STRATHMORE UNIVERSITY BUSINESS SCHOOL
BACHELOR OF SUPPLY CHAIN AND OPERATIONS MANAGEMENT
END OF SEMESTER EXAMINATION
SCM 4105: OPERATIONS RESEARCH

Date: 9th December; 2022 **Time:** 2 Hours

INSTRUCTIONS:

1. This examination consists of **FIVE** questions.
2. Answer question **ONE COMPULSORY** and **ANY** other **TWO** questions
3. Show all your workings clearly in the answer sheet

QUESTION ONE – 30 MARKS (COMPULSORY)

i. Read the Questions **(1 - 7)** keenly, before selecting the appropriate option(s). Do not re-write the question. **[7 marks]**

1. A feasible solution to a linear programming problem

a. Must satisfy all the constraints of the problem simultaneously
b. Need not satisfy all the constraints, only some of them.
c. Must be a corner point of the feasible region.
d. Must optimize the value of the objective.

2. If any coefficient in the indicator row of the simplex tableau is negative, then the solution is _____.

a. Infeasible
b. Feasible
c. Bounded
d. No solution

3. The difference between total and head event slack is _____

a. Free float
b. Independent float
c. Interference float
d. Linear float

4. An optimal assignment requires that the maximum number of lines which can be drawn through squares with zero opportunity cost should be equal to the number of __.

a. Rows or columns
b. Rows and columns
c. Rows +columns-1
d. Rows-columns

5. Corner points of a feasible region are located at the intersections of the region and coordinate axes.

a. True
b. False

6. A finite optimal solution can be not unique.

a. True
b. False

7. Technical coefficients may or may not be greater than 1.

a. True
b. False

ii. The feasible region of a linear programming problem has extreme points: A(0,0), B(1,1), C(0,1), and D(1,0). Identify an optimal solution for the objective function $Minimize\ z = 2x - 2y$ **[2 Marks)**

iii. Consider a 2-sector economy. For 1-unit final output of sector A, it requires 0.2 units from itself and 0.05 units from sector B. For 1-unit final output of sector B, it requires 0.15 units from itself and 0.09 units from sector A. determine the Leontief inverse matrix for this economy. **[4 Marks]**

iv. Consider the following transportation problem:

	1	2	3	4	Supply
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Demand	6	10	12	15	43

Using LCM, obtain the starting solution to solve the problem. **[4 Marks]**

v. A company has three factories and three distribution centres for a particular product. If the capacities of each factory, the requirements of each centre and the costs of transporting an item from any factory to any centre are known.

	Centres			
	1	2	3	Availability
D	55	30	40	40
E	35	30	100	20
F	40	60	95	40
	50	20	30	100

Required

Form a Linear Optimization Model **[3 Marks]**

vi. A small toy manufacturing firm has 200 square of felt, 600 ounces of stuffing and 90 feet of trim available to make two types of toys, a small bear and a monkey. The bear requires 1 square of felt and 4 ounces of staffing. The monkey requires 2 squares of felt, 3 ounces of stuffing, and 1 foot of trim. The firm makes $1 profit on each bear and $1.50 profit on each monkey. The linear program to maximize profit is

Maximize $z = x_1 + 1.5x_2$

Subject to:
$$x_1 + 2x_2 \leq 200$$
$$4x_1 + 3x_2 \leq 600$$
$$x_2 \leq 90$$
$$x_1,\ x_2 \geq 0$$

What is the corresponding dual problem? **[3 Marks]**

vii. Three machine shops A, B, C produce three types of products X, Y, Z respectively. Each product involves operation of each of the machine shops. The time required for each operation on various products is given as follows:

	Machine Shops			Profit per unit
Products	A	B	C	
X	10	7	2	\$12
Y	2	3	4	\$3
Z	1	2	1	\$1
Available Hours	100	77	80	

Formulate the LOM and set up the first TWO simplex tableaus that would be use in finding the optimal solution **[8 Marks]**

