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The extent to which pension funds can be used to address the affordable housing gap in Kenya

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

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This Research Project has been submitted for examination with my approval as the Supervisor.

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Abstract

The government of Kenya developed the Big Four Agenda in 2017, which is inclusive of affordable housing provision among the other three agendas. However, challenges with housing provision have led to an estimated 2 million-unit deficit. This study investigates how pension funds can address Kenya's affordable housing gap through constructing optimal hypothetical portfolios and evaluating the effectiveness of investments in private equity and real estate. The Shapiro-Wilk normality test was used to determine if asset class data follows a normal distribution to justify Mean-CVaR method optimisation in place of MVO. The asset classes were optimised and subsequently portfolios inclusive and exclusive of private equity and real estate asset classes were optimised, with performance compared. The returns of the optimal portfolio were used in a model to estimate the number of housing units the pension fund could purchase. The analysis showed that equity data alone follows a normal distribution hence justifying the use of Mean-CVaR optimisation for the multi-asset portfolio and that the traditional asset portfolio is the most profitable, rendering private equity and housing as ineffective investments. Further research is required on expanding pension portfolio asset class range and identification of better proxies for private equity and real estate data.

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List of Abbreviations

AHP	Affordable Housing Programme
CAPM	Capital Asset Pricing Model
EAC	East African Community
CVaR	Conditional Value at Risk
DFI	Development Finance Institution
REITs	Real Estate Investment Trusts
MPT	Modern Portfolio Theory
NHDF	National Housing Development Fund
RBA	Retirement Benefits Authority
LPE	Listed Private Equity

Chapter 1: Introduction

1.1 Background

1.1.1 Affordable housing in Kenya

Affordable housing is property sold at about KES 1 million (Noppen, 2013). Like Kenya, many countries in both the developed and developing world aim to provide housing for their population. Through the big four agenda, Kenya envisions providing affordable housing for its citizens, considering the increase in population and urbanization rate to 4.4 percent (World Bank Group, 2017).

The right to housing is established as a socio-economic right in the Constitution of Kenya 2010. This implies that every Kenyan citizen should have the opportunity to afford adequate shelter and sanitation. The government of Kenya introduced some initiatives such as M-Akiba to raise funds to finance infrastructure in 2017. M-Akiba is a government bond auction platform aimed at mobilizing funds to be used for government infrastructure projects. It has been estimated that 61,000 Kenyans have registered on the platform (World Bank Group, 2017). The government also introduced the National Housing Development Fund which is designed to invest members' incomes saved into the fund, which they can withdraw from the fund in the future to purchase housing.

However, the government has met challenges trying to increase the formal supply of affordable housing in Kenya. Between 2009 and 2012 only 3,000 units were provided, falling short of the initial target of 200,000 units to be provided annually as underlined by the Vision 2030 strategy (World Bank Group, 2017). The following section further explores the problems faced in the attempt to provide affordable housing for Kenyan citizens.

1.1.2 Challenges faced by affordable housing

Affordable housing is out of reach for many Kenyans despite the high demand for housing. Mortgages are expensive to attain for a vast majority of the population, hence a low mortgage uptake. There are fewer than 25,000 mortgages outstanding in Kenya, and mortgages from the banking sector make up less than 10 percent of housing credit. On the other hand, SACCOs account for 90 percent of affordable housing finance (World Bank Group, 2018a). They also provide mortgages to members, proving to be a bridge between low-income earners and the attainment of affordable housing. However, deposits from members are the source of SACCO funds, which constrains the amount they can lend (World Bank Group, 2017).

The majority of Kenyans earn informal incomes, contributing to their inability to afford adequate housing. In 2015, only 10.5 percent of urban households could afford the cheapest newly constructed houses. As a result, many people seek shelter in informal settlements. An estimated 61 percent of the urban population in Kenya live in slums (World Bank Group, 2017).

