



Strathmore Institute of Mathematical Sciences (SIMS)
Special Examination for the Degree of Bachelor of Business
Science in Financial Economics/Financial Engineering
BSF 4130: Foundations of Asset Pricing

DATE: 27th March, 2024

Time: 2 Hours

Instructions

- **This examination consists of FIVE questions.**
 - **Answer Question ONE (COMPULSORY) and any other TWO questions.**
1. (a) Provide a formal definition of risk aversion **(2 Marks)**
- (b) Assume the utility function $u(w) = 11w - 5w^2$, with $w_0 = \$1$ million, $r_f = 0$, $E[\tilde{r}] = 0.1$, and $\sigma_{\tilde{r}}^2 = 0.2^2$. Find the optimal amount invested in the risky asset assuming that the investor is risk-averse. **(6 Marks)**
- (c) Consider the utility function of the form
- $$u(w) = -\frac{1}{v}e^{-vw}$$
- Derive the absolute risk-averseion coefficient for this utility function. **(4 Marks)**
- (d) Provide an intuitive explanation of the following concepts as used in asset pricing literature.
- (i) Pricing kernel **(4 Marks)**
 - (ii) Complete Markets **(2 Marks)**
- (e) The Arbitrage Pricing Theory (APT) is agnostic about what factors should determine asset returns. However, factor identification has become an empirical issue. Describe three approaches often used in factor identification. **(6 Marks)**
- (f) "*Inference about the risk premia is equivalent to inference about the stochastic discount factor (SDF)*". Briefly explain if you agree with this statement (Hint: Mathematically show the equivalence). **(6 Marks)**
2. (a) Consider an investor with initial income equal to $Y = 10$, who must choose between two assets. The first is a risky asset, let's call it a "stock", that pays off $d^G = 3$ in a good state that occurs with probability $\pi = 1/2$ and pays off

$d^B = 0$ (nothing) in a bad state that occurs with probability $1 - \pi = 1/2$. The second is a safe asset, let's call it a "bond," that pays of $r = 1$ no matter what, in both the good and bad states of the world. Thus, if the investor chooses the stock, his or her total income will be $Y + d^G = 13$ with probability $1/2$ and $Y + d^B = 10$ with probability $1/2$, whereas if the investor chooses the bond, his or her total income will be $Y + r = 11$ for sure. Assume that this investor has a von Neumann-Morgenstern expected utility function, with a Bernoulli utility function of the constant relative risk aversion (CRRA) form

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

- (i) Suppose first that the investor's constant coefficient of relative risk aversion is $\gamma = 1/2$. Which asset will he or she prefer: the stock or the bond? show your workings. **(4 Marks)**
 - (ii) Suppose instead that the investor's constant coefficient of relative risk aversion is $\gamma = 2$. Which asset will he or she prefer: the stock or the bond? **(3 Marks)**
- (b) Consider the same investor described in part (a) above, consider that the certainty equivalent $CE(\bar{d})$ for the stock is the maximum riskless payoff that the investor is willing to exchange for the stock and the risk premium for the stock $\Psi(\bar{d})$ is the difference between its expected payoff $E\bar{d}$ and its certainty equivalent.
- (i) Assuming that the investor's constant coefficient or relative risk aversion is $\gamma = 1/2$, what is the certainty equivalent $CE(\bar{d})$ for the stock? **(4 Marks)**
 - (ii) Assuming instead that the investor's constant coefficient or relative risk aversion is $\gamma = 2$, what is the certainty equivalent $CE(\bar{d})$ for the stock? **(4 Marks)**
 - (iii) What is the risk premium for the stock when $\gamma = 1/2$? what is the risk premium when $\gamma = 2$? Comment on your results. **(5 Marks)**
3. (a) The prediction test of the CAPM plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$. Based on these estimates one may estimate the simple linear regression equation

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \epsilon_i, i = 1, \dots, N \quad (1)$$

- (i) if the CAPM were true, what should the estimated values for γ_0 and γ_1 be? what are their interpretations in the context of CAPM? **4 Marks)**
- (ii) The least squares estimates of equation(1) are summarized below. Using these estimates, test the null hypothesis that the CAPM is true using a 5% significance level. Assume that the average excess return on the market index over the sampled period was 0.0031 **(6 Marks)**

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> summary(sml.fit)

