Strathmore
UNIVERSITY

School of Computing and Engineering Sciences
Bachelor of Informatics and Computer Science (ICS).
Bachelor of Science in Computer Networks and Cyber Security
End of Semester Examination
ICS 1205 - Linear Algebra
CNS 1205 - Linear Algebra

Date: $Monday, 21^{st} March\ 2022$ | Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

a) Are the following statements true or false?

i. A linear system of four equations in three unknowns is always inconsistent. [1 Mark]

ii. A linear system with fewer equations than unknowns must have infinitely many solutions. [1 Mark]

iii. If the system **AX=B** has a unique solution, then A must be a square matrix. [1 Mark]

b) The product of two eigenvalues of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue. [4 Marks]

c) Find the rational number t for which the following system is consistent and solve the system for this value of t. [4 Marks]

$$\begin{aligned} x + y &= 2 \\ x - y &= 0 \\ 3x - y &= t \end{aligned}$$

d) Show that $H = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ satisfies the Cayley-Hamilton theorem.
. [3 Marks]

e) Find the eigenvectors of

$$J = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

[6 Marks]

f) Suppose A is a 2×2 matrix with eigenvalues -1 and 3. Then for each positive integer n, find a_n and b_n such that $A^{n+1} = a_n A + b_n I$ where I is the corresponding identity matrix. [5 Marks]

g) Differentiate between Gaussian elimination and Gauss-Jordan elimination. [2 Marks]

h) Give the geometric interpretation of the following systems; system with one solution, system with no solution and a system with many solutions.
. [3 Marks]

QUESTION TWO [20 Marks]

a) Solve the following system by using the Gauss-Jordan elimination method;

$$\begin{aligned} z + 4y &= 2 \\ 2x + 6y - 2z &= 3 \\ 8y + 4x - 5z - 4 &= 0 \end{aligned}$$

[6 marks]

b) Solve by Gaussian elimination:

$$\begin{aligned} x + 2y &= 2 + 3z \\ -9z + 6x &= -3y + 6 \\ 7x + 14y - 21z &= 13 \end{aligned}$$

[4 Marks]

c) For what values of a and b does the system below have
(i) No solution?
(ii) Many solutions?
(iii) A unique solution?

$$\begin{aligned} x - 2y + 3z &= 4 \\ 2x - 3y + az &= 5 \\ 3x - 4y + 5z &= b \end{aligned}$$

[10 Marks]

QUESTION THREE [20 Marks]

a) Solve for x and y using the Crammers rule given that $ax + by = E$ and $cx + dy = F$ given that $a, b, c, d, E, F \in \mathbb{R}$. [5 Marks]

b) Given $M = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find M^4 and M^{-1} [10 Marks]

c) If 1 and 2 are the eigenvalues of a 2×2 matrix A, what are the eigen values of A^2 and A^{-1}. [5 Marks]

QUESTION FOUR [20 Marks]

a) Diagonalize the matrix $L = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ [7 Marks]

b) From (a) above, give the three possible choices of P and D that will give the result $LP = PD$ where P is the matrix formed from the eigenvectors and D is a diagonal matrix. [3 Marks]

c) Use Crammer's rule to solve for the unknowns in

$$\begin{aligned} 5a - 2b + 3c &= 16 \\ 2a + 3b - 5c &= 2 \\ 4a - 5b + 6c &= 7 \end{aligned}$$

[6 Marks]

d) Using the Cayley-Hamilton theorem find M^4 given that $M = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$
. [4 Marks]

QUESTION FIVE [20 Marks]

a) Use the adjoint method to find the inverse of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -8 \\ -3 & -5 & 8 \end{bmatrix}$ and prove your answer. [10 Marks]

b) Use row operations to find the inverse of $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ and verify your answer. [10 Marks]