On the Banach algebra numerical range

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Let A denote a unital Banach algebra and SA denote its unit sphere. It was proved by F. F. Bonsall and J. Duncan that when the Banach algebra is unital, the algebra numerical range is identical to a subset of itself. Basing on this fact, we set out to investigate the relationship between the set of support functional for the unit ball (used construct the entire numerical range) and the normalized states (used to construct the identical subset of the numerical range). Indeed we have established that the union over all elements of SAof the sets of support functionals for the unit ball at each x ∈ SA i.e. Sx∈S D(A, x) is equal to the set ofe), when the Banach algebra is unital (ˆ is the unit element). The implicationis that even when the algebra is smooth, the set of normalized states i.e. D(, e) is not a singleton. Hence algebra numerical range is not singleton except when the element is a scalar mˆ le of the unit. Further, when defining smoothness for a unital Banach algebra, the unit element of the Banach algebra should beexcluded in the definition because the set of normalized states i.e. D(, e) may not be a singleton. Further, we established conditions under which statements P and P are equiv ˆ t;(P1) The union over all elements of SA of the sets of support functionals for the unit ball at each x ∈ SAi.e. Sx∈S D(A, x) is equal to the set of normalized states i.e. D(A, ecomponent algebra numerical range i.e.(P2) The algebra numerical range i.e. V (A, a) is equal to the ˆV (A, a, ˆ1)