Call: lm(formula = mu.hat ~ betas)
Residuals:
    Min       1Q   Median       3Q      Max
-0.0252 -0.0047  0.00112  0.00936  0.0142

Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept)  0.002   0.007      0.275   0.788
          Betas  0.001   0.009      0.143   0.888

Residual standard error: 0.0115 on 13 degrees of freedom
Multiple R-Squared: 0.00157

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(b) Suppose an expected utility maximizing individual invests her beginning of period wealth, W_0 , in a particular portfolio of assets. Let \tilde{R} be the random return on this portfolio, so that the individual's end-of-period wealth is $\tilde{W} = W_0\tilde{R}_p$. Note that \tilde{R}_p is defined as gross return. Denote this individual's end-of-period utility by $U(\tilde{W})$ and for simplicity express $U(\tilde{W}) = U(W_0\tilde{R}_p)$ as just $U(\tilde{R}_p)$. Provide a Taylor series expansion of $U(\tilde{R}_p)$ around the mean of \tilde{R}_p , ($E[\tilde{R}_p]$) and show that for any probability distribution of the portfolio return \tilde{R}_p , quadratic utility leads to expected utility that depends on the mean and variance of \tilde{R}_p **(10 Marks)**

4. Suppose a typical investor solves the following problem

$$\begin{aligned} \max_{\alpha} U(c_t, c_{t+1}) &= u(c_t) + \beta E_t[u(c_{t+1})] \\ \text{subject to } c_t &= e_t - \alpha p_t \text{ and } c_{t+1} = e_{t+1} + \alpha x_{t+1} \end{aligned} \quad (2)$$

where c_t and c_{t+1} denotes consumption at date t and $t + 1$ respectively. The other parameters in the model are; e_t (investor's endowment at period t and $t + 1$ respectively), p_t (asset price at time t), α (the number of units of the asset purchased by the investor), and x_{t+1} (payoff of the asset at period $t + 1$), β represents the time preference parameter of the investor ($0 \leq \beta \leq 1$). The general functional form $U(\cdot)$ represents the investor's utility function. An often convenient utility function used in many applications is the power utility $u(c_t) = \frac{1}{1 - \gamma} c^{1-\gamma}$

(i) Show that the price of an asset at any given time can be represented as follows: **(3 Marks)**

$$p_t = E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)} x_{t+1}\right]$$

(ii) Provide an intuitive interpretation of the term $\frac{\beta u'(c_{t+1})}{u'(c_t)}$ and briefly discuss how this term influences the asset price p_t **(3 Marks)**

(iii) Show that a simple manipulation of the price equation in (i) can yield the following equation (ignoring time subscripts)

$$1 = E(mR)$$

where R represents the gross return of the asset, while $m = \frac{\beta u'(c_{t+1})}{u'(c_t)}$ (2 Marks)

- (iv) Assuming the power utility function provided and the manipulations in parts (i)-(iii) above, one can show that the risk-free rate can be expressed as

$$R^f = \frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^\gamma$$

Intuitively discuss three factors that influence interest rates in an economy. (6 Marks)

- (v) Show that the basic pricing equation ($1 = E(mR)$) can be rewritten as follows;

$$E(R) = R_f - \frac{Cov(u'(c_{t+1}), R)}{E(u'(c_{t+1}))}$$

Interpret this equation. (6 Marks)

5. (a) Briefly explain the main advantage of the Arrow-Debreu pricing over the capital asset pricing model (3 Marks)
- (b) You are given the following information: Show how you can combine stock and

	Stock	Bond	up.A-D	down.A-D
upstate	1260	1050	1	0
Downstate	840	1050	0	1
Current Price	1050	1000		

bonds in such a way that you obtain an upstate and downstate pure security (7 Marks)

- (c) There are three possible states in the future, states 1,2,and 3, and the consumption at each state is denoted by $c_1, c_2,$ and c_3 . There are one million homogeneous consumers with expected utility function

$$q_1 u(c_1) + q_2 u(c_2) + q_3 u(c_3)$$

, where $(q_1, q_2, q_3) = (0.5, 0.3, 0.2)$, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 0.5$, and with endowment $(y_1, y_2, y_3) = (100, 81, 64)$ Treating the state 1 good as numeraire, find the general equilibrium prices for the state 2 good and for state 3 good. (10 Marks)

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