg during the month of October 2007 and the highest price was KES 77.60 per Kg in the month of July 2022.

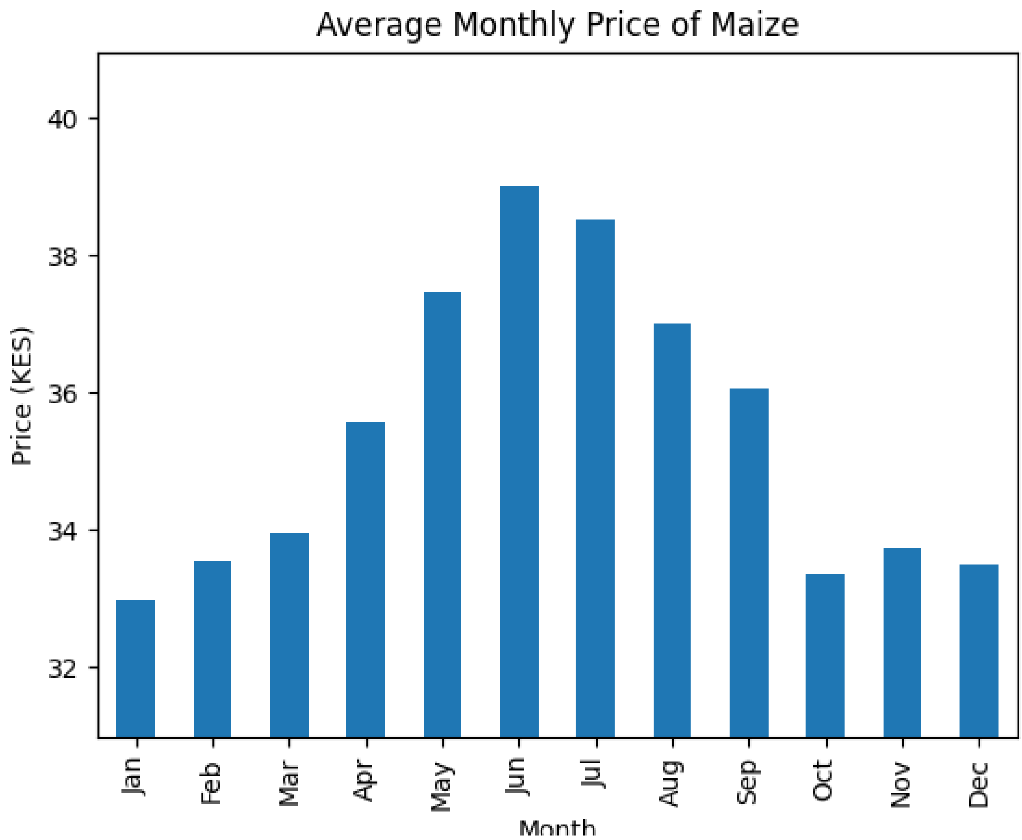

Figure 2.2: Average monthly spot prices of maize in KES per Kg

We observe from figure 2.2 that the average prices of maize are at the lowest in January and at the highest in June. Typically, the harvesting period is between October and December explaining the drastic fall in the prices of maize (corn) in those months. The planting season is around March explaining the rise in in the price of maize. The period between April and July is usually designated for multiple weeding and corn starts developing in August and mature in September and is ready for harvest in October. This explains the seasonal trend in maize (corn) prices.

The second important aspect of maize (corn) prices is the occurrence of sudden moves, often referred to as jumps or spikes. These abrupt changes are typically triggered by unexpected imbalances between supply and demand, which can result from factors such as unpredictable weather changes, disruptions in the supply chain, or inaccuracies in global maize storage estimations. These events are swiftly reflected in spot price dynamics, leading to significant and rapid price swings. The market's storage capacities play a crucial role in absorbing and managing these large price movements.

Couleau et al. (2020) conducted a study on price jump risk in corn futures prices within the context of electronic trading and the shift to real-time announcements of United States Department of Agriculture (USDA) reports. Using tick data for corn transaction prices from CME Group's BBO (Best-Bid-Offer), time-stamped to the nearest second and traded on the electronic platform, they analyzed the period from January 14, 2008, to December 4, 2015, covering 1,983 trading days. They considered corn (maize) futures contracts with delivery months in March, May, July, September, and December, focusing on the nearby series, which represents the contract with the highest trading volume in the nearest delivery month. Their findings provided strong evidence for the presence of price spikes in the maize futures market, especially in the context of market microstructure. Jumps or spikes in commodity prices can have various causes, and this study aims to identify outliers in the time series of the spread between the spot price S_t and the prompt price P_t, given by $x_t := \frac{S_t - P_t}{P_t}$. The analysis distinguishes between positive spikes, often associated with unpredictable weather patterns, and negative spikes, which may result from poor anticipation of market-wide storage levels or corrective measures taken by governments, such as subsidies in the agricultural sector.

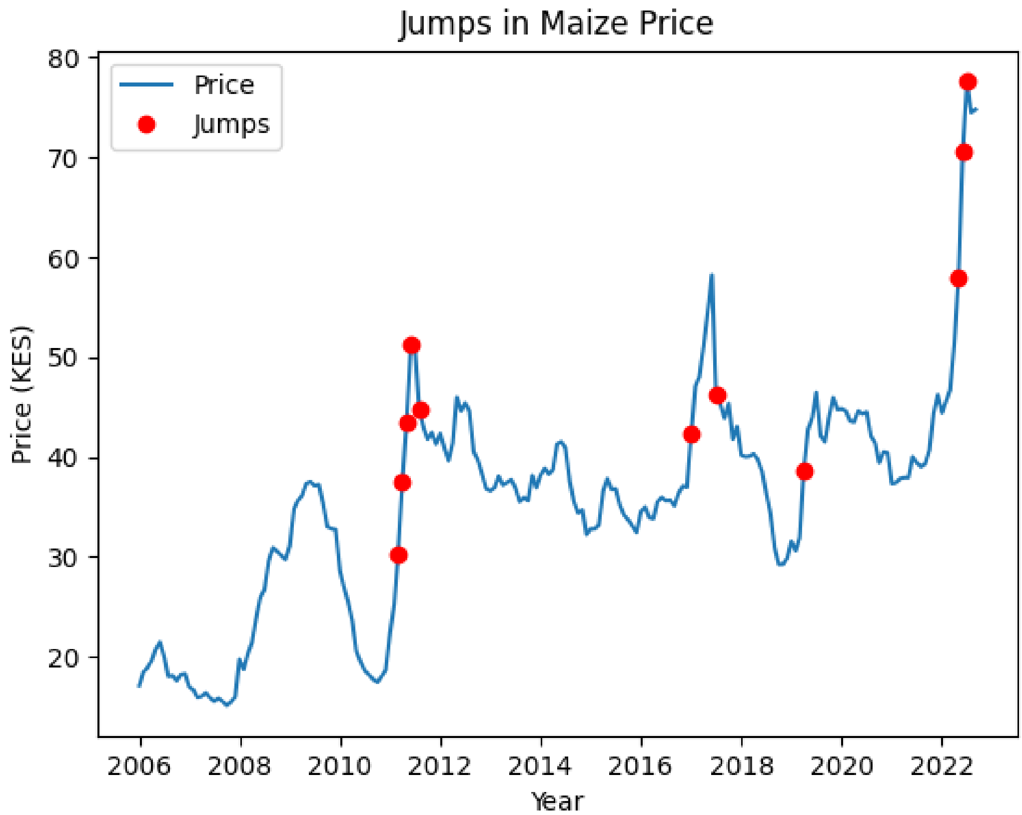

Figure 2.3: Jumps observed in the historical prices of maize

We observe a few jumps clustered around three periods: 2011 - 2012, 2016 - 2017 and 2021 - 2022. Secondly, we observe that there are more positive jumps/spikes than negative ones. Thirdly, we observe that the prices are sticky upwards creating some regimes after every five years (coinciding with Kenyan general elections). The negative jumps/spikes are witnessed in June 2017 and September 2022 coinciding with the government subsidy during the election period in Kenya.

The Futures Curve

To model this, this study adds a seasonality component and introduces parameters to Gabillion's model to have an economic sense. We adopt the name Hénaff et al. (2018) used - the Seasonal Gabillion two-factor model. It is given by

$$\frac{\boldsymbol{d}F(t,T)}{F(t,T)} = e^{-\lambda(T-t)}\phi(t)\sigma_S dB_t^S + (1-e^{-\lambda(T-t)})\sigma_L dB_t^L, \tag{2.16}$$

where B^S and B^L are two correlated Wiener Processes, with $\boldsymbol{d}\langle B^S, B^L\rangle_t = \rho dt$. S and L are Short and Long terms respectively; λ, σ_S and σ_L are positive constants with σ representing volatility. The function $\phi(t) = 1 + \mu_1 cos(2\pi(t - t_1)) + \mu_2 cos(4\pi(t - t_2))$ weights instantaneous volatility with a periodic behaviour. It takes into account the seasonality component.