The unaffordability of housing is also exacerbated by the disparity in provision of housing units for different income segments of the population. It is estimated that 48 percent of housing supply is targeted at upper middle income earners, 35 percent for high income earners in contrast to 2 percent for low income earners (World Bank Group, 2017).

It was reported that both government as well as private sector investments in housing are falling short. Planning applications were only for 15,000 units in 2013 despite a public target of 150,000 to 200,000 properties a year in Nairobi (World Bank Group, 2017).

Developers are unwilling to construct new houses due to the lack of guaranteed purchase for the houses built. Hence, they pass the risks to the buyers by charging high prices, leading to housing being unaffordable for a large proportion of the population. The lowest price in the market for a house in the formal market is KES 2 million, which is above what the low-income population considers an affordable amount (Noppen, 2013).

1.1.3 Pension funds and affordable housing

A pension is a fund or a scheme that provides income to its member at retirement. The Retirement Benefits Authority (RBA) regulates pension funds in Kenya. Through these regulated funds, Kenyans can save amounts that accrue interest, providing them income during retirement. According to the investment and regulation policies on www.rba.go.ke website, 20% of asset value is investable in listed corporate bonds, mortgage bonds, 70% in preference and ordinary shares listed in a securities exchange in the EAC. Among alternative asset classes, 30% is investable in immovable property in Kenya, and 10% invested in private equity and venture capital.

It seems that pension funds can contribute to the financing of affordable housing projects (World Bank Group, 2018b). Some of the supply-side investment opportunities discussed in the World Bank Group report include pension fund investment in housing portfolios. Pension funds in Kenya had assets under management accumulate to KES 1.16 trillion as of 2018, indicating its potential for financing affordable housing projects.

1.1.4 Private equity

Private equity is an investment in companies that are typically not listed on the stock exchange. It is a sector that has grown globally through support from pension funds, with a typical 10-year fund tenure that matches the long-term nature of pension fund investments. Among alternative investments, private equity funds are considered the most favored asset type next to direct real estate investment (Hollenwaeger, 2017). The attractiveness of private equity investment is further augmented when including diversification benefits and increased exposure to the real economy (EAVCA, 2019).

Another report explored the landscape for private equity and venture capital investment in Kenya (Divakaran et al., 2018). They described the critical players in the private equity and venture capital market, which include fund managers, investors, public sector entities, and funds. While DFIs are acknowledged as the primary private equity investors, which was also noted in Ashiagbor et al. (2014), family offices and listed investment vehicles are recognized as effective methods of investing in private equity. Listed investment vehicles include Centum investment, an investment company in East Africa, and TransCentury, which is an infrastructure and investment company. They report that listed investment firms and family offices benefit from greater flexibility to traditional private equity funds since they do not have to report to limited partners. In addition, they can hold some investments longer than other private equity avenues.

Despite its potential in significantly increasing investment returns, private equity has developed slowly in emerging markets (Ashiagbor et al., 2014). This is due to the perceived high risk of investment in private equity. Since the majority of pension fund risk lies with pension fund managers, it is their responsibility to ensure that active scheme members receive their pension income at retirement. Therefore, fund managers act prudently and invest a large sum of pension assets in government bonds and treasury bills. However, Nzomo Mutuku, CEO of RBA Kenya, acknowledges private equity as a way in which pension funds can contribute to the realization of the Big Four Agenda and expand portfolio earnings (EAVCA, 2019).

1.2 Problem statement

Affordable housing supply should equal Affordable housing demand to ensure that every citizen, including the poor, can afford adequate shelter. However, low supply compared to the demand for housing in Kenya led to a housing deficit of 2 million units (World Bank Group, 2017).

To address the gap, ways in which pension funds can be utilized for the provision of housing finance are explored, one which involves investment in housing portfolios (World Bank Group, 2018b). Methods of investment in housing portfolios include housing funds (collateral investment vehicles) where equity stakes can be taken, co-funding can happen between housing and pension funds, or provide risk-sharing for projects proposed by rental housing entities or social institutions. However, the NHDF, as proposed by the government, will only provide 7% returns on investments while pension funds, on average, provide a 10% return on investment, indicating that it is not a profitable venue for investing Kenya's pension funds. Another investment avenue for pension funds under housing portfolios include REITs. However, the only available REIT in Kenya, Stanlib-Fahari I-REIT, has had unappealing returns over the years, falling as low as 10%, and therefore it would not seem profitable for pension funds to invest in them.

