

# **Ensemble Learning for Stochastic Volatility Jump Diffusion Models**

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## **Abstract**

The pricing of securities is indispensable in ensuring the efficiency of financial markets. This is notably important in frontier markets where financial systems are evolving, and asset dynamics exhibit unique characteristics. Accurate pricing ensures market efficiency by enabling the accurate reflection of all available information in asset prices. Incorporation of salient features and characteristics of these assets requires the use of accurate models. Extensive research on these properties in frontier markets is imperative. With the increase in the financial instruments being introduced and developed in these markets, a deeper understanding of the underlying security is cardinal. This study applies a model that accurately describes these features in the Kenyan market. A stochastic volatility jump diffusion model is used to characterize the behaviour of market indices. The Bates model, an extension of the Heston model is applied to market index data from the Nairobi Securities Exchange. Estimation of the model parameters is done through Markov Chain Monte Carlo techniques. The model was not only able to capture stylized facts of stochastic volatility but also the discontinuous price movements by incorporating jumps. The calibration of the Bates model on this market brings to light the presence of jumps on the market indices. A Random Forest ensemble algorithm is then utilized in asset return prediction. The results demonstrate that this ensemble method yields a forecast with very low root mean squared errors. This forecasting technique is thereby able to track the return series closely. The predicted values are close to the actual values. The method is shown to be accurate in the prediction of these returns.

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## **List of Abbreviations**

<b>ARCH</b>	Autoregressive Conditional Homoskedasticity
<b>BS</b>	Black Scholes
<b>CIR</b>	Cox Ingersoll Ross
<b>GARCH</b>	Generalised Autoregressive Conditional Heteroskedasticity
<b>MCMC</b>	Markov Chain Monte Carlo
<b>SV</b>	Stochastic Volatility
<b>SVJD</b>	Stochastic Volatility Jump Diffusion Model

## Acknowledgements

# Chapter 1: Introduction

## 1.1 Background of the Study

The price process is a widely studied facet of the financial system. The movement of prices, the factors that lead to price movements Chen et al. (2013), pricing dynamics Sinha (2015) and predictability of prices Bhattacharjee and Bhattacharja (2019) are some of the various ways in which prices have been researched. Comprehension of fluctuations in the price is fundamental to financial theory, especially how information is assimilated into prices as postulated by the Efficient Market Hypothesis.

Current prices convey information about a particular security. Fama (1970) proposes the efficient market hypothesis. It states that prices incorporate all available information. This theory has three forms: weak form, semi-strong form and strong form efficiency. The first form stipulates that price reflects all information of past prices. It is thereby not possible under this form to earn above average returns by using technical analysis (identifying trends in price by analysing historical price chart and market statistics). Fundamental analysis is the determination of the intrinsic value of a stock to ascertain if it is undervalued or overvalued by the market Bartram and Grinblatt (2018). Fundamental analysis is therefore useful in the identification of mispriced securities in weak form efficiency.

The semi- strong form puts forward that all publicly available information is taken into account in the current price. Neither technical nor fundamental analysis yield higher return than other investors thus not aiding in investment decision making process. Strong form efficiency states that all data, public and private, is reflected in the present-day price. Market inefficiencies are observed in European markets when technical analysis is used Cervelló-Royo et al. (2015).

The Efficient Market Hypothesis postulates rationality in investor behaviour. However, studies on investor behaviour shows biased behaviours that influence investment decisions Konstantinidis et al (2012). Rational investing involves making decisions that will maximize expected utility based on available information Fama (1970). This hypothesis does not hold up when empirical testing is done using to real data. Through momentum investing, investors can be able to recognize certain price patterns that persist over time. This is done by using both technical

and fundamental analysis. The rationality of investors is assumed in the efficient market hypothesis. Behavioural finance studies observe that decision making by investors is guided by psychology as opposed to efficiency and rationality Chaudhary (2013).

Price movements can be seen as a reflection or as the result of incorporation of new information. In economics, the price is dependent on the interaction between demand and supply in a competitive markets Marshall (2013). All else being equal, the law of demand states that if the demand of a security is higher than its supply, the price of the commodity increases. Conversely, when the supply is higher than the demand, the prices decrease.

The seminal work of Black and Scholes (1973) introduced the log normal diffusion process of modelling stock returns. This refers to the log return process having a normal distribution. This process has extensively applied in pricing of options for products in different markets across the world. Through empirical research however, it has been observed that stock return distributions have kurtosis and skewness greater than those of the normal distribution Jin and Zhang (2015). Additionally, there is also evidence of substantial random oscillations such as crashes or sudden upsurges Merton (1976).

As a consequence of the empirical research not aligning with the theoretical framework, modifications as well as other models have been developed to capture these properties. Press (1967) states that the behaviour of log returns can be separated into two. The diffusion element is continuous while the jump is discontinuous. The intuitive interpretation is that the jumps take in when new abnormal important information arrives in the market. On the other hand, new normal information that is not as important is captured by the diffusion element Beinert and Trautmann (1991).

Merton (1976) modified the Black Scholes Geometric Brownian motion by adding Poisson jumps. Jump diffusion models perform better in representing the asset return distributions than normal diffusion models. This introduces sudden and discontinuous movement in the price. Volatility is another salient feature of asset returns. Time varying volatility encompasses volatility clustering. Roger et al. (2000) show that models incorporating jumps while not accounting for varying volatility produce estimates different from actual parameters. Stochastic volatility is incorporated in the model to capture volatility clustering and other stylized facts of volatility. To gain a better understanding characteristics and distribution of assets, a model that merges the jump

component and stochastic volatility needs implementation.

Ultimately stochastic volatility and jump factor play a crucial role in modelling. This can be achieved by adding a jump component to stochastic volatility models as done by Merton (1976). This modification has been done on the Black Scholes, Heston and Bates models. These models perform better than the Black Scholes model Guoqing Yan and Hanson (2006). These models have been taken up as opposed to other time variant counterparts like Autoregressive Conditional Homoskedasticity (ARCH) Engle (1982) because they have a connection to the type of diffusion process Mahieu and Schotman (1998).

The ongoing research on stochastic volatility jump diffusion models provides insight on the complexities that are associated with the proper representation of asset properties. They help address market phenomena such as volatility clustering, heavy tails, and market jumps Jin and Zhang (2015). These models adeptly capture the erratic movements and sudden shifts in asset prices. Market behaviour is represented in a more nuanced way integrating stochastic volatility and jump components Pan and Singleton (2000). The implementation of these models has been formulated and modified to suit a diverse range of securities in securities markets across the world.

Stock indices are indicators of the movements of the stock market or a subset of the market. By studying the stock index movement, an investor is able have a broad range idea on how that market is fairing. Equity indices are used as a benchmark of how a particular asset is performing vis-à-vis the market index or the particular market index Afego (2017).

Campbell et al. (2012) gives the following reasons as to why financial literature models returns as opposed to prices of assets. An asset is a resource owned or controlled that will generate positive economic benefit. It can be owned by an individual, government or corporation. Assets generate cash flow, reduce expenses Returns capture the price movements of a particular asset for a given horizon. Returns can be captured every day, week, month or year depending on the periodicity of the data or according to the investment horizon. Returns are a scale free rundown of the investment opportunity and have attractive statistical properties

Forecasting of asset returns can lead to generation of significant profits and guide investors and policy makers on decision making. Al-hnaity and Maysam Abbod (2015) research found ensemble machine learning techniques were better predictive tools for financial time series.

Ensemble models achieve better predictive results with higher accuracy. These models are able to achieve this performance by reducing the variance and bias. By consolidated multiple base models the technique is able to come up with a powerful model.

## **1.2 Problem Statement**

Modelling of asset returns is an important part of quantitative finance. Numerous theoretical and empirical models have been developed to understand the behaviour, distribution, pricing and the relationship with various variables that affect these returns Chang et al. (2019). Identifying the appropriate statistical distribution for the assets in a market is crucial to investors, asset managers, regulators and risk managers Jayanta K. Pokharel, (2024). Undoubtedly, the precise description of stock price dynamics is of considerable significance in understanding market efficiency, risk management, investment choice, financial stability and in the valuation of contingent claims Brody et al. (2020).

Modelling equity indexes is pivotal for understanding market dynamics, investment decision assessment and development of investment strategies. These securities can be indicators of economic trends therefore providing insights on how different sectors or the economy is performing Pearce (1983). These indexes can be used as underlying securities for various investment products like index funds. Investment managers can use them to benchmark their performance. Pepi (2020) find that indexes help investors to estimate the level of risk associated with investment in particular securities.

Empirical challenges to the Black-Scholes framework have led to the development of alternative models. The on-going research into stochastic volatility jump-diffusion models has further refined asset pricing techniques Nthiwa et al. (2023). These models merge stochastic volatility with jump processes to better capture phenomena such as volatility clustering, heavy tails, and abrupt price movements Kalsheker (2009).

To model these properties, stochastic volatility jump diffusion processes are used. These models combine jump components and stochastic volatility therefore offering a more robust framework for understanding asset return characteristics. This study uses a modified Heston model. The use of SVJD models in the Kenyan market will bring forth a deeper understanding of the properties of the asset returns. The on-going discussion of financial sector deepening and

inclusion will be boosted further by understanding the divergent applications of financial models Bakang (2015). The development of targeted market specific contingent claims will stimulate public participation of citizens. Ensemble methods are highly accurate prediction. Traders or investors with models with higher and more accurate predictive capacity can make well informed financial decisions and aid in optimal portfolio construction.