This study agrees with Hénaff et al. (2018) that this model constitutes an efficient framework, whose parameters are economically viable. We can interpret σ_L and σ_S as 'long-term' and 'short-term' volatility respectively. We are cognisant of the fact that we only have access to monthly spaced futures contracts in the real world despite the model being expressed with a continuous set of maturities.

Estimation of the Futures Curve Model

In this study, the initial (or rough) estimates for the volatility and correlation parameters are derived as initial values for a more rigorous statistical procedure. Two scenarios are considered: one for the long-term volatility (σ_L) and another for the spot volatility (σ_S). For large maturities ($T - t \longrightarrow \infty$), the futures price process converges to a long-term volatility model: $\frac{\boldsymbol{d}F(t,T)}{F(t,T)} \simeq \sigma_L dB_t^L$. We can approximate the long-term volatility (σ_L) as:

$$\sigma_L^2 \simeq \frac{1}{m-1}\sum_{i=1}^{m}(\frac{z_{t_i}^L}{\sqrt{\Delta t_i}} - \bar{\mu}^L)^2,$$

where z_t^L is the log-return of a constant maturity long-dated contract, Δt_i is the time increment, and $\bar{\mu}^L$ is the average of $z_{t_i}^L$, $\bar{\mu}^L = \frac{1}{m}\sum_{i=1}^{m}\frac{z_{t_i}^L}{\sqrt{\Delta t_i}}$.

For short maturities ($T \longrightarrow t$ or rather $T - t \longrightarrow 0$), the futures price process converges to a spot volatility model: $\frac{\boldsymbol{d}F(t,T)}{F(t,T)} \simeq \sigma_S dB_t^S$. The volatility of the rolling prompt contract, i.e., the contract with the nearest maturity, can be used as a good proxy for the spot volatility (σ_S):

$$\sigma_S^2 \simeq \frac{1}{m-1}\sum_{i=1}^{m}(\frac{z_{t_i}^P}{\sqrt{\Delta t_i}} - \bar{\mu}^P)^2,$$

where z_t^P is the log-return of a prompt futures contracts, Δt_i is the time increment, and $\bar{\mu}^P$ is the average of $z_{t_i}^P$, $\bar{\mu}^P = \frac{1}{m}\sum_{i=1}^{m}\frac{z_{t_i}^P}{\sqrt{\Delta t_i}}$.

Additionally, an initial estimate for the correlation parameter (ρ) between spot and long-term contracts can be obtained as:

$$\rho = \frac{1}{m-1} \frac{\sum_{i=1}^{m} (\frac{z_{t_i}^P}{\sqrt{\Delta t_i}} - \bar{\mu}^P)(\frac{z_{t_i}^L}{\sqrt{\Delta t_i}} - \bar{\mu}^L)}{\sigma_S \sigma_L}.$$

To solve for the Maximum Likelihood Estimate (MLE), the likelihood function is formulated based on a time series of futures prices. Let z_t be a vector of price returns at time t, Δt being the corresponding time step $t_{i+1} - t_i$, and θ representing the vector of model parameters: $\theta = (\lambda, \mu_1, \mu_2, \sigma_S, \sigma_L, \rho)$. The components of z_t are given by the differences in futures prices and the futures prices themselves. The drift matrix H_t depends on the parameters λ, σ_L, σ_S, and ρ. An Euler discretization of the stochastic differential equation (SDE) (2.16) on page 20 leads to the relationship $z_t = H_t x_t,\ t \in \{t_1, t_2, ..., t_{m-1}, t_m\}$, where x_t follows a bivariate Gaussian distribution, $x_{t_i} \sim \mathcal{N}\left(\mathbf{0},\, \mathbf{\Sigma}\right),\ 1 \leq i \leq m$ such that $\mathbf{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

The likelihood function is defined as:

$$L(x_{t_1}, x_{t_2}, ..., x_{t_m-1}, x_{t_m}|\theta) = \frac{1}{m} \sum_{i=1}^{m} log\left(det\left(\mathbf{\Sigma}\right)\right) + x_{t_i}^T \mathbf{\Sigma}^{-1} x_{t_i},$$

where $x_t,\ t \in \{t_1, t_2, ..., t_{m-1}, t_m\}$ is obtained as $x_t = (H_t^T H_t)^{-1} H_t^T z_t$. The MLE problem is formulated as:

$$min\ L(x_{t_1}, x_{t_2}, ..., x_{t_m-1}, x_{t_m}|\theta) = \frac{1}{m} \sum_{i=1}^{m} log\left(det\left(\mathbf{\Sigma}\right)\right) + x_{t_i}^T \mathbf{\Sigma}^{-1} x_{t_i}, \tag{2.17}$$

subject to

$$\theta = (\lambda, \mu_1, \mu_2, \sigma_S, \sigma_L, \rho).$$

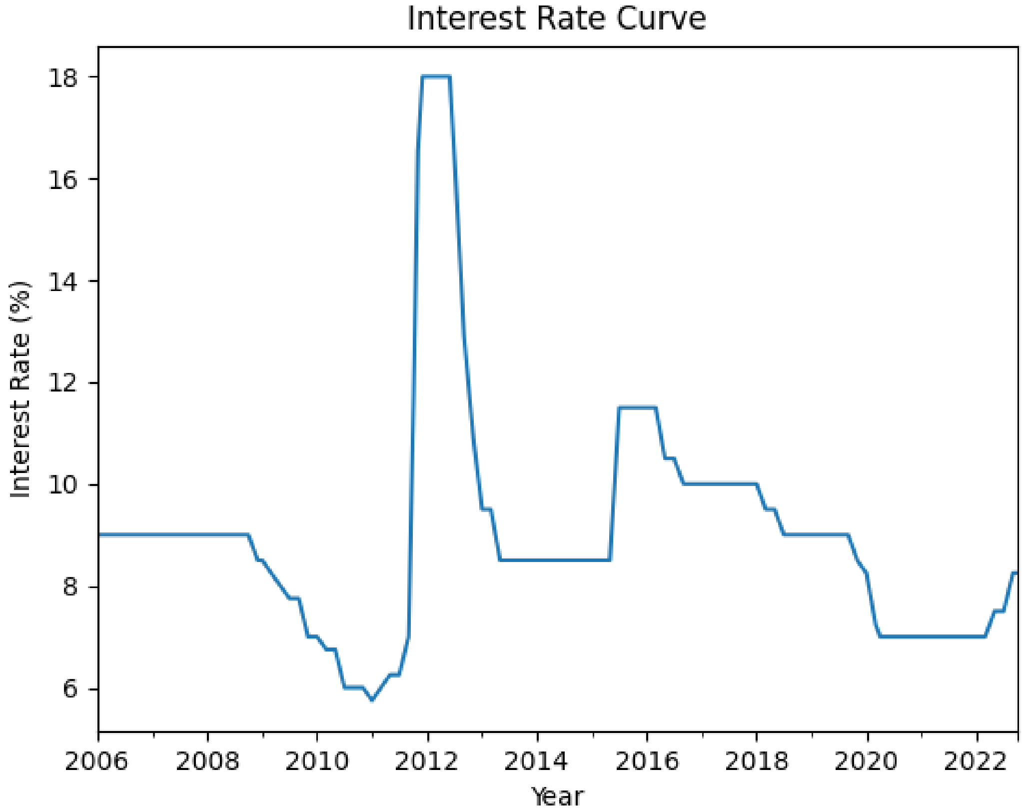

Figure 2.4: Interest rate curve extracted from CBK website

The benchmark interest rates listed on the Central Bank of Kenya (CBK) website guided by the Monetary Policy Committee (MPC) is shown in figure 2.4 above. The rates tend to fluctuate quite a lot. The rate was relatively stable at 9% during the period up to 2009. The rates fell to a low of roughly 6% between 2009 and 2011 before rising drastically to highs of 18% in 2012. This triggered a period of rate cap and this helped stabilise the rate. In 2020, the MPC held the rate at 7% to encourage money circulation. This rate impacts all facets of the economy including the futures prices of commodities.

