



Strathmore
UNIVERSITY

STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)
BBS FENG & BBS ACT & BBS FE
END OF SEMESTER EXAMINATION
BSM 2111: STATISTICAL INFERENCE

DATE: 29th July, 2022

TIME: 2 Hours

INSTRUCTIONS

1. This examination consists of **FIVE** questions.
 2. Answer Question **ONE (COMPULSORY)** and any other **TWO** questions.
 3. You may use a **SIMPLE CALCULATOR**. No **MOBILE PHONES** in the exams room.
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Question One (30 Marks)

- (i) A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1 - x)^2 & \text{for } x = -1, 0, 1, 2. \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{1}{6}$. (3 marks)
- (b) Find $E(X)$. (2 marks)
- (c) Show that $E(X^2) = \frac{4}{3}$. (2 marks)
- (ii) Define confidence interval. (2 marks)
- (iii) Explain three properties of maximum likelihood estimation. (3 marks)
- (iv) The following data have been collected for a sample from a normal population:

5, 10, 8, 11, 12, 6, 15, 13

What is the point estimate of the population mean and standard deviation? (5 marks)

- (v) One source of water pollution is gasoline leakage from underground storage tanks. In Mombasa, a random sample of $n = 74$ gasoline stations is selected and the tanks are inspected; 10 stations are found to have at least one leaking tank. Calculate a 95 percent confidence interval for p , the population proportion of gasoline stations with at least one leaking tank. (4 marks)
- (vi) It is estimated that a particular flight is profitable if the average occupation rate during a year is at least a 60%. An airline is interested in determining whether it is profitable to keep a particular

flight operative. For that, they record the occupation rates of 120 random flights scattered around the year, resulting a mean occupation rate of 58% and a standard deviation of 11%. Considering that the occupation rates (in proportion) have an approximate normal distribution, is there enough evidence to cancel the flight because it is not profitable? Employ a significance level of $\alpha = 0.10$. (4 marks)

- (vii) Suppose the samples are simple random samples taken from normal populations. Calculate the confidence intervals for the ratio of the two population variances and the ratio of standard deviations given the following information. (5 marks)

$$\alpha = 0.05; \quad n_1 = 30; \quad s_1 = 16.37; \quad n_2 = 39; \quad s_2 = 9.88.$$

Question Two (20 Marks)

- (i) Suppose X_1, \dots, X_n are iid from Cauchy $(\theta, 1)$, with the following density:

$$f(x_1, \dots, x_n | \theta) = \prod_{i=1}^n \frac{1}{1 + (x_i - \theta)^2}.$$

Obtain the MLE for θ . (5 marks)

- (ii) Let X_1, X_2, \dots, X_n be normal random variables with mean μ and variance σ^2 . What are the method of moments estimators of the mean μ and variance σ^2 ? (4 marks)
- (iii) Let X_1, X_2, \dots, X_n be a s.r.s from a distribution with mean μ and variance σ^2 . Consider the following estimators of μ :

$$\hat{\mu}_1 = \frac{X_1 + 2X_2 + 3X_3}{6}, \quad \hat{\mu}_2 = \frac{X_1 + 4X_2 + X_3}{6}, \quad \hat{\mu}_3 = \frac{\frac{3}{2}X_1 + \frac{1}{2}X_2 + X_3 + \dots + X_n}{n}.$$

- (a) Which ones are unbiased? (4 marks)
- (b) Among the unbiased ones, which is the most effective? (4 marks)
- (c) Which of them is consistent in squared mean? (3 marks)

Question Three (20 Marks)

- (i) The feeding habits of two species of net-casting spiders are studied. The species, the deinopis and menneus, coexist in eastern Australia. The following data were obtained on the size, in millimeters, of the prey of random samples of the two species:

dinopis	12.9	10.2	7.4	7.0	10.5	11.9	7.1	9.9	14.4	11.3
menneus	10.2	6.9	10.9	11.0	10.1	5.3	7.5	10.3	9.2	8.8

- (a) What is the difference, if any, in the mean size of the prey (of the entire populations) of the two species? (4 marks)
- (b) Estimate, with 95% confidence, the ratio of the two population variances. (4 marks)
- (ii) A gunpowder manufacturer developed a new formula that was tested in eight bullets. The resultant initial velocities, measured in feet per second, were

3005, 2925, 2935, 2968, 2995, 3005, 2937, 2905.