Empirical research on theoretical financial frameworks in frontier security markets is not expansive Thomas et al. (2017). With developed markets having more research done on them, knowledge and comprehension on the behaviour of stock returns is quite extensive. For markets that lag behind, among other factors, the inadequacy of information on the markets is a huge factor that contributes to investors shying away from them Cont (2000).

Nthiwa et al. (2023) show that the assumption of constant volatility is not congruent to the real-world price dynamics. The use of models with these assumptions lead to mispricing and inaccurate hedging strategies for options. Empirical evidence of nonconstant volatility in the Kenyan market has been documented by authors such as Bosire and Maina (2021) as well as Ndwiga and Muriu (2016). Mothi (2019) notes that frontier markets are characterized by smaller markets that are generally less liquid thus more volatile. Mulambula et al. (2020) paper provides evidence of jumps in the Kenyan market. Identification of jumps helps in examining the types of information linked with various events such as political events Gkillas et al. (2018) .Therefore, investigating jumps is valuable for modelling and forecasting asset prices.

Modelling of asset prices in frontier markets is important in understanding the specific dynamics of securities in a market Mothi (2019). A profound comprehension of market dynamics significantly enhances investors' capacity to assess risk with greater precision, leading to more informed investment decisions Haidari (2023). This boosts investor confidence, especially among institutional and foreign investors who rely on accurate risk pricing. Improved market transparency attracts data driven investors thus stimulating market participation. Increased research on pricing dynamics and volatility patterns promotes the development of innovative financial products. Mastery of these features facilitates accurate pricing that is key in derivative pricing such as ones based on market volatility. The model's ability to detect jumps and volatility surges can help regulators implement market surveillance systems or risk-based capital requirements. The calibration of the model in frontier markets has a few challenges including

having limited historical data availability compared to advanced markets. This makes the number of jumps that can be observed for these markets are fewer. Frontier markets are also characterized by infrequent trading and illiquidity that leads to stale pricing. This distorts price signals making calibration challenging.

### **1.3 Research Objective**

The study objectives are to:

- i. To analyse the empirical distributions of frontier markets assets returns
- ii. To develop a continuous stochastic models with jumps to capture the empirical behaviour of these asset returns
- iii. To back test the performance of the developed stochastic models
- iv. To predict asset prices using ensemble methods.

### **1.4 Research Questions**

This study works towards answering the following research questions:

- i. What is the empirical distribution of asset returns in frontier markets?
- ii. How does the stochastic volatility jump diffusion model capture behaviour of asset returns?
- iii. How does the stochastic model perform?
- iv. What is the prediction performance of the ensemble model?

# Chapter 2: Literature Review

## 2.1 Introduction

The model by Black and Scholes (1973) is a contingent claim pricing model that has been vastly studied both theoretically and empirically. A geometric Brownian motion process is proposed. Log returns are modelled to follow a normal distribution. However, asset return distributions in practice display properties that are antithetical to the assumptions of this model Jin and Zhang (2015). Drawing from empirical studies, asset returns have excess kurtosis and skewness than inferred by normal distribution. Volatility clustering is also an important feature present in the data Niu and Wang (2013). To capture these features, this model has been extended other models developed to better explain the behaviour of returns data.

Building upon this research, Merton (1976) made improvements by adding on Poisson jumps to the Black Scholes Geometric Brownian Motion. His model was able to capture the jump component in the data. Studies like Bates (1996); Broadie and Kaya (2006) showed that there is strong empirical indication that supports the presence of jumps as well as stochastic variability. The Black Scholes model is not adequate in terms of capturing these data characteristics thus the need for new types of models emerged.

Press (1967) defines jumps as the discontinuous component. Volatility measures the lack of certainty associated with the yield that will be generated by a stock Hull (2015). Stochastic volatility is a phenomenon where the volatility of a financial instrument price is not constant but varies over time. Time variation was previously modelled using Autoregressive Conditional Heteroskedasticity (ARCH), its variations; Generalised Autoregressive Conditional Heteroskedasticity (GARCH) and Exponential Generalised Autoregressive Conditional Heteroskedasticity (EGARCH). The connection of stochastic volatility models to the sort of diffusion process applied in asset pricing has led to the adoption of these models in recent research Mahieu and Schotman (1998).

Ensemble methods combine base or weak learners to produce models that are superior models. These models are machine learning algorithms that improve prediction performance

accuracy (Zhou, 2014). Ensembles achieve better results by having the base learners complement each other by making different kinds of errors. By training multiple base learners on the same task and then having their outputs combined you create a better output Dietterich (2000).

Ways to combine or ensemble models include but are not limited to bagging, boosting and stacking. Bagging or bootstrap aggregation is a technique where base learners are trained independently on random subsets of the training data Dietterich (2000). The training is done in a parallel manner. The aggregate of the prediction is then done via averaging. Bootstrapping is done when selecting the subset meaning that the data that is sampled is done so with replacement. Individual data points can be picked more than ones with the random subset being a bootstrap sample. Bagging reduces the variance and overfitting.

Boosting is a technique where weak learners are trained sequentially with each model correcting and learning from the errors of the preceding model. This process is rerun until number of predefined models is reached or some other criteria are met Zhou, (2014). While bagging involves simple averaging, boosting assign weights based on accuracy. Instances wrongly classified are assigned higher weights. Learners of both bagging and boosting are homogenous meaning they are of the same type. Improving on the errors of previous model results in the reduction in bias.

Stacking is a heterogeneous technique that uses a separate model called meta learners to combine weak learner model results. As it is a heterogeneous technique, it trains different kinds of base machine learning algorithms. Base model outputs are used as inputs by the meta learners to give prediction of the final output. Stacking ensemble is empirically shown to have higher prediction accuracy than bagging and boosting by Nti et al. (2020) but is a computationally expensive method.

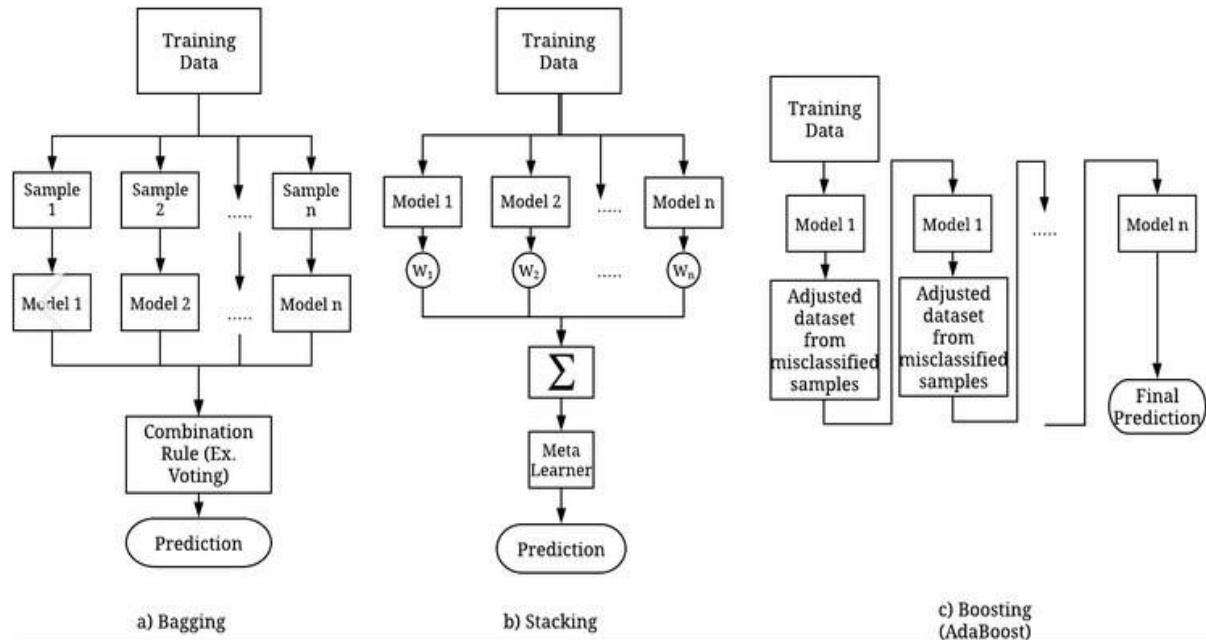


Figure 2.1 Flowchart on Ensemble Methods

Note: From “User Response Prediction in Online Advertising,” by Z. Gharibshah, X. Zhu, ACM Computing Surveys (CSUR), p 24 (<https://arxiv.org/pdf/2101.02342>). Copyright 2020 by Association for Computing Machinery

## 2.2 Jump Diffusion Model

The application of jump diffusion models has been implemented in many studies both in the financial papers and other fields. They have been used in option pricing Cai and Kou (2011) and Ahlip and Prodan (2015), portfolio optimization Zhang and Lang (2023), credit risk modeling Triki and Abid (2022) and Huang and Huang (2012). Jump diffusion models are better in representing asset return distributions than normal diffusion models. They capture huge random fluctuations and the heavy tailed nature of the distribution.

Merton (1976) integrated jumps to the diffusion process. His work found the lack of suitability of the Black Scholes in modelling option prices whose underlying asset returns are not continuous. His work used compound Poisson jump process with jump size being normally

distributed and constant intensity. Monte Carlo simulation was the technique used in the estimation of option prices. This model captured negative skewness, excess kurtosis and the effects of discontinuous stock returns on option prices.

Kou (2002) used jump diffusion model to accurately price options. Leptokurtic features as well as volatility smile observed in S&P 500 were addressed by the model. The model shows how jumps affect option prices. Using Monte Carlo simulation, this model's application can be done on option types such as European, American and barriers. The model was found to produce analytically tractable and simple applicability for various option pricing problems.