The incorporation of alternative asset classes such as housing and private equity into the pension fund portfolio requires analysis of the best method of risk management. Previous literature evaluated the profitability of pension fund portfolio investment in housing using the Mean-Variance framework, which implies that assets follow a normal distribution. This is generally not the case, particularly for alternative asset classes, propagating the need to explore other methods of portfolio risk management of pension funds such as the Mean-CVaR framework.

This study contributes to the knowledge gap about pension funds' roles in reducing the affordable housing gap in the following ways: (i) It aims to identify the most appropriate method of evaluating pension portfolio performance with the inclusion of immovable assets and private equity as asset classes in the portfolio. (ii) It will determine the number of units of affordable housing that can be purchased each year through the portfolio's returns.

1.3 Research questions

1. Do asset returns, particularly housing, and private equity, follow a normal distribution?
2. How profitable will it be for pension funds to invest in a combination of asset classes including housing and private equity?
3. How can pension funds be utilized to reduce the affordable housing finance gap?

1.4 Research objectives

The research objectives are:

1. To determine the statistical distribution of the asset class data.
2. To obtain the optimal weights of portfolios, including asset allocations in housing and private equity.
3. To determine the number of affordable housing units that can be purchased through available and permissible pension portfolio funds using the returns obtained in objective three above.

1.5 Significance of research

The affordable housing agenda is critical in improving the welfare of Kenya's citizens in terms of standard of living and quality of life. It will have a ricocheting effect since it will lead to improvements in the overall sanitation of environments, which should lead to improvements in the general health of the citizens and hence a more productive workforce (improved human capital). Therefore, the provision of affordable housing will collectively contribute to the increased economic growth of the nation. This study will contribute to existing literature through an analysis of pension fund potential in lowering the housing finance gap through investment in alternative asset classes such as real estate and private equity.

Chapter 2: Literature Review

2.1 Introduction

Affordable housing as a topic is vastly researched, and many studies recognize the various challenges surrounding its provision. Some past literature addresses issues surrounding affordable housing from a demand-side perspective, which includes providing solutions related to developing the mortgage market. However, this study aims to explore the viability of investment portfolios of pension funds in reducing the affordable housing gap, particularly with the inclusion of non-traditional investment classes such as real estate investment and private equity.

Alternative investment classes such as real estate and equity rarely have returns that follow a normal distribution; therefore, this study aims to explore portfolio selection using Mean-CVaR optimization. It should be noted that this study looks at a lack of finance as the challenge faced in affordable housing provision. Other challenges are better addressed in other studies/topics.

2.2 Theoretical studies

2.2.1 Determining the distribution of data

It is essential to identify the distribution of the data within asset classes of a portfolio, particularly when alternative investment classes are involved, to determine the suitability of assuming normality of asset returns. For example, under the Mean-Variance portfolio theory, asset returns are assumed to follow a normal distribution. A statistical test (famously known as the Shapiro-Wilk test) can evaluate this assumption, where the values $X_1, \dots, X_n$ are said to be sampled from a normally distributed population under the null hypothesis (Shapiro & Wilk, 1965). The test statistic used is

$$W = \frac{[\Sigma_{i=1}^{n} a_i y_i]^2}{\Sigma_{i=1}^{n} y_i^2} \tag{2.1}$$

where

a_i = normalized Shapiro-Wilk coefficients

y_i = ordered sample values

If the p-value of the test is less than the significance level selected for the test, we reject the null hypothesis. Otherwise, we fail to reject the null hypothesis. This test can be accompanied by a normal probability plot (Quantile-Quantile Plot), which is a visualization of how close the sample quantiles are to the theoretical quantiles.