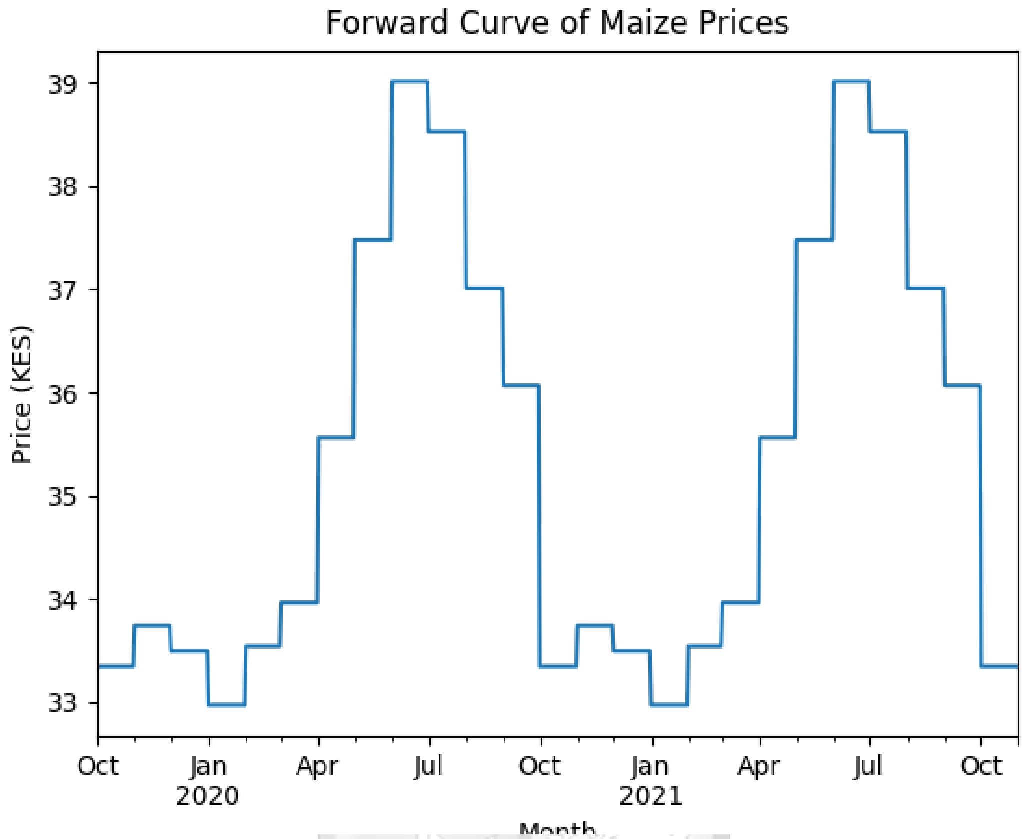

Figure 2.5: Futures price of maize

Typically, the Futures curve of maize (corn) can be modelled to mirror the seasonal variations in the price of maize. The prices peak during the wet season (May - September) and drops during the dry seasons (October - March).

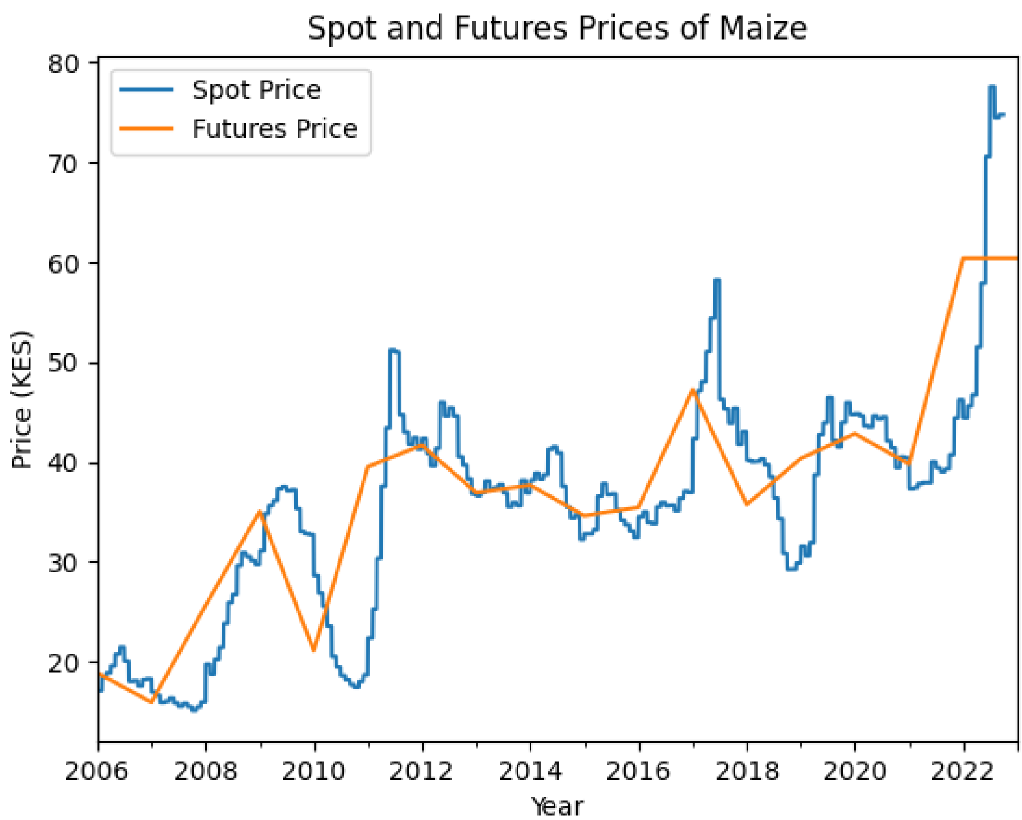

Figure 2.6: Spot and Futures prices of maize

From figure 2.6 we observe that the futures curve mirrors the spot prices through time. Entering into a futures contract can be used as a hedging strategy in the agricultural sector.

The Spot Price

We consider the spot price as a separate stochastic process but correlated to the prompt price. The spot dynamics is based on the spot log-return $y_t = log(\frac{S_t}{S_{t-1}})$ and is given by:

$$log(\frac{S_t}{S_{t-1}}) = a_1 + a_2 log(\frac{P_{t-1}}{S_{t-1}}) + a_3 log(\frac{P_t}{P_{t-1}}) + \epsilon_t \tag{2.18}$$

where ϵ_t follows a GARCH(p, q) process. Here, P_t is the prompt price, and the spot price is mean-reverting around a stochastic level equal to the prompt price. The GARCH(p, q) process captures the heteroscedasticity of the spot price and the correlation between the spot price and the prompt futures price.

Alternatively, the spot process can be modeled by considering the return of the spot to prompt spread $y_t = \frac{S_t - P_t}{P_t}$, using the front-back spread as the independent variable:

$$\frac{S_t - P_t}{P_t} = a_1 + a_2 \frac{S_{t-1} - P_{t-1}}{P_{t-1}} + a_3 \frac{P_{t-1} - B_{t-1}}{B_{t-1}} + \epsilon_t, \tag{2.19}$$

where B_t is the price of the second nearby futures (back contract) and ϵ_t is a GARCH(p, q) process. This model directly handles the spread between the spot and the prompt prices. The spread is a key variable in storage management.

Both spot models incorporate seasonality transmitted by the prompt price through the futures curve dynamics. Seasonal patterns are captured in the futures curve dynamics, and there is no need for a separate seasonal element in the spot dynamics.

The spot model also accounts for price spikes. A fast mean-reverting jump process is added to the spot model, distinguishing between positive and negative spikes. Positive spikes are caused by unpredictable weather changes, while negative spikes are attributed to poor market anticipation of the storage situation during 'shoulder months'.

In summary, the spot process for storage valuation is given by $\tilde{S}_t = S_t exp(Y_t^+ + Y_t^-)$, where Y_t^+ and Y_t^- represent the positive and negative spike processes, respectively. This formulation incorporates seasonality in both futures and spot prices, integrating positive and negative spikes into the spot process, each generated by distinct jump processes.

Chapter 3

Simulation of Future and Spot Prices

3.1 Numerical Results

This study uses futures-spot models to value various storage contracts and compare the results to the *intrinsic* value of storage units. We note that emptying a storage facility is faster than filling it i.e. the withdrawal rate is higher than the deposit rate. The table below is a summary of the characteristics of a storage unit.

Characteristic	Storage Facility
Total capacity	5,000 Kg
Deposit rate	580 Kg
Withdrawal rate	720 Kg
Initial volume	0
Final volume	0
Lease duration	24 months

Table 3.1: Storage Units Characteristics

The experiments were conducted using *Python* involving 500 simulations for the Monte Carlo (MC) method. The simulations followed the Longstaff and Schwartz algorithm. The procedure included both backward and forward phases. Under the backward phase, a set of spot and futures paths was simulated. Subsequently, the dynamic programming algorithm (2.7) was applied to estimate the optimal spot strategy. Following this, the hedging strategy based on futures contracts was evaluated using either equation (2.10) or (2.11). Under the forward phase, a new set of spot and futures paths was re-simulated by applying the estimated optimal spot strategy combined with the futures hedging strategy to the new trajectories. The cumulative cash flows resulting from these physical and financial operations, denoted as $Wealth_{spot+futures}(d^*)$ were stored for each sample path.

Empirical mean and standard deviations of the cash flows are then computed based on the stored data. The mean of the cumulative wealth provided an estimate of the Expected Value $EV\ J^*$ of the storage unit, as given in (2.7). The standard deviation served as an indicator of the dispersion of realized cash flows around the EV, reflecting the uncertainty faced by the manager on a single realization of spot and futures prices.