Kou, 2002, extended his work on a double exponential model used to analytically approximate finite models for American options Kou and Wang (2004). This model was used to also get the analytical solution to path dependent options. This model captures the effects of both small and large jumps that are present in option prices. This model is thereby more realistic than both the BS and jump diffusion model.

Fu et al. (2017) incorporated arbitrary discrete jump size and nonparametric distribution to the model. The flexibility in jump size allows for capturing of leptokurtic features present in the data. An exact solution for European options pricing is obtained which captured all the features of the data. This solution was easy to implement with properties of fast convergence. Accuracy, efficiency, robustness and flexibility of this model are corroborated by the numerical results.

On the other hand, Mulambula et al. (2020) used a European logistic type jump diffusion model for option pricing to estimate volatility. The use of this model is to cater towards improving the reliability of skewed implied volatility surface that tends to flatten over longer maturities. It priced options more accurately than the BS model and capture how jumps affect the volatility.

Beinert and Trautmann (1991) used maximum likelihood estimation to get parameter estimates for the model. A Poisson type diffusion processes was found to explain the behaviour of common and preferred German stocks. The model explained excess kurtosis and skewness of

the data. They were also able to show the significance of jumps for most of the sample sub periods.

In contrast, Ball and Torous (1983) modelled common stocks in the New York Stock Exchange using a Bernoulli jump model. The model maintained desirable economic properties of the returns and was empirically tractable. They applied method of cumulants and maximum likelihood estimates for estimation of parameters. The maximum likelihood estimation was the most powerful test to detect the existence of jumps. This study was able to confirm the presence of jumps in most of the data.

### **2.3 Stochastic Volatility Models**

Volatility is a measure of uncertainty of the price movements. A high volatility means that there is a high potential variability in a security's value. It is associated directly with the risk associated with holding a financial security thus affecting the investment decision making process and portfolio choice. Higher volatility is therefore indicative of a riskier security.

Volatility is observed to be varying over time exhibiting clusters with periods associated with high volatility and others with low volatility. Models like Black and Scholes (1973) model that assume constant volatility misprice options by underestimating the risk and therefore having lower risk premiums on their options as shown by Jin and Zhang (2015). Volatility contributes to the ascertainment of the risk level and the valuing options and as well as other derivatives.

Stochastic volatility models are used when volatility is not constant over time. One of the first models to be proposed to yield a time varying volatility was done by Clark (1973). He used a time deformation approach where asset returns were modelled as a function of the arrival of information that occurs in a random manner. These models account for leptokurtosis, persistent and time variable volatility in comparison to their ARCH counterparts. The performance differences of SV models and GARCH models have been extensively documented.

Stochastic volatility models have been shown to appropriately capture empirical properties as shown by many authors like Andersen and Benzoni (2010) and Broto and Ruiz (2004) leading to their popularization.

Cisana et al. (2007) looked at the performance of SV models in the Italian market. Exponential Ornstein- Uhlenbeck, Vasicek and Heston models were implemented for this empirical study. Each of the models' theoretical results were verified with all the models maintaining their features and able to reproduce empirical properties of the prices. However, the exponential Ornstein- Uhlenbeck model was able to best characterize stylized facts of volatility and log return distributions of the Italian market.

Luo et al. (2018) developed a neural stochastic volatility model for volatility estimation and forecasting. This model adapted the fundamentals of stochastic volatility models and incorporated recurrent neural network formulations. Empirical application of this model on data shows that it outperforms GARCH family models as well as some pure stochastic volatility models. Although this model has slower convergence, is it highly flexible and accurate in volatility modelling and prediction on China's daily stock prices.

Research conducted on the Kenyan market has mainly focused on ARCH type models. Volatility clustering and non-constant volatility was documented by Ndwiga and Muriu (2016). They modelled using symmetric and asymmetric GARCH models. Maqsood et al.(2017) ascertain that an asymmetric GARCH model characterizes NSE index return data better than the symmetric type. They also found a positive correlation between volatility and stock returns. However, they found presence of leverage effect unlike the findings of Ndwiga and Muriu (2016). Application of stochastic volatility models in this frontier market is not extensive with recent research Novat (2020) applying Merton jump diffusion model that does not account for varying volatility.

These models have been applied on diverse instruments such as equities, exchange rates, options and market indices. Broto and Ruiz (2004) used a stochastic volatility model with an autoregressive component on the S&P 500 while the Heston model is used on VIX and S&P 500 indices. The Italian stock market is best modelled using the exponential Ornstein- Uhlenbeck process as shown by Cisana et al. (2007). Mahieu and Schotman(1998) applied the Hull and White model on exchange rates of different currencies while Turfus (2019) extended the application on Libor caps and floors.

## 2.4 Stochastic Volatility Jump Diffusion Models

The principal limitation of jump diffusion studies is their inability to capture the effects of clustering of volatility as noted by Kou (2002). These models assume independent structure and are therefore not able to capture dependence structure among asset returns. Nevertheless, incorporating the jump factor is fundamental in modelling. Chan (2005) found that models that fail to consider the jump factor led to model misspecification while incorporating them together with GARCH yield accurate estimates. Random fluctuation of volatility is accommodated for by stochastic volatility models. Moreover, these models integrate volatility stylized facts such as volatility skew and smile, reversion to the mean and leverage Malhotra et al. (2019).

Eraker et al. (2003) examined stochastic models that allowed for jumps and volatility dynamics in returns. They found evidence of the presence of jumps. They study the return data of the S&P 500 and Nasdaq 100 indices. Misspecifications occur in models that fail to account for jumps. These jumps have a significant effect on price of options. A single factor square root model is modified to encompass jumps. Markov Chain Monte Carlo (MCMC) estimation is the parameter approximation technique. The method yielded verifiable computationally efficient results through simulation.

Kaeck and Alexander (2013) studied European equity indices and the S&P 500. The former is found to be well represented using an affine model. It has both jumps in the price and variability. A GARCH volatility process lacking jumps is also suitable for this data. For the latter, these properties are arduous to represent using the JD process. They examine how twelve continuous time jump diffusion models are in encapsulating market dynamics of the data sets.

Ignatieva et al. (2015) carried out an extensive analysis on equity index returns. The study was done on continuous time models to look into the features of commonly used model classes, notably affine and non-affine stipulations in the variation that is reinforced by the jump factor. Jump factors are found to be crucial model components whether the setup is affine or non-affine. Nevertheless, non-affine specifications outperform affine ones.

Balajewicz and Toivanen (2016) used Reduced order Moments to price American and European options under various SVJD models. This specification on Heston, Merton

and Bates models is derived by carrying out proper orthogonal conditions. Experimentation shows that this method is faster within range in the parameter variant on of a given model, in comparison to simplified versions of original models. They demonstrated that this approach is fast, accurate and flexible making it suitable for an assortment of problems.

Kirkby and Nguyen (2020) proposed a transform-based regime switching jump diffusion model which is more efficient in the pricing of Asian options. Their method encompasses Heston, Hull- White, Stein- Stein processes. They came up with an amalgamated way to price options under general SVJD models by combining Fourier pricing techniques and continuous time Markov chain approximation. This method effectively solves pricing of options like barrier and American options.

Guoqing, Yan and Hanson (2006) model stock prices with a variant of the SVJD model. They use a square root mean reverting for SV together with a log uniform jumps. This model was compared to the Black Scholes model. Their option prices of this model are higher because stochastic volatility and jumps are associated with higher premium on risk especially for near the money options as well as options with longer maturity. To verify results, the implementation of this formula was achieved by fast Fourier transforms as well as more precise discrete Fourier transforms.

Saranya and Prasanna (2018) conducted a study on the consolidated effect of jumps, asymmetry and variability in the prediction of market volatility. In the five Asian stock markets studied, the persistence of the stochastic volatility process is observed. Leverage effect, size and jump frequency engenders the computation of the unpredictability in the market. The SV model with jumps and non-symmetric aspect remarkably help in measurement of volatility in tumultuous periods.

Novat (2020) used the Merton's type of jump diffusion model and estimated parameters using maximum likelihood estimation. He used selected stock market data from securities exchanges in Nairobi, Dar es Salam and Uganda. The model's results were compared to those of the Black Scholes. The incorporation of the Poisson process by the Merton model generated realistic option prices for the East African market. The model was incorporated the volatility and leptokurtic features of the data.

Jin and Zhang (2015) bring into line jump diffusion long run risk models. The efficacy of the models is on their capacity to elucidate the variance and equities risk premiums in the US financial markets. In financial markets, excess return measures on equity and volatility are affected by jump risk in volatility of long-term growth in consumption. Significant chunk of equity premium is found to be from the compensation of jump risk in volatility and growth.

Roger et al. (2000) used a simulation-based approach on Norwegian- British exchange rate. In simulation-based estimation, the model is the data generating process. They estimated a SVJD that nests a Geometric Brownian motion and jump diffusion specifications. They compared the results from the model with a Geometric Brownian motion plus a Poisson process. This model was estimated using maximum likelihood. For maximum likelihood, an assumption is made on the probability density function for this rate.

The study found that while the SVJD model could match both conditional and unconditional moments, the other model could only match the unconditional moments.

He and Chen (2022a) develop a hybrid of the Heston CIR model for pricing exchange options. In this model volatility, domestic and non-domestic exchange interest rate parameters are taken to be regime switching. For this model, the exact and analytical solutions are derived and numerically tested. The quantitative impact of regime switches on the security is looked into and formula accuracy is verified. This model is found to be a good substitute for the original model and outperforms it in test cases.