Another goodness of fit test is the Anderson-Darling test where the null hypothesis states that the sample comes from a specified distribution. The test statistic is (Anderson & Darling, 1954)

$$W_n^2 = -n - \frac{1}{n}\Sigma_{j=1}^{n}(2j-1)\left[log u_j + \log(1-u_{n-j+1})\right] \quad (2.2)$$

Where

$u_j = F(x_i)$, the cumulative distribution function of the specified distribution

$n =$ the sample size

for a sample $X_1, \ldots, X_n$ ordered from the smallest to the largest number.

However, determining distributions has become easier with the availability of statistical software. For example, various ways of determining data distribution in R include the usage of graphics, estimation of parameters, and goodness of fit tests (Ricci, 2005). Graphical methods include histograms and normal Q-Q plot for visualization of data distribution. Among parametric estimations explored is the Maximum Likelihood Estimation method, which is employed with mle() or fitdistr() functions in stats4 and MASS packages, respectively. Goodness-of-fit (normality tests) explored include the Shapiro-Wilk normality test and the Anderson-Darling test, among others, like the Jacque-Bera test and Lilliefors test.

A new package fitdistrplus, which contains various methods of fitting distributions to univariate data, was introduced for different types of data such as continuous censored, discrete and non-censored data (Delignette-Muller & Dutang, 2015). The package expanded on fitdistr package to provide other methods of parametric estimation other than maximum likelihood estimation, such as moment matching estimation, quantile matching estimation, and maximum goodness-of-fit estimation.

This study will utilize visual methods along with the Shapiro-Wilk normality test to determine the distribution of the asset returns within the chosen asset classes. The more the methods used, the more reliable the results, and the Shapiro-Wilk test is selected for its reputation as a powerful normality test (Ricci, 2005).

2.2.2 Portfolio Theory and Optimisation

2.2.2.1 Modern Portfolio Theory

A significant contribution to the existing knowledge of portfolio choice occurred with the introduction of Modern Portfolio Theory (Markowitz, 1952). The study suggested that investors should not only consider maximizing returns of individual assets in their portfolio but also the covariance of returns between different assets. This emphasized the importance of diversification of portfolios, where the investor selects assets that ideally do not move in the same direction during the same stage of economic cycles. Markowitz's (1952) work also acknowledged the importance of risk, defined as the variance of portfolio returns, and its relations with the expected portfolio return. The relationship between risk and return implied that portfolio selection involved optimization of portfolio returns subject to risk exposure.

Markowitz's work on portfolio selection in 1952 led to the derivation of the capital market line (Tobin, 1958). It was done by considering portfolio investment in two monetary assets at times of uncertainty, which implies returns following a given probability distribution. This paper further expanded the risk-return dynamics of portfolio investment. The capital market line displays different combinations of risk and return, measured similarly in Markowitz's (1952) paper as variance and the expected value of asset returns, respectively.

The capital asset pricing model (CAPM) was derived a few years later (Lintner, 1965; Sharpe, 1964). Sharpe extended the idea of a capital market line to show how investor's expectations about assets would affect the holdings (demand) of these assets in the portfolios, hence affecting their market prices. He based the model on the assumptions that investors have homogeneous expectations and a risk-free rate of interest with which investors can borrow and lend at. The changes in prices would change investors' positions in their portfolios because of the perceived changes in expected returns from the portfolio. Hence the attractiveness of assets for investments changes with the investors' perceptions over time.

The mean-variance portfolio optimization approach that resulted from the above contributions to modern portfolio theory is as follows. Let there be a portfolio of n assets. The expected return of a portfolio is the weighted sum of individual returns of the portfolio's asset returns given by the formula

$$E(R_p) = \boldsymbol{W}^T\boldsymbol{R} \tag{2.3}$$

Where

$\boldsymbol{W}$ is the weight vector of assets in the portfolio

$\boldsymbol{R}$ is the return vector of the portfolio's assets

The portfolio risk, represented by variance, is calculated as

$$Var(R_p) = \boldsymbol{W}^T\boldsymbol{\Sigma}\boldsymbol{W} \tag{2.4}$$

Where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of assets in the portfolio.