QUESTION TWO – 20 MARKS (OPTIONAL)

a. Define the following terms: **[6 Marks]**

i. Objective function
ii. Model
iii. Constraints
iv. State matrix
v. Open economic system
vi. Dummy activity

b. Suppose that in a certain city the probability that a woman with less than a high school education has a daughter with less than a high school education, a high school education, or more than a high school education is $0.2, 0.6$ and 0.2 respectively. The probability that a woman with a high school education has a daughter with less than a high school education, a high school education, or more than a high school education is $0.1, 0.5$ and 0.4 respectively. The probability that a woman with more than a high school education has a daughter with less than a high school education, a high school education, or more than a high school education is $0.1, 0.1$ and 0.8 respectively. If the population of women in the city is now 60% with less than high school education, 30% with high school education, and 10% with more than high school education.

i. What is the transition matrix for this information? **[2 Marks]**
ii. What is the likely distribution of women according to education level two generations from now? **[4 Marks]**
iii. What will be the steady state for this city? **[8 Marks]**

QUESTION THREE – 20 MARKS (OPTIONAL)

A food producer uses two processing plants P_1 and P_2 that operates 7 days a week. After processing, beef is graded into high-, medium-, and low-quality foodstuffs. High-quality beef is sold to butchers, medium-quality beef is used in super-market ready-meals and low-quality beef is used in dog food. The producer has been contracted to provide $120kg$ of high-, $80kg$ of medium- and $240kg$ of low-quality beef each week.

It costs \$4000 per day to run plant P_1 and \$3200 per day to run plant P_2. Each day P_1 processes $60kg$ of high-quality beef, $20kg$ of medium-quality beef and $40kg$ of low-quality beef. The corresponding quantities for P_2 are $20kg, 20kg$ and $120kg$, respectiveely. How many days each week should the plants be operated to fulfill the beef contract most economically? **[20 Marks]**

QUESTION FOUR – 20 MARKS (OPTIONAL)

a. To stimulate interest and provide an atmosphere for intellectual discussion, the faculty of Strathmore Institute of Mathematical Sciences decides to hold special seminars on four contemporary topics – Statistics, Linear programming, Discrete mathematics, Matrices. Each such seminar is to be held once a week. However, scheduling these seminars (one topic and not more than one seminar per day) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Statistics	Linear programming	Discrete mathematics	Matrices
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

Find an optimal schedule for the seminars, hence the number of students who will be missing at least one seminar **[7 Marks]**

b. The linear optimization mode for a company having factories F_1, F_2 and F_3 that supply products to warehouses at W_1, W_2 and W_3 is given as below (shipping costs in millions of Kshs):

Minimize $x_0 = 16x_{11} + 20x_{12} + 12x_{13} + 14x_{21} + 8x_{22} + 18x_{23} + 26x_{31} + 24x_{32} + 16x_{33}$

Subject to :
$$x_{11} + x_{12} + x_{13} = 200$$
$$x_{21} + x_{22} + x_{23} = 160$$
$$x_{31} + x_{32} + x_{33} = 90$$
$$x_{11} + x_{21} + x_{31} = 180$$
$$x_{12} + x_{22} + x_{32} = 120$$
$$x_{13} + x_{23} + x_{33} = 150$$
$$x_{ij} \geq 0 \quad i, j = 1,\ 2,\ 3$$

Use North-West corner rule to determine the initial transportation cost for the company, hence find the optimal distribution for this company in order to minimize its total shipping cost. **[13 Marks]**

QUESTION FIVE – 20 MARKS (OPTIONAL)

a. The following details are available regarding a project:

Activity	Predecessor activity	Duration (weeks)
A	–	3
B	A	5
C	A	7
D	B	10
E	C	5
F	D, E	4

Determine the critical path using ESTs and LSTs. **[5 Marks]**

b. Determine, using PERT, the duration and standard deviation for the project with the below time estimates in weeks: **[15 Marks]**

Activity	t_0	t_m	t_p
$1-2$	3	6	9
$1-6$	2	5	8
$2-3$	6	12	18
$2-4$	4	5	6
$3-5$	8	11	14
$4-5$	3	7	11
$6-7$	3	9	15
$5-8$	2	4	6
$7-8$	8	16	18