Chan (2005) modelled Australian short-term interest rates. To capture the short rate features the model integrated linear mean-reverting drift, time-varying volatility and GARCH as well as jumps. When the jump factor is included, the model captured the GARCH-induced volatility.

## 2.5 Models

Evidently, the Heston model and its variants have been used to model different securities. Eraker et al. (2003); Kaeck and Alexander (2013) apply a Heston model for equity index dynamics. Stock prices have been modelled using a square root process where the mean reverts for stochastic volatility with jumps that are log uniform by Guoqing Yan and Hanson (2006). Makate and Sattayatham (2011a) use a square root diffusion process with jump with mean reversion properties on European call option. He and Chen (2022b) used a hybrid version of the Heston CIR model. Their model has regime switching properties to price foreign exchange options accounting for the regime switching that significantly impacts these options. Studies show that the Heston model is efficient, precise and easily adjustable to cater to specifications thus can be used for various contingent claims.

The MCMC is a Bayesian estimation procedure. Given that it is a Bayesian estimation method, no assumptions are made regarding the type of distribution of asset returns. This approach is particularly well-suited for complex and high-dimensional distributions due to its ability to generate samples from virtually any probability distribution. The method relies on computer-driven sequential sampling. Each random sample is generated based on the preceding one, forming a Markov chain Cape et al. (2015). The algorithm begins with an initial set of parameter values and iteratively samples from the posterior distributions of each parameter using a likelihood function to update these estimates. Over successive iterations, the sampling process converges toward a stationary distribution. This as a result approximates the joint posterior distribution of the model parameters. This convergence allows for the estimation of distributional properties with increasing accuracy. As noted by Ravenzwaaij (2018), the efficiency and flexibility of MCMC make it especially effective for inference in high-dimensional settings.

With consideration for estimation risk, this approach offers estimates of the latent volatility, jump times, and size of jumps Eraker et al. (2003). It has also been demonstrated in similar contexts to have better sampling properties to rival approaches in addition to being computationally efficient to verify the method's accuracy using simulation. MCMC is able to draw from complex distributions, therefore ideal for estimation of this model. The MCMC technique will be utilized in the parameter estimation for the model.

### 2.5.1 Ensemble Algorithm

Ensemble learning is a class of machine learning technique that merges a group of base learners to have a reliable model whose performance is improved. The algorithms or learners are strategically combined to obtain models which make different kinds of errors that are less correlated between the models. These models reduce the bias and variance thus boosting the overall accuracy or reducing the likelihood of an unfortunate selection of a bad model for prediction.

Traditional methods may suffer from bias due to restrictive assumptions. GARCH and stochastic volatility models rely on parametric assumptions that may not capture complex nonlinearities and do not capture sudden jumps accurately. Ensemble learning methods can extract nonlinear patterns from data, improving predictive power. Single models can lead to overfitting of training data. Ensemble methods mitigate this by aggregating and learning from different models trained on various subsets Yang et al. (2024).

Bootstrap aggregation is used for out of sample stock market forecasting of G7 and eleven Asian countries Fu et al. (2017). In both markets, the stock returns forecasts yield economically significant returns. Returns from bagging outperform the benchmark and are adequate to offset trading costs. Fundamental ratios, macro variables and technical variables are the three types of predictors that make the combined methods. Bagging performs better than combined forecast methods.

Nti et al. (2020) employed Decision Trees, Support Vector Machine and Neural Network to construct ensemble classifiers. They used stock market indices data from stock exchanges in Ghana, Johannesburg, Bombay and New York. An investigative comparison is carried out to find out the best ensemble methods such as stacking, boosting, bagging and blending. Stacking and blending ensemble techniques offered better prediction results in contrast to bagging and boosting.

Al-hnaity and Maysam Abbod (2015) proposed an integrated ensemble model for prediction of FTSE 100 closing price. Cross validation, bagging and cross validation bagging methods were used to improve the forecasting outcome of the model. This model was found to be efficient in the prediction of the index closing price and can be put to use for other financial time series data.

Gyamerah et al. (2019) used stacking ensemble methods on Kenyan stocks. They concluded that machine learning algorithms can be used for optimal decision making on investments. They compared the performance of stacking ensemble performance against the individual Adaptive Boosting and k- Nearest Neighbour classifiers. Ensemble methods outperform individual classifiers.

Li and Pan (2022) put forward a blended ensemble technique that combines two current neural networks with a fully connected neural network. They used the S&P 500 index data to evaluate these techniques. The blended model was built to capture how the change in the data series affects the price of securities. The model was found to outperform other models by having a lower mean squared error and increasing the rate of precision. With a better prediction model investors are able to make better investment decisions.

Pavlicko et al. (2021) used ensemble methods to predict the financial state of Central Europe. They tested the ability of these models to predict the financial distress of these companies. They used current ratio, return on equity, return on assets, debt ratio and net working capital as predictor variables. It combined RobustBoost, CART, and k-NN methods to construct a hybrid approach. The proposed model was able to generate a one-period-ahead prediction to approximate the uncertainty of financial hardship that was more accurate and interpretable than other prediction models.

Bagging is the ensemble algorithm chosen for this study. This approach has been shown to yield accurate results when applied to stock data especially market indices Al-hnaity and Maysam Abbod (2015). By training weak learners independently, this process reduces the variance of the prediction by smoothing out the noise models as well as improving forecast in unstable markets Lee et al. (2020). Random forests enhance the model by adding random feature selection boosting the model's ability to generalize. Bagging coupled with random forests increase forecast accuracy, makes model more robust and is suitable for complex financial models.

## **2.1 Research Gap**

Research on stochastic volatility jump diffusion models has been mainly carried out on mature exchange markets especially the US and Europe. Owing to the fact that these markets have diverse securities, are highly developed and have more data available. There have been

applications of the Merton jump diffusion as researched by Novat (2020) who focuses on the East African market.

However, the application of stochastic volatility jump diffusion models on market indices has not been explored. Jump diffusion and stochastic volatility have been modelled separately. By building on previous research, these two concepts are merged in one model to better capture asset properties. Using Monte Carlo evidence, Roger et al. (2000) demonstrates that pure jump diffusion models are badly biased in the existence of jumps along with stochastic volatility.

The application of stochastic volatility jump diffusion models has been done to different types of securities like equity indices, interest rates, foreign exchange rates. The forecasting ability of the model is done by applying ensemble methods. In the Kenyan market, these methods have not been widely implemented and applied in investment decision making processes.

## **Chapter 3: Methodology**

### **3.1 Introduction**

This section gives a rundown on a mathematical research methodology used in the solving of the research questions to achieve the stated objectives. This paper will use a quantitative research design to model the asset price characteristics and forecast these prices.

### **3.2 Research Design**

A quantitative research design is implemented. A stochastic volatility jump diffusion model is fit to stock return data set in the Kenyan stock market. The model describes the behaviour and distribution of these returns. The design is also descriptive in nature by investigating the characteristics of asset returns.

### **3.3 Population and Sampling**

This study examines the Kenyan equity index market. We seek to study the distribution of stock indices in this market and generate a model that accurately tracks their behaviour. The Nairobi Securities Exchange has three stock indices. The indices used will be the NSE all-share index (NASI), NSE 25 share index and NSE 20 share index covering the period between 2016 and 2023.

### **3.4 Data Collection Methods**

Kenyan equity indices are studied. The study utilizes secondary data that will be obtained from the Nairobi Securities Exchange website. The daily prices of the stock indices will be extracted, cleaned and converted into daily returns.

### 3.5 Data Analysis

The improved Heston model will be used to represent the distribution and properties of the market data. The traditional Heston model which is a SV model will be improved to take into account for the jump dynamics. The estimation technique used in parameter approximation is the Markov Chain Monte Carlo. Finally, prediction of the market return will be done using bagging ensemble methods.

#### 3.5.1 The Heston model

In this framework, we start with a SV model that is the Heston model. It is then modified to incorporate the jump dynamics. Evidence overwhelmingly concludes that this model is precise and more reliable in capturing price distribution as illustrated by Puig (2017) and Gatheral (2011).

Heston (1993a) assumes that volatility follows a Cox Ingersoll Ross process Cox et al (1985). The CIR model is a stochastic volatility model thereby capturing stylized facts such as non constant nature of the volatility and volatility clustering. This process yields volatility values that are greater than zero which is consistent with empirical data. The process also captures mean reversion components and volatility clustering that is present in real world data Jin and Zhang (2015). The CIR model also has a mean reverting term, that reflects how volatility fluctuates but tends to be stable average over time.

$(\Omega, F, P)$  be a probability space with filtration. We define the Heston stochastic volatility model below as

$$\begin{aligned}dS_t &= \mu S_t dt + \sqrt{V} S_t dW_t^S \\dV_t &= \kappa(\theta - V) dt + \sigma \sqrt{V} dW_t^V, \\dW_t^S dW_t^V &= \rho dt.\end{aligned}$$

where  $S_t$  is the price of the asset,  $\mu$  in  $\mathfrak{R}$  the real rate of return and  $V$  is the volatility.  $\kappa > 0$  is the mean reverting rate  $\theta$  in  $\mathfrak{R}$  is the long-term variance  $\sigma > 0$  is the volatility of volatility  $W_t^S$  is

the Brownian motion for the asset and  $W_t^v$  is standard Brownian motions corresponding to the volatility.  $k$  is the rate of mean reversion and  $\rho$  is the correlation between Brownian motions of asset price and its variance.  $k$  shows how quickly the variance returns to its long-term average level; with high values showing that variance returns quickly after shocks thus spikes are short lived. Conversely, when the value of  $k$  volatility is more persistent leading to volatility clustering.  $\theta$  shows the equilibrium level that variance reverts to. This is the average level variance fluctuates to over time Gruszka & Szwabiński (2023). High values imply highly volatile security with lower values infer a volatile asset. Parameter  $\sigma$  is called the volatility of the volatility, and it is generally responsible for the of randomness of the volatility process. High  $\sigma$  shows that the market has frequent and extreme volatility shifts for example in the event of a financial crisis.