The two most common optimization problems with relation to these measures are

1. Minimize variance subject to expected target returns
2. Maximize the portfolio returns subject to risk constraints

However, this method has been subject to criticism over the years. For example, asset returns following a normal distribution is a strict assumption and may lead to inaccuracies on the results obtained from portfolio optimization. Also, institutional investors are more interested in the downside risk of the portfolio, which may not be best represented by variance or standard deviation.

2.2.2.2 CVaR as a measure of risk

The limitations associated with Markowitz's Mean-Variance Framework led scholars to find other suitable measures of portfolio risk. Value at Risk, among the alternative risk measures, determines the occurrence of losses with a given probability level and therefore is an implication of downside risk on a portfolio. However, it lacks the property of subadditivity and hence cannot be considered a coherent measure of risk (Artzner et al., 1998).

This led to the exploration of CVaR as a measure of risk in portfolio optimisation (Rockafellar & Uryasev, 2000). CVaR, also known as expected shortfall, was considered to be a better measure of risk compared to VaR since it contains the properties of coherency (Artzner et al., 1998). The technique of portfolio optimization employed would calculate VaR and optimize CVaR simultaneously. They further expound on their proposed portfolio optimization method, where CVaR is used instead of variance as a risk measure.

The approach is as follows (Rockafellar & Uryasev, 2000). Let $f(\boldsymbol{x}, \boldsymbol{y})$ be the loss associated with some vector $\boldsymbol{x}$ chosen from a subset $\mathbf{X}$ of $\mathbb{R}^n$ and a random vector $\boldsymbol{y}$ of $\mathbb{R}^m$. $\boldsymbol{x}$ is representative of a portfolio, with $\mathbf{X}$ being the set of available portfolios subject to various constraints. The vector, $\boldsymbol{y}$, on the other hand, is representative of uncertainties such as market prices. For each $\boldsymbol{x}$, $f(\boldsymbol{x}, \boldsymbol{y})$ is a random variable whose distribution is affected by the distribution of $\boldsymbol{y}$, denoted as $p(\boldsymbol{y})$. Therefore, the probability of $f(\boldsymbol{x}, \boldsymbol{y})$ not exceeding a threshold θ (specific loss amount) is:

$$\psi(\boldsymbol{x}, \theta) = \int_{f(x,y)\leq\theta} p(y)dy \tag{2.5}$$

Where $\psi(\boldsymbol{x}, \boldsymbol{y})$ is the cumulative distribution function associated with $\boldsymbol{x}$, as a function of θ for fixed $\boldsymbol{x}$. The function is assumed to be non-decreasing and continuous everywhere with respect to θ.

Denote the α-Var as $\theta_\alpha(\boldsymbol{x})$ and α-CVaR as $\phi_\alpha(\boldsymbol{x})$, where α is the probability level between 0 and 1. Therefore

$$\theta_\alpha(\boldsymbol{x}) = \min\{\theta \in \mathbb{R} : \ \psi(\boldsymbol{x}, \theta) \geq \alpha\} \tag{2.6}$$

and

$$\phi_\alpha(\boldsymbol{x}) = (1-\alpha)^{-1} \int_{f(x,y)\geq\theta_\alpha(x)} f(\boldsymbol{x}, \boldsymbol{y})p(y)dy \tag{2.7}$$

$\theta_\alpha(\boldsymbol{x})$ is the left-end point of the non-empty interval consisting of the values θ such that $\psi(\boldsymbol{x}, \boldsymbol{y}) = \alpha$ in equation (2.6).

Therefore $\phi_\alpha(\boldsymbol{x})$ is the conditional expectation of loss associated with $\boldsymbol{x}$ relative to the loss being $\theta_\alpha(\boldsymbol{x})$ or greater.