Numerical results confirmed that the hedging strategy significantly reduced the variance of cumulative cash flows. However, the analysis was contingent upon the selected model, as both the backward and forward phases relied on sample paths generated by the model. To mitigate the dependency on the model and make the comparison less model-dependent, cumulative cash flows of the estimated optimal strategy were computed based on historical spot and futures paths. This real-case test for the optimal strategy underscored the importance of spot modeling by showcasing the profit that would have accrued to the storage manager in an actualized path. This real case test for the optimal strategy reinforced the relevance of spot modeling by providing the profit that would have been accumulated by the storage manager in a realized path.

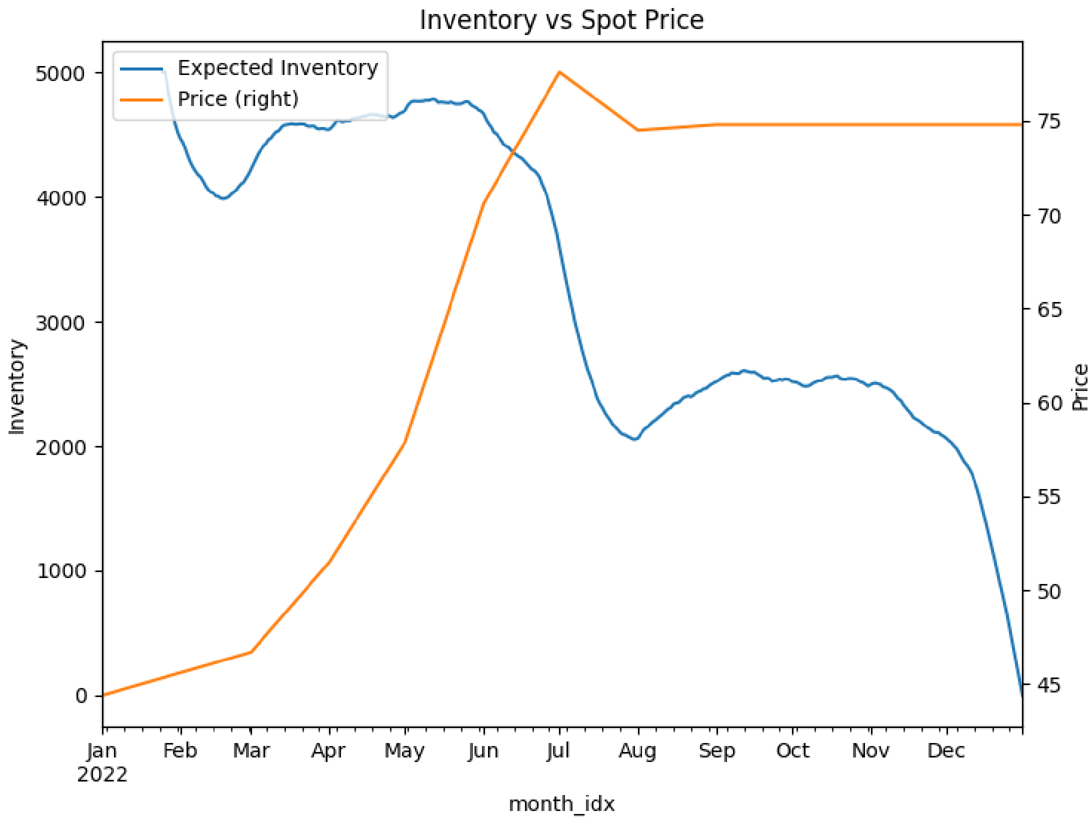

Figure 3.1: Spot Price vs Inventory

We observe in figure 3.1 that when the spot prices are high, the amount of maize (corn) in storage is limited. The unit managers (or traders) sell their commodities (maize) in the spot market when the prices are high. We summarize the results of the valuation algorithm in Table 3.2 on page 30. The table reports the net present value (NPV) realised from *Full* valuation, *IV*, and *EV* strategies. As anticipated, the *extrinsic* spot-based strategy is expected to yield a larger value than the *intrinsic* physical futures-based strategy. This outcome suggests that the *extrinsic* strategy, which involves actively managing the spot market based on its financial characteristics, enables better financial exploitation of the rights (without obligation) of depositing or withdrawing maize (commodities). In essence, the *extrinsic* strategy facilitates a more effective extraction of the optionality inherent in storage.

On the other hand, the *intrinsic* physical futures-based strategy, which relies on holding futures contracts without active spot management, tends to be more conservative. Consequently, it may not capture the full financial potential associated with dynamic spot market decisions.

Furthermore, the financial hedging strategy is designed to reduce the uncertainty of storage cash flows. The empirical results indicate a significant reduction in the variance of cumulative cash flows when employing the hedging strategy. This reduction in variance suggests that the hedging strategy effectively mitigates the uncertainty associated with both spot and futures prices, contributing to a more stabilized financial performance in storage operations.

484,468	380,086	104,382

Table 3.2: Valuation Results

We observe in Table 3.2 that the NPV of the storage contracts are KES 484,468, KES 380,086, and KES 104,382 based on *Full*, *intrinsic*, and *extrinsic* valuation strategies respectively. This implies that *EV* is the most efficient strategy in realising storage value.

The numerical results in this study underscore the significance of a joint modeling approach for maize spot prices and futures curves in the valuation and hedging of storage units. This integrated strategy empowers storage unit managers to strategically leverage storage optionality by capitalizing on the inherent volatility and seasonality of spot prices.

A pivotal observation is the consistent outperformance of *Extrinsic Value* (*EV*) over *Intrinsic Value* (*IV*) in historical back-testing. The dynamic management of spot prices, grounded in their financial characteristics, consistently yields superior financial outcomes compared to a more conservative *intrinsic* strategy.

The adoption of a realistic framework for spot and futures markets is a notable strength of the joint modeling approach. By incorporating risk factors for both markets, the model accounts for the seasonality of the futures curve and acknowledges the non-convergence of futures prices to spot prices. This departure from assumptions in some literature enhances the model's accuracy in representing actual market conditions.

Furthermore, the joint model facilitates the development of a more relevant hedging strategy based on futures contracts. The consideration of risk factors specific to the futures curve aligns the hedging strategy more closely with real market conditions, contributing to enhanced risk management practices.

In conclusion, the joint modeling approach not only enables better tracking of the $Extrinsic\ Value$ (EV) of storage in real market conditions but also provides storage unit managers with a robust framework for optimizing financial performance through dynamic spot market decisions.

3.2 Model Risk

This section of the study delves into the critical role of spot modeling in storage valuation and hedging. Notably, the narrowing of seasonal spreads over time emphasizes the *extrinsic* nature of storage value, necessitating a closer examination of spot modeling and its impact. The contention is that uncertainties surrounding storage value primarily stem from spot process modeling, as the evolution of the spot process significantly influences optimal trading strategies. On the other hand, the futures model is posited to predominantly impact hedge quality or variance reduction rather than the expected value of the storage unit. This aligns with the findings of other authors, such as Bjerksund et al. (2011), who reached similar conclusions using the *rolling intrinsic valuation* method.

The objective of this section is to formalize these assertions within the framework of model risk measurement. The discussion is divided into two parts: firstly, a performance comparison of the two spot models, (2.18) and (2.19), proposed in this study is conducted using historical data. The focus is on evaluating the impact of various modeling assumptions and assessing the sensitivity of storage estimated value concerning model parameters. Subsequently, a model risk measure is introduced to quantitatively analyze these uncertainties, following the methodology proposed by Cont (2006).

This study looks at the daily deltas against the projected inventory. Daily deltas refer to the changes in the inventory levels of a product over a given time period, typically on a daily basis. These changes can be due to various factors, such as sales, purchases, and returns. Projected inventory, on the other hand, refers to the estimated inventory levels of a product at a future point in time, based on assumptions and forecasts.

In storage valuation, daily deltas are important because they determine the amount of inventory that is available for sale or use at any given time. This information is crucial in determining the value of the inventory, as the value of the inventory is often based on its cost or market value at the time of sale or use. Projected inventory is also important in storage valuation because it provides insight into the future value of the inventory. By estimating the inventory levels at a future point in time, one can determine whether there will be enough inventory to meet future demand or whether there will be excess inventory that may need to be sold at a discount.

The link between daily deltas and projected inventory lies in their impact on the overall value of the inventory. By tracking daily deltas, one can adjust inventory levels as needed to meet demand or avoid excess inventory. By projecting inventory levels, one can anticipate future demand and adjust inventory levels accordingly to maximize the value of the inventory. Ultimately, the goal of storage valuation is to maximize the value of the inventory by balancing current and future demand with inventory levels and pricing.