The volatility directly modulates by the price process. This means that when the level of volatility is high the movement in the price is also high with lower volatility causing the price movement to be low. The presence of correlation between the two Brownian motions implies a correlation between the asset price and variance seeing that are driven by those processes. The rate of mean reversion and long-term variance ensures that volatility reverts to the long-term value. This mechanism effectively models the phenomenon of volatility clustering.

This model is then discretized. Using Ito's lemma,

$$\begin{aligned} d \ln S &= \left( \mu - \frac{1}{2} V_t \right) dt + \sqrt{V_t} dW_t^S, \\ dV &= \kappa(\theta - V_t) dt + \sqrt{V_{\sigma V}} W_t^V, \\ dW_t^S dW_t^V &= \rho dt. \end{aligned}$$

To discretise the equation, noting the Feller condition  $2\kappa\theta \geq \sigma_V^2$  is satisfied, it is very unlikely to get a negative variance. Let  $Y_t = \ln S_t - \ln S_{t-1}$

$$\begin{aligned} Y_t &= \left( \mu - \frac{1}{2} V_{t-1} \right) \Delta t + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^S, \\ V_t - V_{t-1} &= \kappa(\theta - V_{t-1}) \Delta t + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^V, \end{aligned}$$

$$\begin{aligned}\epsilon_t^S &\sim \mathcal{N}(0,1), \\ \epsilon_t^V &\sim \mathcal{N}(0,\sigma_V^2), \\ \text{Corr}(\epsilon_t^S \epsilon_t^V) &= \rho.\end{aligned}$$

By assuming  $\psi = \rho\delta_V$  and  $\Omega = \delta_V^2(1 - \rho^2)$  we can simplify the equations above to:

$$\begin{aligned}Y_t &= \mu\Delta t - \frac{1}{2}V_{t-1}\Delta t + \sqrt{V_{t-1}}\sqrt{\Delta t}\epsilon_t^S, \\ V_t &= \kappa\theta\Delta t + (1 - \kappa\Delta t)V_{t-1} + \sqrt{V_{t-1}}\sqrt{\Delta t}\epsilon_t^V.\end{aligned}$$

Now let

$$\begin{aligned}\epsilon_t^S &= \frac{Y_t - \mu\Delta t + \frac{1}{2}V_{t-1}\Delta t}{\sqrt{V_{t-1}}\sqrt{\Delta t}}, \\ \epsilon_t^V &= \frac{V_t - \kappa\theta\Delta t - (1 - \kappa\Delta t)V_{t-1}}{\sqrt{V_{t-1}}\sqrt{\Delta t}}, \\ \epsilon_t^S \epsilon_t^V &\sim \mathcal{N}\left((0,0), \begin{pmatrix} 1 & \rho\delta_V \\ \rho\delta_V & \delta_V^2 \end{pmatrix}\right) = \mathcal{N}\left((0,0), \begin{pmatrix} 1 & \psi \\ \psi & \psi^2 + \Omega \end{pmatrix}\right).\end{aligned}$$

### 3.5.2 The Bates Model

Bates (1996) improved on Heston's model by introducing the jump- diffusion dynamics. This specification was able to capture the discontinuous part of price process which happens when new abnormal important information arrives in the market. This feature is not present in the Heston model that assumes that the price process is continuous. Many authors like Pan and Singleton (2000) and Jin and Zhang (2015) have shown that a model incorporating jumps as well as stochastic volatility is able to capture nuances in the behaviour of market data thus providing a more robust way of modelling assets.

By adding log normal jumps to the model, we get the Bates model. The Heston model can therefore be seen as a case of the Bates model where the jump parameter is equal to 0. The interpretation of the parameters is the same as the Heston model as it maintains the stochastic volatility structure. The presence of discontinuities in the price process may account for

leptokurtic and skewness features in the data. The log-normal assumption ensures positive jump sizes thus ensuring the asset price remains strictly positive. The incorporation of jump dynamics help capture sudden, discontinuous price movements as well as fat tailed distribution and skewness. Bates(1996) also finds that stochastic volatility models like Heston are not able to produce asymmetric skews therefore thus capturing some characteristic of assets.

$$dS_t = \mu S dt + \sqrt{V} S dW_1 + d\left(\sum S_{\tau_j} - [e^{Z_j} - 1]\right) - \lambda \left(e^{\mu_s + \frac{1}{2} \delta_s^2} - 1\right) S dt ,$$

$$dV_t = \kappa (\theta - V_t) dt + \sqrt{V_t} \sigma_V dW_2 ,$$

$$dW_1 dW_2 = \rho dt .$$

where  $S_t$  is the price of the asset,  $\mu$  in  $\mathfrak{R}$  the real rate of return and  $V$  is the volatility.  $\kappa > 0$  is the speed at which the mean reverts,  $\theta$  in  $\mathfrak{R}$  is the long-term variance  $\sigma > 0$  is the volatility of volatility.  $W_1$  is the Brownian motion for the asset and  $W_2$  is standard Brownian motions corresponding to the volatility and  $N_t^j$  specifies the jump in the process.  $\rho$  is the correlation between Brownian motions of asset price and its volatility. The jump size  $e^{Z_j}$  is a log normal distribution with  $Z_j \sim \mathcal{N}(\mu_S, \sigma_S^2)$  with the number of jumps per period  $[0, t]$  defined as  $N(t)$ .  $N(t) \sim \text{Poisson}(\lambda t)$  and  $\tau_j$  is the time for the  $j^{\text{th}}$  jump.

By Ito's lemma,

$$d \ln S = \left( \mu - \lambda \left( e^{\mu_S + (1/2) \sigma_S^2} - 1 \right) - \frac{1}{2} V \right) dt + \sqrt{V} dW_1 + d\left( \sum_{j=1}^{N(t)} Z_j \right) ,$$

$$dV_t = \kappa (\theta - v_t) dt + \sqrt{V_t} \sigma_V dW_2 ,$$

$$dW_1 dW_2 = \rho dt .$$

By discretizing it;

$$Y_t = \left( \mu - \lambda \left( e^{\mu_S + (1/2)\sigma_S^2} - 1 \right) - \frac{1}{2} V_{t-1} \right) \Delta t + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^S + Z_t B_t ,$$

$$V_t - V_{t-1} = k(\theta - V_{t-1}) \Delta t + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^V ,$$

$$\epsilon_t^S \sim \mathcal{N}(0,1) ,$$

$$\epsilon_t^V \sim \mathcal{N}(0, \sigma_V^2) ,$$

$$\text{Corr}(\epsilon_t^S \epsilon_t^V) = \rho ,$$

$$Z_t \sim \mathcal{N}(\mu_S, \sigma_S^2) ,$$

$$B_t \sim \text{Bernoulli}(\lambda \Delta t) .$$

By assuming  $\psi = \rho \delta_V$  and  $\Omega = \delta_V^2 (1 - \rho^2)$  and  $\bar{\mu} = \mu - \lambda \left( e^{\mu_S + \frac{1}{2} \delta_S^2} - 1 \right)$ .

We also assume that  $\lambda \Delta t = \lambda_D$  where  $\lambda_D$  is the daily intensity rate of the Poisson process. We therefore simplify the equations to:

$$Y_t = \left( \bar{\mu} - \frac{1}{2} V_{t-1} \right) \Delta t + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^S + Z_t B_t ,$$

$$V_t = k\theta \Delta t + (1 - k\Delta t) V_{t-1} + \sqrt{V_{t-1}} \sqrt{\Delta t} \epsilon_t^V .$$

Now let

$$\epsilon_t^S = \frac{Y_t - \bar{\mu} \Delta t + \frac{1}{2} V_{t-1} \Delta t - Z_t B_t}{\sqrt{V_{t-1}} \sqrt{\Delta t}} ,$$

$$\epsilon_t^V = \frac{V_t - k\theta \Delta t - (1 - k\Delta t) V_{t-1}}{\sqrt{V_{t-1}} \sqrt{\Delta t}} ,$$

$$\epsilon_t^S \epsilon_t^V \sim \mathcal{N} \left( (0,0) , \begin{pmatrix} 1 & \psi \\ \psi & \psi^2 + \Omega \end{pmatrix} \right) ,$$

$$Z_t \sim \mathcal{N}(\mu_S, \sigma_S^2) ,$$

$$B_t \sim \text{Bernoulli}(\lambda_D) .$$

### 3.5.3 Markov Chain Monte Carlo

A MCMC technique is used in the estimation of parameters of the stochastic volatility jump diffusion model. This technique can be used for complex distributions because it is able to draw from any probability distribution. The sampling is a computer driven sampling method. Sequential generation of random samples is done.