$\phi_\alpha(\boldsymbol{x})$ and $\theta_\alpha(\boldsymbol{x})$ are characterized in terms of F_α on $\mathbf{X}$ x $\mathbb{R}$ that is now modified by:

$$F_\alpha(\boldsymbol{x},\theta) = \theta + (1-\alpha)^{-1}\int_{\boldsymbol{y}\in\mathbb{R}^n} [f(\boldsymbol{x},\boldsymbol{y}) - \theta]^+ p(\boldsymbol{y})d\boldsymbol{y} \quad (2.8)$$

Where $[f(\boldsymbol{x},\boldsymbol{y}) - \theta]^+ = \max\{f(\boldsymbol{x},\boldsymbol{y}) - \theta, 0\}$

$F_\alpha(\boldsymbol{x},\theta)$ is proved to be convex and continuously differentiable (Rockafellar & Uryasev, 2000). The α-CVaR of the loss associated with any $\boldsymbol{x} \in \mathbf{X}$ is determinable from the formula

$$\phi_\alpha(\boldsymbol{x}) = \min_{x\in\mathbb{R}} F_\alpha(\boldsymbol{x},\theta) \quad (2.9)$$

$$A_\alpha(\boldsymbol{x}) = \underset{\theta\in\mathbb{R}}{\operatorname{argmin}}\, F_\alpha(\boldsymbol{x},\theta) \quad (2.10)$$

$$\theta_\alpha(\boldsymbol{x}) = left\ endpoint\ of\ A_\alpha(\boldsymbol{x}) \quad (2.11)$$

and

$$\phi_\alpha(\boldsymbol{x}) = F_\alpha(\boldsymbol{x},\theta_\alpha(\boldsymbol{x})) \quad (2.12)$$

The proof above is significant since it allows for the calculation of α-CVaR without having to calculate α-VaR first, which can be a complicated task. A theorem in the study also proved that minimizing the α-CVaR of the loss associated with $\boldsymbol{x}$ all over $\boldsymbol{x} \in \mathbf{X}$ is equivalent to minimizing the $F_\alpha(\boldsymbol{x},\theta)$ over all $(\boldsymbol{x},\theta) \in \mathbf{X} \text{ x } \mathbb{R}$ (Rockafellar & Uryasev, 2000) where

$$\underset{\boldsymbol{x}\in\mathbf{X}}{\operatorname{Min}}\, \phi_\alpha(\boldsymbol{x}) = \min_{(x,\theta)\in\mathbf{X}x\mathbb{R}} F_\alpha(\boldsymbol{x},\theta) \quad (2.13)$$

Note that when $f(\boldsymbol{x},\boldsymbol{y})$ is convex with respect to $\boldsymbol{x}$, $F_\alpha(\boldsymbol{x},\theta)$ is convex with respect to $(\boldsymbol{x}^*,\theta^*)$ and $\phi_\alpha(\boldsymbol{x})$ is convex with respect to $\boldsymbol{x}$. In addition, the joint minimisation problem becomes a convex programming problem when $\mathbf{X}$ is a convex set.

However, later studies extended Rockafellar and Uryasev's work in 2000 to provide other methods of portfolio optimization with CVaR as the risk measure (Krokhmal et al., 2001). The study conducted by Rockafellar and Uryasev in 2000 only explored the minimization of CVaR subject to other constraints. However, an investor may wish to maximize their returns as a primary objective while keeping the risk to a minimum specified level. As seen with MPT, maximizing returns subject to a risk constraint is equivalent to minimizing risk subject to a return constraint.