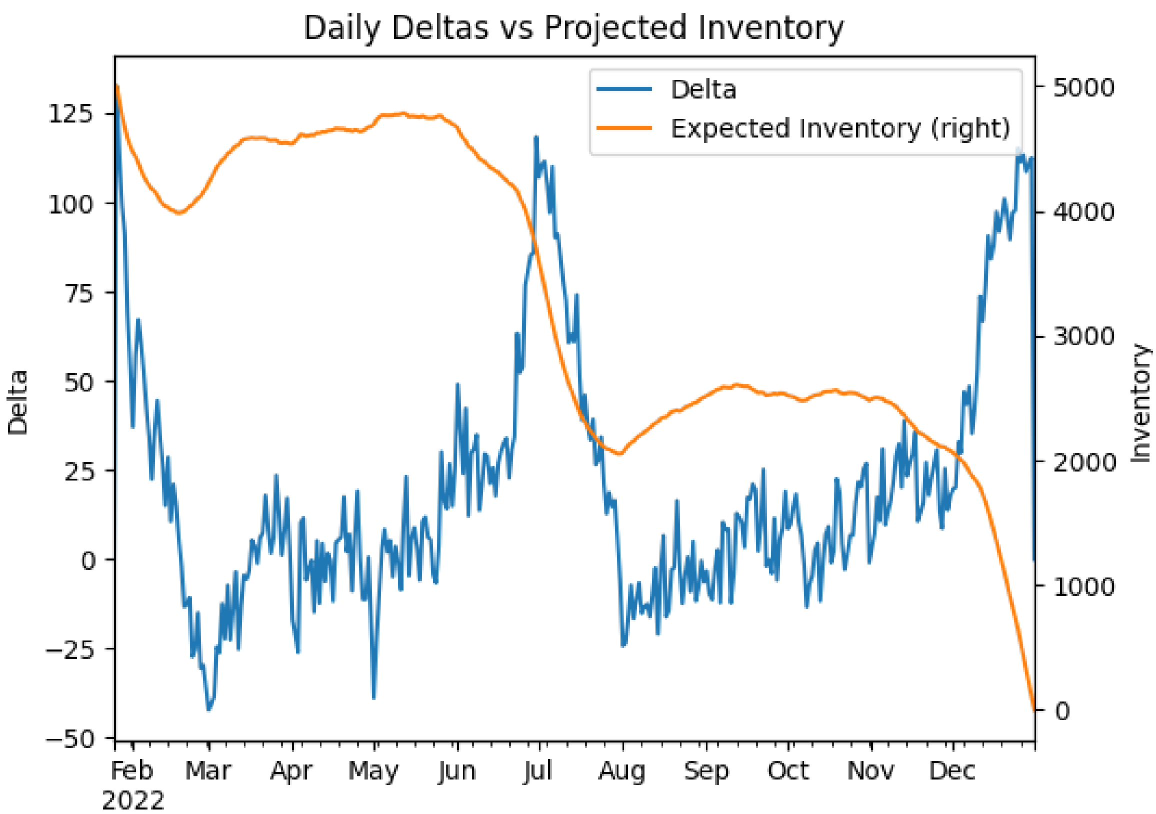

Figure 3.2: Daily Deltas vs Projected Inventory

We observe in Figure 3.2 that the projected inventory falls through time because the storage facility is rented for a specific period. However, the daily deltas sharply reduce between February and March 2022 and then stabilise for a few months before hitting another peak in July. The daily deltas follow an observable pattern. There are two peaks around January/February and July/August. The other months are fairly smooth with deltas ranging from -25 to +25.

We then proceed to look at the impact of the futures curve on the trigger prices. In storage valuation, trigger prices refer to the price levels at which it becomes profitable to inject or withdraw inventory from storage. These trigger prices are influenced by a number of factors, including the futures curve. The futures curve represents the expected prices of a commodity at various points in the future. The shape of the futures curve can have a significant impact on trigger prices because it affects the profitability of storing inventory.

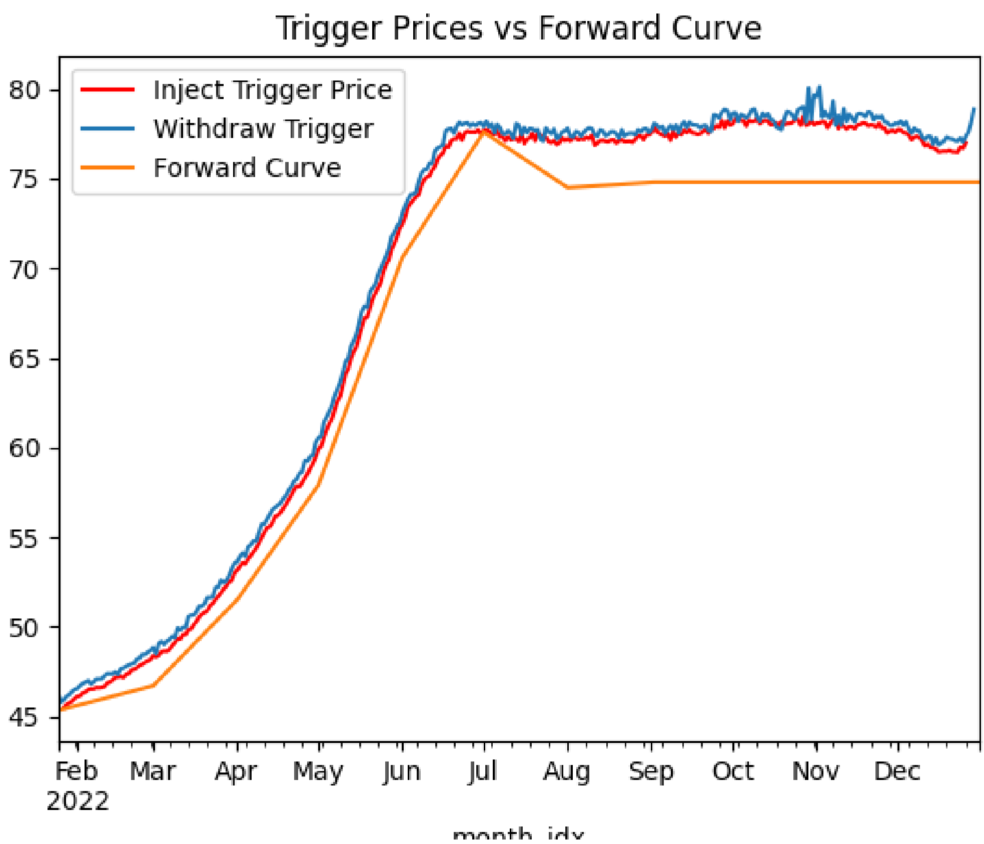

Figure 3.3: Trigger Prices vs Futures Curve

We observe that the deposit and withdrawal trigger prices mirror the futures curve in Figure 3.3. Generally, if the futures curve is upward-sloping, indicating that future prices are higher than current prices, it may be profitable to inject inventory into storage and sell it at a later date when prices are higher. In this case, the trigger price for injection would be lower than the current market price. Conversely, if the futures curve is downward-sloping, indicating that future prices are lower than current prices, it may be profitable to withdraw inventory from storage and sell it in the current market. In this case, the trigger price for withdrawal would be higher than the current market price.

In addition to the shape of the futures curve, other factors such as storage costs, interest rates, and market volatility can also affect trigger prices. As such, it is important to regularly monitor and adjust trigger prices based on changing market conditions to ensure optimal storage valuation.

Chapter 4

Conclusions

This study has addressed the maize storage valuation and hedging problem, specifically exploring the implications of transitioning from an *intrinsic* to an *extrinsic* valuation and risk management method. The experimental framework developed for this purpose includes a novel model, an adaptation of Hénaff et al. (2018)'s model, capturing the joint dynamics of the futures curve and spot prices, a back-testing engine for pricing storage units and evaluating various hedging strategies, and a method for quantifying model risk. The investigation involved extensive back-testing using 17 years of historical data on futures and spot prices. The numerical tests have confirmed the anticipated outcome: the *extrinsic* method, on average, extracts more value from storage units compared to the traditional *intrinsic* method. To assess the stability of valuation estimates in the face of model uncertainty, two model risk measures were introduced, drawing inspiration from the work of Cont (2006). It is noteworthy that the models used in this study were estimated based on historical data rather than market data, leading to a redefined concept of "benchmark data."

The results from the risk measures highlighted the substantial sensitivity of storage value to modeling assumptions. The size of the price range, reflecting model uncertainty, constitutes a significant proportion of the storage value. This underscores the need for more attention in the literature on discussing and scrutinizing modeling assumptions, rather than solely focusing on determining an optimal valuation strategy.

The term "*extrinsic*" used to characterize the valuation based on stochastic optimization and dynamic hedging is questioned in the context of this study. While borrowed from financial options theory, where *extrinsic value* can be extracted through a self-financed hedging strategy, the analogy falls short in the context of storage valuation due to the residual risk left by the dynamic hedging protocol. Consequently, the expected discounted cash flow computed by an *extrinsic* method should be regarded not as a "price" but as a market index, from which a market price could be derived, likely at a significant discount. This study suggests that the model risk measurement framework developed could be valuable, as it provides a distribution of possible values for storage units, acknowledging the inherent uncertainties in the valuation process.

In the Kenyan context, there is a need to set up a derivatives or futures market to act as a hedging tool for agricultural commodities. The idea of establishing a commodity exchange market is highly welcomed at this juncture. More work can be done in institutionalizing the commodity exchange market to create value for stakeholders.