A likelihood function is used in the calculation of posterior distributions of each of the variables. The first random sample is used to generate the next sample and this goes on iteratively. The algorithm starts with an initial set of parameter values and sequentially samples from the posterior distributions of each parameter. A random sample is only dependent on the one that immediately precedes it Ravenzwaaji (2018). This process is iterated over each parameter, forming a Markov chain whose stationary distribution approximates the joint posterior. The random sample from the distribution is examined to gradually move toward desired probability distribution to estimate the distribution properties. The desired probability distribution is a stationary one we want to reach. Once reached, the posterior probability is given. This technique provides a lot of flexibility and are efficient if high- dimension sampling. For reliability of the posterior distribution, a burn in period necessary whereby generated samples not representative of the target distribution are discarded MCMC-based estimation inherently supports model validation because it allows for direct comparison between simulated model outputs.

Gibbs sampling and Metropolis Hasting are the specific algorithms used in drawing samples from the posterior distributions. Gibbs sampling is particularly effective for parameters whose full conditional distributions are analytically tractable, often due to conjugate prior choices. It iteratively updates the value of a target variable while holding the remaining variables at their current values. This generates a Markov chain. After multiple iterations, the chain converges to the joint posterior distribution of all variables. Parameters such as the jump intensity, mean reversion rate, and long-term variance typically fall into this category, allowing for efficient and direct sampling. This approach reduces computational burden and enhances convergence speed when updating these parameters sequentially Robert et al. (2004)

Conversely, the Metropolis-Hastings algorithm is utilized for parameters lacking closed-

form conditional distributions or exhibiting strong dependencies with other parameters. It proposes a candidate value for a parameter based on a proposal distribution and then accepts or rejects this new value based on an acceptance probability derived from the ratio of posterior densities. A Markov Chain is constructed whose distribution is stationary and matches the target distribution. It is effective for estimation of the correlation parameter and the volatility of variance. Combining the two algorithms leverages the computational efficiency of Gibbs sampling while maintaining the flexibility of Metropolis-Hastings to handle complex posterior distributions.

### **3.5.4 Ensemble Methods**

Ensembles combine different weak learners that complement each other yielding better results like a higher accuracy of prediction. Nti et al. (2020) finds higher prediction accuracy of equity indices when bagging is used in comparison to the boosting technique. Bagging train weak learners independent while boosting has sequential trains the weak learners.

Cross validation is a machine learning procedure that utilizes sample variation of training data. It involves partitioning of available data into subsets using one of the subsets as a validation test. The remaining set is for model training. In K fold cross validation, the dataset is divided into k subsets. Training is done on all but one of the subsets on the training data. The one remaining dataset is the validation set. This process is repeated k times with a different subset reserved for each time. K fold cross validation is found to be faster than only leaving one training data set.

Bootstrap aggregation is a technique that for generating ensemble algorithms to enhance the performance of the ensemble classifier. The base learner of our bagged model is random forest. Random forest built on decision trees. A voting classifier is used for aggregation to get the bagged model. Majority voting is used. Bagging involves the generation of a ton of bootstrap samples of the initial data. This method is effective in reducing the variance and overfitting. Overfitting occurs when there is low bias and high variance. An over fit model is too well trained on training data and performs poorly on the validation set.

To evaluate the accuracy of the forecast predictions, the root mean square error is used. It

shows how far predicted values fall from the true values. The distance between the regression model and data points is the residual. It measures how dispersed these residuals are, showing how observed data points cluster around the predicted values. The formula for root mean squared error is as below

$$RMSE = \sqrt{\frac{\sum_{i=1}^N \| y(i) - \hat{y}(i) \|^2}{N}}$$

Low values of root mean square error are associated with models that are able to closely track the observed values. Higher values indicate that the models predictions deviate more substantially from observed values, thus less accurate model.

# Chapter 4: Data Analysis

## 4.1 Introduction

This chapter presents how data was analysed for this study. Section 4.2 highlights the descriptive statistics of the data while section 4.3 explores how the Bates model parameters are calculated. Section 4.4 presents the returns forecasts.

## 4.2 Descriptive Statistics

This section shows the summaries and describes the main features and measures of adjacency of the variables in the dataset. Table 4.1 presents the findings.

Variable	Observations	Mean	Standard deviation	Skew	Kurtosis
NSE 20	1945	2521.392	809.574	0.382	- 1.313
NSE 25	1945	3612.488	544.281	-0.126	- 0.291
NSE All share index	1945	146.391	22.239	-0.504	- 0.008

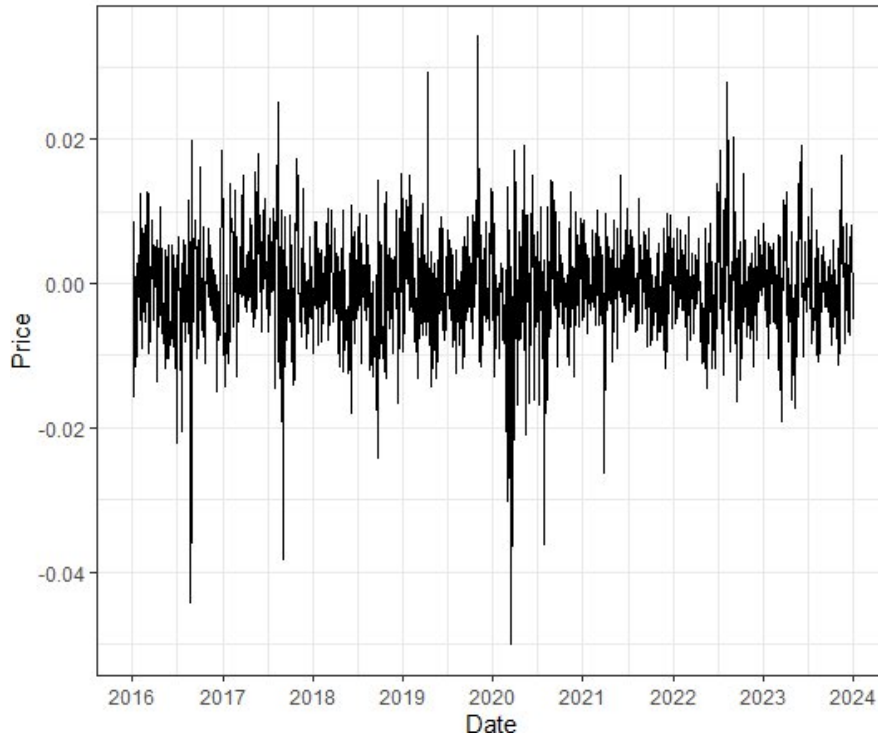
Table 4.1 Summary of all the variables

The table above shows balanced panel with all variables that the Bates model is fit on. All variables have their corresponding data points enabling for seamless analysis.

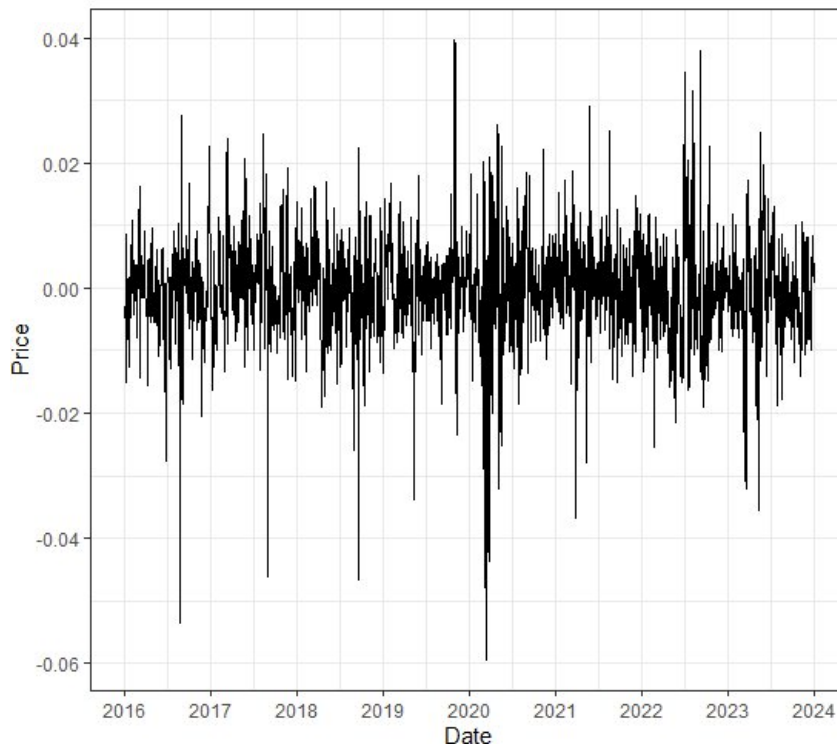
NSE 20 share index has a mean of KES 2521.392 with a standard deviation of 809.574, skewness of 0.382 and an excess kurtosis of -1.313. This has the largest dispersion among all the variables. The price changes for this index are the highest since it has the largest twenty companies in the Nairobi Securities Exchange that are traded very highly meaning its price movement is also high.

