STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN STATISTICAL SCIENCE
MASTER OF SCIENCE IN MATHEMATICAL FINANCE & RISK ANALYTICS
END OF SEMESTER EXAMINATION

STA 8101: PROBABILITY THEORY/
MFI 8102: MEASURE & PROBABILITY THEORY

DATE: 9TH DECEMBER 2024 | Time: 3 Hours

INSTRUCTIONS
Answer QUESTION ONE and ANY OTHER TWO questions

Question ONE (30 Marks)

(a) Define the following set operations as defined in probability theory

i. Indicator function [2 marks]

ii. Measure space [2 marks]

iii. Cartesian product [2 marks]

(b) Define convergence in probability and give necessary and sufficient conditions for its existence [5 marks]

(c) Prove that if $X_n \xrightarrow{P} X$, then $X_n - X \xrightarrow{P} 0$. [5 marks]

(d) Show that $(\lim A_n)^c = \lim A_n^c$ [5 marks]

(e) State Fubini's and Fatou's theorems. [4 marks]

(f) Explain the statistical (limiting) concept of probability and give an illustration [5 marks]

Question TWO (15 Marks)

(a) State and prove the three axioms of a probability measure [6 marks]

(b) Let X_n and X be random variables, where $X_n \xrightarrow{P} X$. Let f be a real-valued continuous function. Prove that $f(X_n) \xrightarrow{P} f(X)$ [5 marks]

(c) State and prove Jensen's inequality [4 marks]

Question THREE (15 Marks)

(a) State the six properties of conditional expectations [6 marks]

(b) Let $X_1, X_2, \ldots$ be i.i.d. with mean μ and variance σ^2. Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Prove that

$$Z = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \xrightarrow{d} N(0,1) \text{ as } n \to \infty.$$

[9 marks]

Question FOUR (15 Marks)

(a) Prove that if A_n is an increasing sequence of events, then $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$ [6 marks]

(b) State and prove Markov's inequality. Hence, or otherwise, use your results to prove Chebyshev?s inequality [9 marks]