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Appendix A

The Codes Run

The first set of codes for stylized facts

```
# Common Imports
#from cmdty_storage import CmdtyStorage, RatchetInterp
#import pandas as pd
#import numpy as np
#import matplotlib.pyplot as plt

## Common Imports
from cmdty_storage import CmdtyStorage, three_factor_seasonal_value,
multi_factor_value, RatchetInterp, value_from_sims
import pandas as pd
import numpy as np
# np.seterr(all='raise') # increasing numerical tolerance
import ipywidgets as ipw
from IPython.display import display
######
import matplotlib.pyplot as plt

# !pip install openpyxl seaborn
# installing relevant packages for reading excell documents (!pip
install openpyxl) and plotting (!pip install seaborn)

maize = pd.read_excel(r'C:\Users\46092439\Desktop\Storage\Data\dry-
maize-retail-prices_2006-2022.xlsx')
#print(maize)
maize
```

```
maize.info()

# maize.groupby(maize['Month']).sum('Price (KES)')
price_mean = maize.groupby(maize['Month'].dt.strftime('%b'), sort =
False)['Price (KES)'].mean()
price_mean.plot(kind ='bar', title = 'Average Monthly Price of Maize')
# average across months after converting dates to months
# .dt.strftime('%b') converts time to months and plot
plt.ylim(bottom=price_mean.min()-2)
plt.xlabel("Month")
plt.ylabel("Price (KES)")
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Average Monthly
Price of Maize.png") # saving image in png to a particular location

price_mean

month_yr = maize['Month'].dt.strftime('%y-%b')
maize.plot(title = 'Historical Monthly Prices of Maize (2006 - 2022)',
x = 'Month', y = 'Price (KES)')
plt.xlabel("Year")
plt.ylabel("Price (KES)")
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Historical
Monthly Prices of Maize (2006 - 2022).png")

# Identifying spikes/jumps
diffs = maize['Price (KES)'].diff() # Compute first differences
diffs

threshold = 2 * np.std(diffs) # Define a threshold for jump detection
(e.g., 2 standard deviations)

jump_locs = np.where(np.abs(diffs) > threshold)[0]
```

```
# Find jump locations

# Plot the data and highlight jump locations
Title = 'Jumps in Maize Price'
plt.plot(maize['Month'], maize['Price (KES)'], label='Price')
plt.plot(maize['Month'][jump_locs], maize['Price (KES)'][jump_locs],
'ro', label='Jumps')
plt.title(Title)
plt.legend()
plt.xlabel("Year")
plt.ylabel("Price (KES)")
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\{Title}.png")
# saving image in png file format to a particular location
#plt.show()

# forward/ futures curve
monthly_index = pd.period_range(start='2019-10-01', periods=25,
freq='M')
monthly_fwd_prices = [33.346875, 33.74125, 33.49625, 32.97294118,
33.54470588, 33.96529412, 35.56411765, 37.47823529, 39.01529412,
38.52882353, 37.01, 36.06941176, 33.346875, 33.74125, 33.49625,
32.97294118, 33.54470588, 33.96529412, 35.56411765, 37.47823529,
39.01529412, 38.52882353, 37.01, 36.06941176, 33.346875]
fwd_curve = pd.Series(data=monthly_fwd_prices,
index=monthly_index).resample('D').fillna('pad')
%matplotlib inline
fwd_curve.plot(title='Forward Curve of Maize Prices')
plt.xlabel("Month")
plt.ylabel("Price (KES)")
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Forward
Curve of Maize Prices.png")
```

```
# we need to make our decisions daily
from cmdty_storage import FREQ_TO_PERIOD_TYPE
FREQ_TO_PERIOD_TYPE

# delivery date is the 20th day of the following month
def settlement_rule(delivery_date):
    return delivery_date.asfreq('M').asfreq('D', 'end') + 20

# interest rate curve
rates = [0.085, 0.0825, 0.07, 0.07, 0.07, 0.07, 0.07, 0.07]
# random figures based on CBR from Central Bank of Kenya
(https://www.centralbank.go.ke/rates/central-bank-rate/)
rates_pillars = pd.PeriodIndex(freq='D', data=['2019-12-25',
'2020-02-01', '2020-05-01', '2020-06-01', '2020-07-01',
'2020-08-01', '2020-09-01', '2020-10-20'])
ir_curve = pd.Series(data=rates, index=rates_pillars).resample('D').
asfreq('D').interpolate(method='linear')
ir_curve.plot(title='Interest Rate Curve')
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Interest Rate Curve.png")
```

Spot & future price of maize (corn)

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from cmdty_storage import CmdtyStorage
from cmdty_storage import three_factor_seasonal_value

maize_df = pd.read_excel(
    r"C:\Users\46092439\Desktop\Storage\Data\dry-maize-retail-
    prices_2006-2022.xlsx"
```

```
)
rates_df = pd.read_csv(
    r"C:\Users\46092439\Desktop\Storage\Data\CBR-2006---2022.csv"
) #rates from CBK website

rates_df["Date"] = pd.to_datetime(rates_df["Date"], format="%d/%m/%Y")

rates_df.columns = ["date", "rate"]
maize_df.columns = ["month", "price"]

maize_df["day_idx"] = maize_df["month"].dt.to_period("D")
maize_df["month_idx"] = maize_df["month"].dt.to_period("M")
maize_df["year_idx"] = maize_df["month"].dt.to_period("Y")

patch_range = pd.date_range(start="2006-01-01", end="2008-08-01",
freq="M")
patch_rate = pd.DataFrame({"date": patch_range, "rate": [9.00
for _ in patch_range]})

rates_df = pd.concat([patch_rate, rates_df])

rates_df["date"] = pd.to_datetime(rates_df["date"].astype("string"))
rates_df["day_idx"] = rates_df["date"].dt.to_period("D")
rates_df["month_idx"] = rates_df["date"].dt.to_period("M")

fwd_curve = maize_df.groupby("month_idx")["price"].mean().
resample("D").fillna("pad")
ir_curve = (
     rates_df.groupby("month_idx")["rate"]
     .mean()
     .resample("D")
     .asfreq("D")
```

```
    .interpolate(method="linear")
)

print("monthly_avg_maize_price:\n", fwd_curve.sample(10), end="\n\n")
print("monthly_avg_interest:\n", ir_curve.sample(10))


maize_df.groupby("month_idx")["price"].mean().plot(title=
'Forward Curve of Maize Prices')
#plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Forward Curve of Maize Prices.png")

maize_df.groupby("month_idx")["price"].mean().plot(title=
'Spot and Futures Prices of Maize')
#plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Forward Curve of Maize Prices.png")
maize_df.groupby('year_idx')["price"].mean().plot()
plt.xlabel("Year")
plt.ylabel("Price (KES)")
plt.legend(['Spot Price','Futures Price'])
#plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Spot and Futures Prices of Maize.png")

ir_curve.plot(title='Interest Rate Curve')
plt.xlabel("Year")
plt.ylabel("Interest Rate (%)")
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Interest Rate Curve.png")

next_year = pd.DataFrame({"month": [pd.to_datetime("2023-12-31")]})
# forecasting
```

```
patched_year = pd.concat([maize_df[["month", "price"]], next_year])
patched_year["year_idx"] = patched_year["month"].dt.to_period("Y")


maize_df.groupby("month_idx")["price"].mean().resample("D").fillna(
"pad").plot()
(
    patched_year.groupby("year_idx")["price"]
    .mean()
    .resample("Y")
    .fillna("pad")
    .interpolate(method="linear")
    .plot(title='Spot and Futures Prices of Maize')
)
plt.xlabel("Year")
plt.ylabel("Price (KES)")
plt.legend(['Spot Price','Futures Price'])
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Spot and Futures
Prices of Maize.png")

# simulations
sim_spot_valuation_mean = three_factor_results.sim_spot_valuation.
aggregate(func='mean', axis='columns')
sim_spot_valuation_90th = three_factor_results.sim_spot_valuation.
aggregate(func=np.percentile, q=90, axis='columns')
sim_spot_valuation_10h = three_factor_results.sim_spot_valuation.
aggregate(func=np.percentile, q=10, axis='columns')

sim_spot_valuation_mean.plot(title = 'Statistics of Simulated
Spot Prices', legend=True)
fwd_curve['2021-04-25' : '2022-04-01'].plot(legend=True)
sim_spot_valuation_10h.plot(legend=True)
```

```
ax = sim_spot_valuation_90th.plot(legend=True)
ax.legend(['Mean', 'Forward Curve', '10th Percentile',
'90th Percentile'])
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\
Statistics of Simulated Spot Prices.png")