The NSE 25 share index has a mean of KES 3612.488, a standard deviation of 544.281, a skewness of -0.126 and excess kurtosis of -0.291. Although it has the greatest mean value, its standard deviation though high, is not the highest of the three. The NSE all share index has a mean of KES 146.391, a standard deviation of 146.391, a skewness of - 0.504 and excess kurtosis level of

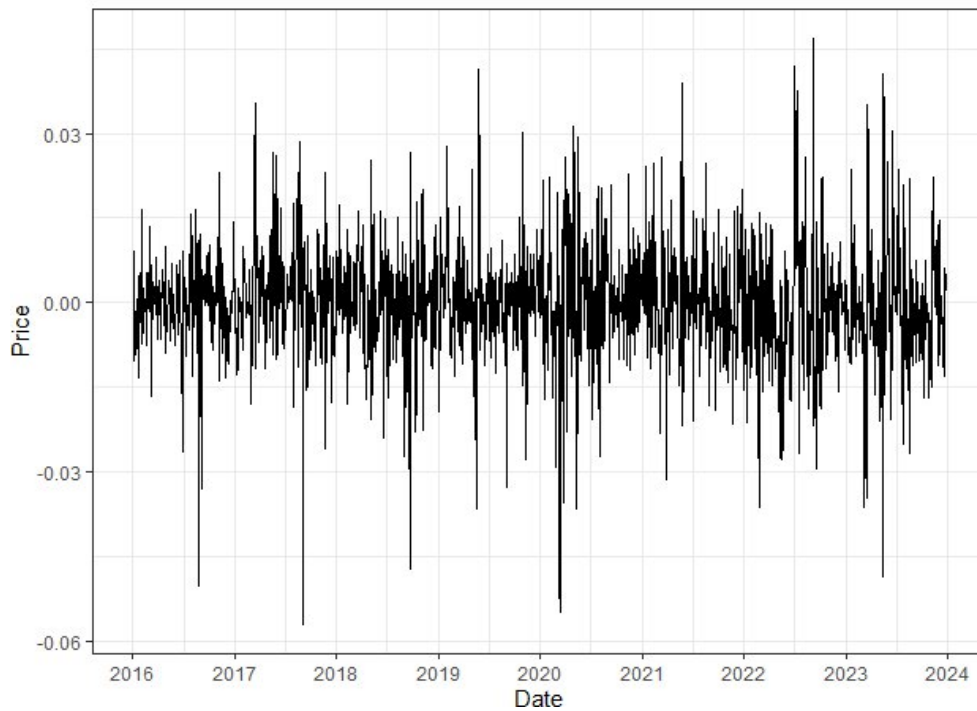
-0.008. Excess kurtosis is derived by subtracting 3(kurtosis level for a normal distribution). All the data series are platykurtic, which means they have lower peaks than that of the normal distribution. The plots of the returns for each of the market indices are given below.



**Figure 4.1 NSE 20 Share Index from 2016- 2023**



**Figure 4.2 NSE 25 Share Index from 2016- 2023**



**Figure 4.3 NSE All Share Index from 2016- 2023**

The three indices have high price movements during the period of the study. There are periods of high returns and low returns. The period with the largest dip across the board was 2020 where the economy was hit hard by the global pandemic. This was in tandem with the country's economic growth rate of -0.3% (*World Bank Open Data, n.d.*).

In Figure 4.1 and 4.2 the highest spike was in 2019 while the same was observed in 2022 for Figure 4.3 for the NSE All Share index. The return series have spikes in different periods. The spikes and dips have different magnitudes and time frames indicating that the different stocks in the indices are affected differently shocks in the same market.

### **4.3 MCMC Estimation**

The estimation of parameters is done by calculating all posterior distributions of and state space as defined by Cape et al. (2015). The prior distribution of the parameters is known from previous literature; from these we obtain the posterior distribution. Prior distributions integrate existing literature and expert insight thus guiding the estimation process. These serve as form of regularization ensuring parameter estimates stay within plausible bounds. Simulation takes place by

using Gibbs sampling and Metropolis-Hasting algorithm. The model is discretized and likelihood functions found then the posterior distribution is derived. The MCMC obtains draws from this distribution by generating a Markov chain over the parameters and state space. The parameters obtained for the indices are shown below.

Parameters	NSE20	NSE25	NSE All share
$\mu$	-0.39 (0.432)	0.00024 (0.475)	-0.92 (0.49)
$\bar{\mu}$	0.0708 (0.031)	-0.0085 (0.057)	0.0027 (0.048)
$\kappa$	4.604 (0.499)	4.461 (0.559)	4.881 (0.477)
$\theta$	0.0772 (0.033)	0.063 (0.036)	0.059 (0.032)
$\nu$	0.0322	0.025	0.022
$\delta$	0.4835 (0.312)	0.394 (0.288)	0.392 (0.32)
$\rho$	-0.0517 (0.00254)	-0.02 (0.00322)	0.049 (0.00252)
$\lambda$	0.0245 (0.028)	0.0230 (0.21)	0.0360 (0.035)

**Table 4.2 MCMC Estimates of the Model**

The results present the mean and standard deviation of each of the parameter estimates. The average daily expected returns ( $\mu$ ) for the series are -0.39, 0.00024 and 0.92 with standard deviations of 0.432, 0.475 and 0.49. The volatility ( $\nu$ ) of the three indices are 0.0322, 0.025 and 0.022 with their volatility of these volatilities ( $\delta$ ) being 0.4835, 0.394 and 0.392.

The mean speed of reversion ( $\kappa$ ) is from the range between 4.46 and 4.881. The long run ( $\theta$ ) mean of variance is 0.077, 0.063 and 0.059. NSE 20 and NSE25 share indices have negative correlation coefficients while the All share index is positive. The daily jump intensity rate ( $\lambda$ ) is

0.0245, 0.0230 and 0.0360. The correlation between the Brownian motions ( $\rho$ ) is -0.0517, -0.02 and 0.049 for NSE 20, NSE 25 and NSE all share index respectively.

### 4.4 Forecasting

Random forest coupled with bagging ensemble methodology is applied to predict returns of the indices. Lag values of each series are used in the prediction. Validation of the forecasting model is assessed using root mean squared error. The root mean square error (RMSE) shows the difference between the predicted values and actual values. The root mean square errors are 0.0049, 0.0087 and 0.0079 for NSE 20, NSE 25 and NSE all share index respectively. These low values show that the forecasted values are able to track the returns closely. The plots below present the forecasted returns compared to the actual returns for each of the market indices

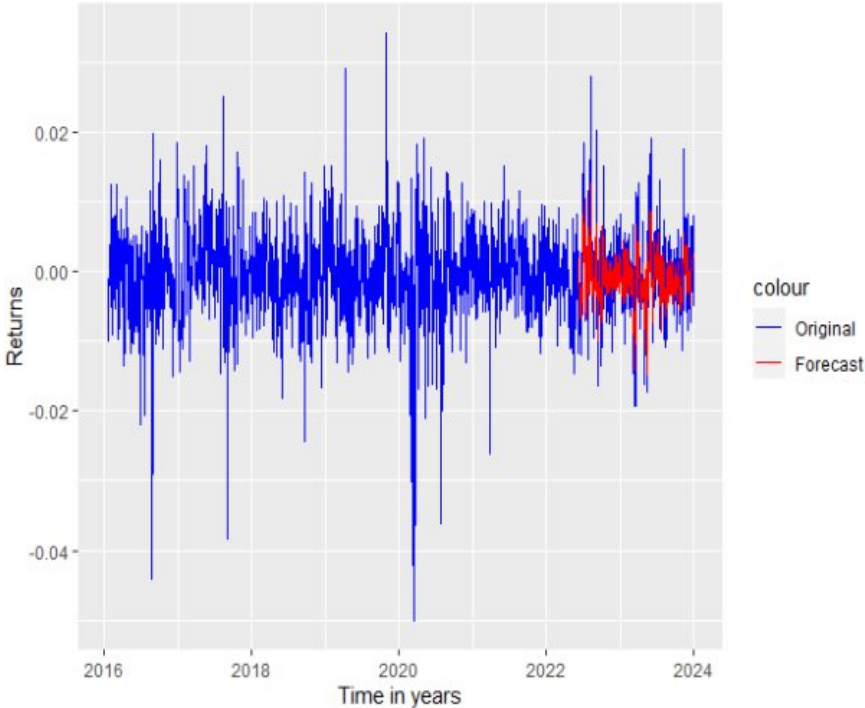
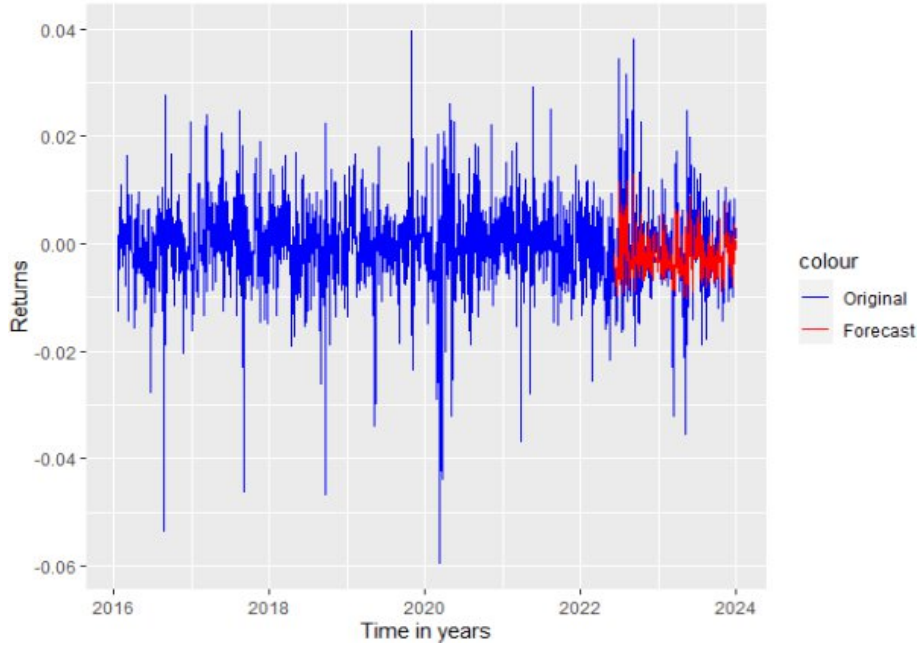
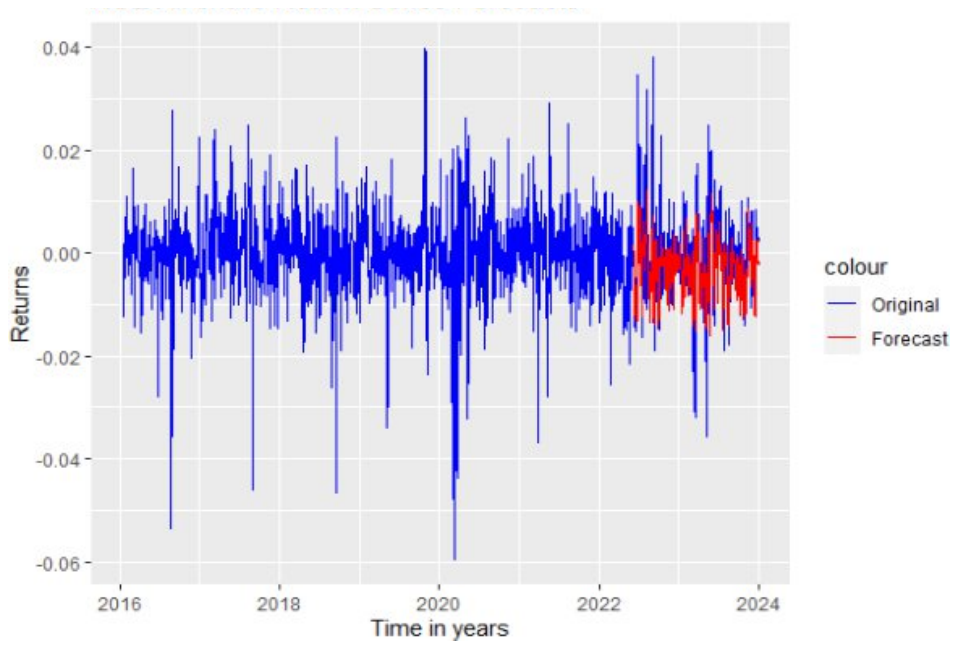