Assuming that the initial velocities have normal distribution with $\sigma = 39$ feet per second, Find a confidence interval at level $\alpha = 0.05$ for the initial mean velocity of the bullets that employ the new gunpowder. (3 marks)

- (iii) A random sample of 20 nominally measured 2mm diameter steel ball bearings is taken and the diameters are measured precisely. The measurements, in mm, are as follows:

2.02, 1.94, 2.09, 1.95, 1.98, 2.00, 2.03, 2.04, 2.08, 2.07
 1.99, 1.96, 1.99, 1.95, 1.99, 1.99, 2.03, 2.05, 2.01, 2.03.

Assuming that the diameters are normally distributed with unknown mean, μ , and unknown variance, σ^2 . find a two-sided 95% confidence interval for the variance, σ^2 . (4 marks)

- (iv) In a biomedical experiment, we would like to estimate the population mean remaining life μ of healthy rats that are given a certain dose of a toxic substance. Suppose that we would like to write a 95 percent confidence interval for μ with a margin of error equal to $E = 2$ days. From past studies, remaining rat lifetimes have been approximated by a normal distribution with standard deviation $\sigma = 8$ days. How many rats should we use for the experiment? (2 marks)

- (v) Acute exposure to cadmium produces respiratory distress and kidney and liver damage (and possibly death). For this reason, the level of airborne cadmium dust and cadmium oxide fume in the air, denoted by X (measured in milligrams of cadmium per m^3 of air), is closely monitored. A random sample of $n = 35$ measurements from a large factory are given below:

0.044, 0.030, 0.052, 0.044, 0.046, 0.020, 0.066, 0.052, 0.049, 0.030, 0.040,
 0.045, 0.039, 0.039, 0.039, 0.057, 0.050, 0.056, 0.061, 0.042, 0.055, 0.037,
 0.062, 0.062, 0.070, 0.061, 0.061, 0.058, 0.053, 0.060, 0.047, 0.051, 0.054,
 0.042, 0.051.

Find a 99 percent confidence interval for μ , the mean level of airborne cadmium. (3 marks)

Question Four (20 Marks)

- (i) If 8% of a particular canned product is known to be underweight, what is the probability that a random sample of 100 cans will contain

- (a) 14 or more underweight cans? (2 marks)
 (b) 4 or fewer underweight cans? (2 marks)
 (c) 5 or more underweight cans? (2 marks)
 (d) more than 4 but less than 15 underweight cans? (2 marks)

- (ii) Questions (a) and (b) are based on the following random sample of 12 data points from a population distribution : 7, 15, 15, 19, 26, 27, 29, 29, 30, 33, 38, 53.

- (a) Find both the biased and the unbiased sample variances. (4 marks)
 (b) Suppose that the distribution variance is 100.

- (i) Find the mean square error of the sample mean as an estimator of the distribution mean. (2 marks)
- (ii) Find the bias in the biased form of the sample variance as an estimator of the distribution (2 marks) variance.
- (iii) Compute the mean and standard deviation of the sampling distribution of the sample mean when you plan to take an SRS of size 64 from a population with mean 44 and standard deviation 16. (4 marks)

Question Five (20 Marks)

- (i) A manufacturer of chocolate topping uses machines to dispense liquid ingredients into bottles that move along a filling line. The machine that dispenses toppings is working properly when 8 grams are dispensed. The standard deviation of the process is 0.15 gram. A sample of 50 bottles is selected periodically and the filling line is stopped if there is evidence that the average amount dispensed is actually less than 8 grams. Suppose that the average amount dispensed in a particular sample of 50 bottles is 7.983 grams. At the 0.05 level of significance, using the critical value approach to hypothesis testing, is there evidence that the average amount dispensed is less than 8 grams? (5 marks)
- (ii) The head of the Statistics department is interested in the difference in writing scores between freshman Statistics students who are taught by different teachers. The incoming freshmen are randomly assigned to one of two Statistics teachers and are given a standardized writing test after the first semester. We take a sample of eight students from one class and nine from the other. Is there a difference in achievement on the writing test between the two classes? (5 marks)

Class1	35	51	66	42	37	46	60	55	53
Class2	52	87	76	62	81	71	55	67	

- (iii) In certain food experiment to compare two types of baby foods A and B, the following results of increase in weight (lbs) we observed in 8 children as follows.

Food A	49	53	51	52	47	50	52	53
Food B	52	55	52	53	50	54	54	53

Examine the significance of increase in weight of children due to food B. (5 marks)

- (iv) A university has found over the years that out of all the students who are offered admission, the proportion who accept is 0.70. After a new director of admissions is hired, the university wants to check if the proportion of students accepting has changed significantly. Suppose they offer admission to 1200 students and 888 accept. Is this evidence at the $\alpha = .05$ level that there has been a real change from the status quo? How about at the 0.02 level? (5 marks)

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