# 3 factors
sim_number=125
st_factor = three_factor_results.sim_factors_regress[0][sim_number]
lt_factor = three_factor_results.sim_factors_regress[1][sim_number]
seas_factor = three_factor_results.sim_factors_regress[2][sim_number]

st_factor.plot(title = 'Simulation of 3 Factors', legend=True)
lt_factor.plot(legend=True)
ax_factors = seas_factor.plot(legend=True)

ax_factors.legend(['Short-Term Factor', 'Long-Term Factor',
'Seasonal Factor'])
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Simulation of
3 Factors.png")

three_factor_results.expected_profile
```

Kenyan data findings

```
import pandas as pd
import numpy as np

from cmdty_storage import CmdtyStorage, RatchetInterp
from cmdty_storage import value_from_sims

val_date = '2022-01-25'
inventory = 10000.0/2
```

```
discount_deltas = True

num_sims = 500
seed = 12
fwd_sim_seed = 25

storage = CmdtyStorage(
    freq='D',
    storage_start = '2021-01-01',
    storage_end = '2022-12-31',
    injection_cost = 0.1,
    withdrawal_cost = 0.25,
    ratchets = [
            ('2021-01-01', # For days after 2021-04-01 (inclusive)
            until 2022-10-01 (exclusive):
                [
                    (0.0, -150.0, 250.0),
                    # At min inventory of zero, max withdrawal
                    of 150, max injection 250
                    (2000.0, -200.0, 175.0),
                    # At inventory of 2000, max withdrawal
                    of 200, max injection 175
                    (5000.0, -260.0, 155.0),
                    # At inventory of 5000, max withdrawal
                    of 260, max injection 155
                    # (10000.0, -275.0, 132.0),
                    # At max inventory of 7000, max withdrawal
                    of 275, max injection 132
                ]),
            ('2022-01-01', # For days after 2022-10-01
            (inclusive):
                [
```

```
                                (0.0, -130.0, 260.0),
                                # At min inventory of zero, max withdrawal
                                of 130, max injection 260
                                (2000.0, -190.0, 190.0),
                                # At inventory of 2000, max withdrawal
                                of 190, max injection 190
                                (5000.0, -230.0, 165.0),
                                # At inventory of 5000, max withdrawal
                                of 230, max injection 165
                                # (10000.0, -245.0, 148.0),
                                # At max inventory of 7000, max withdrawal
                                of 245, max injection 148
                            ]),
                    ],
    ratchet_interp = RatchetInterp.LINEAR
)

def settlement_rule(delivery_date):
    return delivery_date.asfreq('M').asfreq('D', 'end') + 20

maize_df = pd.read_excel(
    r"C:\Users\46092439\Desktop\Storage\Data\dry-maize-retail-
    prices_2006-2022.xlsx"
)
rates_df = pd.read_csv(
   r"C:\Users\46092439\Desktop\Storage\Data\CBR 2006 - 2022.csv"
)


rates_df["Date"] = pd.to_datetime(rates_df["Date"], format="%d/%m/%Y")


rates_df.columns = ["date", "rate"]
```

```
maize_df.columns = ["month", "price"]

maize_df["day_idx"] = maize_df["month"].dt.to_period("D")
maize_df["month_idx"] = maize_df["month"].dt.to_period("M")

patch_range = pd.date_range(start="2006-01-01", end="2008-08-01",
freq="M")
patch_rate = pd.DataFrame({"date": patch_range, "rate": [9.00 for _ in
patch_range]})

rates_df = pd.concat([patch_rate, rates_df])

rates_df["date"] = pd.to_datetime(rates_df["date"].astype("string"))
rates_df["day_idx"] = rates_df["date"].dt.to_period("D")
rates_df["month_idx"] = rates_df["date"].dt.to_period("M")

# fwd_curve = maize_df.groupby("month_idx")
["price"].mean().resample("D").fillna("pad")
fwd_curve = maize_df.groupby("month_idx")
["price"].mean().resample("D").asfreq('D').interpolate(method='linear')
x = (rates_df['rate']).groupby(rates_df['date'].dt.year).mean().
round(2).to_list()
y = pd.period_range(start='2006-12-31', end='2023-12-31', freq='Y')
irc = pd.Series(data=[*x, 7.45],
index=y).resample('D').asfreq('D').interpolate(method='linear')

irc.plot()

# 2-Factor model parameters
lt_vol_2f = 0.2
st_vol_2f = 0.6
mr_2f = 0.3
```

```python
start_sim = pd.Period(storage.start, freq='D')
sim_periods = pd.period_range(start=start_sim, end=storage.end)
num_sim_periods = len(sim_periods)

sqrt_dt = np.sqrt(1/365.0)
lt_step_stan_dev = lt_vol_2f * sqrt_dt
def generate_2f_sims():
    norms_factor_1 = np.random.normal(loc=0.0, scale=lt_step_stan_dev,
    size=(num_sim_periods, num_sims))
    factor_1_sims_array = np.cumsum(norms_factor_1, axis=0)
    factor_1_sims_data_frame = pd.DataFrame(data=factor_1_sims_array,
    index=sim_periods)
    st_step_stan_dev = st_vol_2f * sqrt_dt
    norms_factor_2 = np.random.normal(loc=0.0, scale=st_step_stan_dev,
    size=(num_sim_periods, num_sims))
    factor_2_sims_array = np.zeros(shape=(num_sim_periods, num_sims))
    factor_2_sims_array[0] = norms_factor_2[0]
    for i in range(1, num_sim_periods):
        factor_2_sims_array[i] = factor_2_sims_array[i-1]*(1.0 - mr_2f)
        + norms_factor_2[i]
    factor_2_sims_data_frame = pd.DataFrame(data=factor_2_sims_array,
    index=sim_periods)
    # No drift adjustment despite exponential, so not fitted to curve,
    and of no use in practice
    sims_spot_prices_data = np.matmul(np.diag(fwd_curve[sim_periods]
    .values), np.exp(factor_1_sims_array + factor_2_sims_array))
    sims_spot_prices_data_frame = pd.DataFrame(data=
    sims_spot_prices_data, index=sim_periods)
    return sims_spot_prices_data_frame, (factor_1_sims_data_frame,
    factor_2_sims_data_frame)
```

```
sim_spot_regress, sim_factors_regress = generate_2f_sims()
sim_spot_valuation, sim_factors_valuation = generate_2f_sims()
two_factor_basis_funcs = '1 + x0 + x1 + s + x0**2
+ x1**2 + s**2 + x0*x1 + x1*s + x0*s'

value_from_sims_results = value_from_sims(
    cmdty_storage = storage,
    val_date = val_date,
    inventory = inventory,
    fwd_curve = fwd_curve,
    interest_rates = irc,
    settlement_rule = settlement_rule,
    discount_deltas=discount_deltas,
    sim_spot_regress = sim_spot_regress,
    sim_factors_regress = sim_factors_regress,
    sim_spot_valuation = sim_spot_valuation,
    sim_factors_valuation = sim_factors_valuation,
    basis_funcs = two_factor_basis_funcs
)
'{0:,.0f}'.format(value_from_sims_results.npv)

print("Full NPV:\t{0:,.0f}".format(value_from_sims_results.npv))
print("Intrinsic NPV:
\t{0:,.0f}".format(value_from_sims_results.intrinsic_npv))
print("Extrinsic NPV:
\t{0:,.0f}".format(value_from_sims_results.extrinsic_npv))

%matplotlib inline
ax_deltas = value_from_sims_results.deltas.plot(title='Daily Deltas vs
Projected Inventory', legend=True, label='Delta')
ax_deltas.set_ylabel('Delta')
inventory_projection =
```

```
value_from_sims_results.expected_profile['inventory']
ax_inventory = inventory_projection.plot(secondary_y=True, legend=True,
ax=ax_deltas, label='Expected Inventory')
h1, l1 = ax_deltas.get_legend_handles_labels()
h2, l2 = ax_inventory.get_legend_handles_labels()
ax_inventory.set_ylabel('Inventory')
ax_deltas.legend(h1+h2, l1+l2, loc=1)

ax_triggers =
value_from_sims_results.trigger_prices['inject_trigger_price'].plot(
    title='Trigger Prices vs Forward Curve', legend=True, color=['r'])
value_from_sims_results.trigger_prices['withdraw_trigger_price'].
plot(legend=True)
fwd_curve['2022-01-25' : '2022-12-31'].plot(legend=True)
ax_triggers.legend(['Inject Trigger Price', 'Withdraw Trigger',
'Forward Curve'])

rates = [0.005, 0.006, 0.0072, 0.0087, 0.0101, 0.0115, 0.0126]
rates_pillars = pd.PeriodIndex(freq='D', data=['2021-04-25',
'2021-06-01', '2021-08-01', '2021-12-01', '2022-04-01',
'2022-12-01', '2023-12-01'])
ir_curve = pd.Series(data=rates,
index=rates_pillars).resample('D').asfreq('D').
interpolate(method='linear')
ir_curve.plot(title='Interest Rate Curve')
#ir_curve.to_csv("Interest Rate Curve.csv#)

val_date = '2022-01-25'
inventory = 10000.0/2
discount_deltas = True

num_sims = 500
```

```
seed = 12
fwd_sim_seed = 25

storage = CmdtyStorage(
    freq='D',
    storage_start = '2021-01-01',
    storage_end = '2022-12-31',
    injection_cost = 0.1,
    withdrawal_cost = 0.25,
    ratchets = [
                ('2021-01-01', # For days after 2021-04-01 (inclusive)
                until 2022-10-01 (exclusive):
                    [
                        (0.0, -150.0, 250.0),
                        # At min inventory of zero, max withdrawal of
                        150, max injection 250
                        (2000.0, -200.0, 175.0),
                        # At inventory of 2000, max withdrawal of 200,
                        max injection 175
                        (5000.0, -260.0, 155.0),
                        # At inventory of 5000, max withdrawal of 260,
                        max injection 155
                        # (10000.0, -275.0, 132.0),
                        # At max inventory of 7000, max withdrawal of
                        275, max injection 132
                    ]),
                ('2022-01-01', # For days after 2022-10-01 (inclusive):
                    [
                        (0.0, -130.0, 260.0),
                        # At min inventory of zero, max withdrawal of
                        130, max injection 260
                        (2000.0, -190.0, 190.0),
```