Figure 4.4 NSE 20 Index Return Series Forecasts



**Figure 4.5 NSE All Share Index Return Series Forecast**



**Figure 4.6 NSE All Share Index Return Series Forecast**

# Chapter 5: Summary and Recommendations

## 5.1 Introduction

This chapter encompasses the summary of the study, the conclusions made and suggested recommendations for further studies. Section 5.2 highlights the summary of the study and data presented while section 5.3 explores the recommendations and proposed application of the model.

## 5.2 Summary of the Study

This study explores the calibration of the Bates model on Kenyan market indices. The presence of stochastic volatility are features observed in frontier markets. The Bates model is well suited in calibrating these properties. Traditional models that do not take these characteristics into account are mis specified. Table 2 presents the values of the parameters of this model. The model is discretized, and likelihood function is obtained. The posterior distributions are obtained then simulated. The returns are then forecast using ensemble bootstrap aggregation methodology. The Bates model parameters are estimated using MCMC. The particular MCMC methods used are Gibbs Sampling and Metropolitan Hastings algorithms. The parameters of the model are consistent with literature on Bates models.

Over the period of the study, on average, NSE 20 and NSE all share indices have negative growth rates while NSE 25 has a positive growth rate ( $\mu$ ). From table two, there is a presence of jumps ( $\lambda$ ) in the data for each of the return series. By incorporating jumps we enhance the model's ability to capture discontinuities present in the data therefore making forecasting accuracy better as it aligns with empirical return distribution. The parameter ( $\delta$ ) the magnitude of random fluctuations of the variance. Higher values of ( $\delta$ ) means that the variance ( $v$ ) can experience more frequent and larger movements indicating more pronounced stochastic volatility. NSE 20 share index has the greatest ( $\delta$ ) and ( $\delta$ ) value of the three indices making it more unstable and with higher uncertainty.

The correlation ( $\rho$ ) between asset price and instantaneous variance process in all the return series is negative. Two phenomena that can be used to explain this observation is volatility feedback effect and leverage effect. The volatility effect, where higher expected volatility raises the risk premium demanded by investors, leading to a drop in the asset price. The asymmetric volatility or

leverage effect is where negative returns lead to larger increases in future volatility than positive returns. This observation corresponds to the results widely documented in previous literature. This negative correlation can be explained by asymmetric volatility and volatility feedback. The positive correlation is contrary to what has been observed in previous work. These results are in line with observations in other markets where negative correlation coefficients are

The values of the different parameters are not too dispersed when comparing the results of each series. The NSE all share index has the greatest coefficient for mean reversion and jump intensity rate. Jumps are present in the all the series that were analysed. Underestimating the jump intensity leads to under pricing tail risk. It is important to account for the frequency of jumps as it increases the model's responsiveness to abrupt market changes and thereby enhancing its capacity to forecast volatility spikes and risk metrics. The annual jump intensity rates are higher than those observed in other markets. This might be due to the risk level and higher volatility that frontier markets have in comparison to developed markets.

The ensemble method used in forecasting is accurate and has low root mean squared errors for each of the series. From the comparison between the forecasted values and the actual values, we are able to see that the forecast values are close to the true values. The use of ensemble methods, particularly random forests, helps in improving the accuracy of predictions.

### **5.3 Recommendations**

The application and calibration of the Bates model in various market data is limited partially due to the complexity of parameter estimation. With the development of methods like Markov Chain Monte Carlo, the estimation procedure has become less cumbersome. Most of the studies on this model are concentrated on the US and European markets leaving a gap in literature on the application of this model to frontier and emerging markets. Research in frontier markets should explore the use of the Bates model as well as other hybrid jump diffusion structures. Moreover, jump dynamics can be applied to alternative stochastic volatility model specifications and comparative analyses done across different and models.

The development of derivative securities in the Kenyan market represents a pivotal advancement in the deepening financial instruments in the market. More research into stochastic volatility dynamics can bring about the development of volatility indices that measure expectations on the volatility in assets in different industries as well as the entire market. Increased research on

SVJD models can spark innovation of products that are explicitly linked to volatility or jump structures. These new products can be tailored hedge and speculate on market discontinuities. Further research on the application of these models, will enhance the understanding of empirical properties and the implications on various market stakeholders and policy makers.

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# Appendices

## Appendix A: Similarity Report

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**Ivy Kamau Wanjiru**

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## Appendix B: Strathmore University Institutional Ethics Review Clearance Certificate



24<sup>th</sup> April 2024

Ms Kamau Ivy,  
ivy.kamau@strathmore.edu

Dear Ms Kamau,

**RE: Ensemble Learning for Stochastic Volatility Jump Diffusion Models**

This is to inform you that SU-ISERC has reviewed and approved your above SU-masters research proposal. Your application reference number is SU-ISERC2102/24. The approval period is from 24<sup>th</sup> April 2024 to 23<sup>rd</sup> April 2025.

This approval is subject to compliance with the following requirements:

- i. Only approved documents including (informed consents, study instruments, MTA) will be used.
- ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-ISERC.
- iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-ISERC within 72 hours of notification.
- iv. Any changes anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-ISERC within 72 hours.
- v. Clearance for the export of biological specimens must be obtained from relevant institutions.
- vi. Submission of a request for renewal of approval at least 60 days prior to the expiry of the approval period. Attach a comprehensive progress report to support the renewal.
- vii. Submission of an executive summary report within 90 days of completion of the study to SU-ISERC.

Before commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology, and Innovation (NACOSTI) <https://research-portal.nacosti.go.ke/> and obtain other clearances needed.

Yours sincerely,

Mr Ambrose Rachier,  
Chairperson; SU-ISERC



## Appendix C: R Codes for the Bates Model

```
##### BATES MODEL #####

#### NSE 20 RETURNS

BURNIN = 5000           # Burnin period
N = 20000              # number of simulations

Y = returns1$NSE20_returns # input daily returns
T = length(Y) - 2

mu = rep(NA, BURNIN+N)
psi = rep(NA, BURNIN+N)
omega = rep(NA, BURNIN+N)
kappa = rep(NA, BURNIN+N)
theta = rep(NA, BURNIN+N)
lambda = rep(NA, BURNIN+N)
mus = rep(NA, BURNIN+N)
sigma2s = rep(NA, BURNIN+N)
epsilons = rep(NA, (BURNIN+N)*T)
epsilons = matrix(epsilons, BURNIN+N)
epsilonv = rep(NA, (BURNIN+N)*T)
epsilonv = matrix(epsilonv, BURNIN+N)
V = rep(NA, (BURNIN+N)*(T+2)) # parameter initializations
V = matrix(V, BURNIN+N)
Z = rep(NA, (BURNIN+N)*(T+2))
Z = matrix(Z, BURNIN+N)
B = rep(NA, (BURNIN+N)*(T+2))
B = matrix(B, BURNIN+N)

#### Posterior dist of Parameter
Kappa = na.omit(kappa)
#View(Kappa)
Theta = na.omit(theta)
#View(Theta)
Psi = na.omit(psi)
#View(Psi)
Lambda = na.omit(lambda)
#View(Lambda)
BATES25 = View (c(mustar, mu[i],Kappa,Theta,V[i,j],sigma2v,Psi, Lambda ,mus[i],sigma2s[i]))
```