```
                    # At inventory of 2000, max withdrawal of 190,
                    max injection 190
                    (5000.0, -230.0, 165.0),
                    # At inventory of 5000, max withdrawal of 230,
                    max injection 165
                    # (10000.0, -245.0, 148.0),
                    # At max inventory of 7000, max withdrawal of
                    245, max injection 148
                ]),
            ],
    ratchet_interp = RatchetInterp.LINEAR
)

def settlement_rule(delivery_date):
    return delivery_date.asfreq('M').asfreq('D', 'end') + 20

# 2-Factor model parameters
lt_vol_2f = 0.2
st_vol_2f = 0.6
mr_2f = 0.3

start_sim = pd.Period(storage.start, freq='D')
sim_periods = pd.period_range(start=start_sim, end=storage.end)
num_sim_periods = len(sim_periods)

sqrt_dt = np.sqrt(1/365.0)
lt_step_stan_dev = lt_vol_2f * sqrt_dt
def generate_2f_sims():
    norms_factor_1 = np.random.normal(loc=0.0, scale=lt_step_stan_dev,
    size=(num_sim_periods, num_sims))
    factor_1_sims_array = np.cumsum(norms_factor_1, axis=0)
    factor_1_sims_data_frame = pd.DataFrame(data=factor_1_sims_array,
```

```
    index=sim_periods)
    st_step_stan_dev = st_vol_2f * sqrt_dt
    norms_factor_2 = np.random.normal(loc=0.0, scale=st_step_stan_dev,
    size=(num_sim_periods, num_sims))
    factor_2_sims_array = np.zeros(shape=(num_sim_periods, num_sims))
    factor_2_sims_array[0] = norms_factor_2[0]
    for i in range(1, num_sim_periods):
        factor_2_sims_array[i] = factor_2_sims_array[i-1]*(1.0 - mr_2f)
        + norms_factor_2[i]
    factor_2_sims_data_frame = pd.DataFrame(data=factor_2_sims_array,
    index=sim_periods)
    # No drift adjustment despite exponential, so not fitted to curve,
    and of no use in practice
    sims_spot_prices_data =
    np.matmul(np.diag(fwd_curve[sim_periods].values),
    np.exp(factor_1_sims_array + factor_2_sims_array))
    sims_spot_prices_data_frame =
    pd.DataFrame(data=sims_spot_prices_data, index=sim_periods)
    return sims_spot_prices_data_frame, (factor_1_sims_data_frame,
    factor_2_sims_data_frame)

sim_spot_regress, sim_factors_regress = generate_2f_sims()
sim_spot_valuation, sim_factors_valuation = generate_2f_sims()
two_factor_basis_funcs = '1 + x0 + x1 + s + x0**2 + x1**2 + s**2 +
x0*x1 + x1*s + x0*s'

value_from_sims_results = value_from_sims(
    cmdty_storage = storage,
    val_date = val_date,
    inventory = inventory,
    fwd_curve = fwd_curve,
    interest_rates = ir_curve,
```

```
    settlement_rule = settlement_rule,
    discount_deltas=discount_deltas,
    sim_spot_regress = sim_spot_regress,
    sim_factors_regress = sim_factors_regress,
    sim_spot_valuation = sim_spot_valuation,
    sim_factors_valuation = sim_factors_valuation,
    basis_funcs = two_factor_basis_funcs
)
'{0:,.0f}'.format(value_from_sims_results.npv)

print("Full NPV:\t{0:,.0f}".format(value_from_sims_results.npv))
print("Intrinsic NPV:
\t{0:,.0f}".format(value_from_sims_results.intrinsic_npv))
print("Extrinsic NPV:
\t{0:,.0f}".format(value_from_sims_results.extrinsic_npv))

ax_deltas = value_from_sims_results.deltas.plot(title='Daily Deltas vs
Projected Inventory', legend=True, label='Delta')
ax_deltas.set_ylabel('Delta')
inventory_projection =
value_from_sims_results.expected_profile['inventory']
ax_inventory = inventory_projection.plot(secondary_y=True, legend=True,
ax=ax_deltas, label='Expected Inventory')
h1, l1 = ax_deltas.get_legend_handles_labels()
h2, l2 = ax_inventory.get_legend_handles_labels()
ax_inventory.set_ylabel('Inventory')
ax_deltas.legend(h1+h2, l1+l2, loc=1)
import matplotlib.pyplot as plt
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Daily Deltas vs
Projected Inventory.png")

ax_triggers = value_from_sims_results.trigger_prices[
```

```
'inject_trigger_price'].plot(
    title='Trigger Prices vs Forward Curve', legend=True, color=['r'])
value_from_sims_results.trigger_prices['withdraw_trigger_price'].plot(
legend=True)
fwd_curve['2022-01-25' : '2022-12-31'].plot(legend=True)
ax_triggers.legend(['Inject Trigger Price', 'Withdraw Trigger',
'Forward Curve'])
import matplotlib.pyplot as plt
plt.savefig(fr"C:\Users\46092439\Desktop\Storage\Plots\Trigger Prices
vs Forward Curve.png")
```

Simulated data findings

```
import pandas as pd
import numpy as np

from cmdty_storage import CmdtyStorage, RatchetInterp
from cmdty_storage import value_from_sims

maize_df = pd.read_excel(
    r"C:\Users\46092439\Desktop\Storage\Data\dry-maize-retail-
    prices_2006-2022.xlsx"
)

maize_df.columns = ["month", "price"]

maize_df["day_idx"] = maize_df["month"].dt.to_period("D")
maize_df["month_idx"] = maize_df["month"].dt.to_period("M")

patch_range = pd.date_range(start="2006-01-01", end="2008-08-01",
freq="M")
patch_rate = pd.DataFrame({"date": patch_range, "rate": [9.00 for _ in
patch_range]})
```

```
fwd_curve = maize_df.groupby("month_idx")
["price"].mean().resample("D").asfreq('D').interpolate(method='linear')
fwd_curve
```

Appendix B

Turnitin Report & Ethics Clearance

We checked the similarity score for this thesis in Turnitin and found it was 24%. Likewise, we sought ethical clearance from the Ethics Department under the Graduate School since we used secondary data from public websites like CBK and KNBS.

B.1 Turnitin Report

MSc MathFin.pdf

ORIGINALITY REPORT

24%	20%	22%	1%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.econstor.eu Internet Source	15%
2	Patrick Hénaff, Ismail Laachir, Francesco Russo. "Gas Storage Valuation and Hedging: A Quantification of Model Risk", International Journal of Financial Studies, 2018 Publication	3%
3	tel.archives-ouvertes.fr Internet Source	2%
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6	Anabelle Couleau, Teresa Serra, Philip Garcia. "Are Corn Futures Prices Getting "Jumpy"?", American Journal of Agricultural Economics, 2020 Publication	1%
7	www2.mdpi.com Internet Source	1%

B.2 Ethics Clearance

Strathmore UNIVERSITY

24th April 2024

Gitonga Njoroge Simon

113478

njoroge.gitonga@strathmore.edu

Dear Simon,

RE: Valuation of Maize Storage Facility

This is to inform you that the Office of Graduate Studies on 24th April 2024 received your acknowledgement of breach in ethical processes given that you have already collected data and proceeded to write the Thesis prior to obtaining Ethical clearance. The ethics approval process is ONLY done before any collection of primary or secondary data.

This is a letter for you to proceed with the next steps of your academic requirements.

Please be advised, that in future, all research proposals should be submitted to the SU-ISERC through the RHInnO Ethics platform: https://strathmoreuniversity.rhinno.net/login

Disclaimer: *1) This is not in any way an ethical approval letter. 2) Should there be any legal implications/actions emanating from the research in terms of any ethical violations, you will be personally liable.*

Yours sincerely,

Dr. Bernard Shibwabo
Director of Graduate Studies

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