Therefore, to show the equivalence of three formulations of the optimization problem, they prove that the following equations will generate the same efficient frontier if $\phi(\boldsymbol{x})$ is convex, $R(\boldsymbol{x})$ is concave, and the set **X** is convex (Krokhmal et al., 2001):

$$\min_{x} \phi(\boldsymbol{x}) - \mu_1 R(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbf{X}, \qquad \mu_1 > 0 \tag{2.14}$$

$$\min_{x} \phi(\boldsymbol{x}), R(\boldsymbol{x}) \geq \rho, \qquad \boldsymbol{x} \in \mathbf{X} \tag{2.15}$$

$$\min_{x} \; - R(\boldsymbol{x}), \;\; \phi(\boldsymbol{x}) \leq \omega \;, \boldsymbol{x} \in \mathbf{X}, \tag{2.16}$$

The efficient frontiers are obtained by varying $\rho,\ \mu_1\ and\ \omega$. This contribution to the literature was of importance since it extended optimization problems beyond minimizing CVaR subject to target returns to allowing for maximization of portfolio returns subject to CVaR constraints. The method of maximizing portfolio returns subject to CVaR constraints would be more sensible for this study as the main objective is to maximize the pension portfolio returns while maintaining a reasonably small amount of portfolio risk.

2.2.2.3 Investment performance

The Sharpe ratio is among the most used ratios to measure the investment performance of a portfolio (Sharpe, 1966). It is an indicator of how good an investment is through a reward to risk ratio. The Sharpe ratio is mathematically represented as:

$$Sharpe\ Ratio = \frac{E(R_p) - r_f}{\sigma_p} \tag{2.17}$$

Where

$E(R_p)$ = the expected return on a portfolio

r_f = the risk-free rate

σ_p = the standard deviation of portfolio returns

The risk factor in this ratio is the standard deviation of the portfolio returns, while the return factor is the mean returns of the portfolio evaluated. If there is an assumption that there is no risk-free rate, the ratio is simply the expected portfolio returns over the variation of returns. Concerning the capital market line, the Sharpe ratio is its slope.

However, estimating risk using standard deviation assumes a symmetrical distribution of portfolio returns, which may not be the case for a given portfolio. Hence the Sharpe ratio may not be suitable for evaluating the performance of portfolios with non-symmetrical distributions of portfolio returns. Hence this study will utilize a variation of the Sharpe ratio known as the conditional Sharpe ratio, which will replace the standard deviation as a measure of risk with CVaR.

2.3 Empirical studies

2.3.1 Pension fund investment in real estate and private equity

A study evaluated the profitability of investment in housing for pension funds in an attempt to establish it as a possible source of funding for Kenya's affordable housing gap (Kisia & Muthoni, 2019). They execute this through the comparison of returns from two hypothetical portfolios, one with traditional asset classes such as equity and fixed income and another with housing as an additional asset class. They used Mean-Variance optimization to obtain the weights of the optimal portfolio. The study concluded that it is profitable for pension funds to invest in housing. However, this required the assumption that asset classes follow a normal distribution, including housing. This contradicts studies that dispute the assumption of the normal or symmetrical distribution of alternative asset classes (Hollenwaeger, 2017).

A report investigated the effects of private equity investment in pension fund portfolios in Switzerland (Hollenwaeger, 2017). In order to carry out the empirical study, the paper employed the linearized mean-CVaR method for portfolio selection (Rockafellar & Uryasev, 2000). The rationale behind this decision was to relax the normal distribution assumption on the alternative assets included in the portfolio, which were hedge funds and private equity. Within the asset allocation, listed private equity was utilized as a proxy for traditional private equity. It seems appropriate, as noted in the study, for pension funds to invest in listed private equity (LPE) since daily pricing is available, and it is more liquid than traditional methods of private equity investment.

The hybrid nature of listed private equity as a small-cap stock and its relation to limited partnerships' core business means that it can fall within the regular equity, small-cap stock, and alternative asset class. As a result, the study tests the effect of LPE's hybrid nature on portfolio optimal allocation and returns by varying the asset classes in which private equity falls under (Hollenwaeger, 2017). The study was not conclusive of the asset class that LPE should be invested under though empirical results show that LPE improves the value of pension fund portfolios.

Hence, this study will use listed vehicles on Kenya's Nairobi Stock Exchange as a proxy for private equity investment in the pension fund portfolios to be designed. This seems appropriate since listed vehicle stocks are robust to the liquidity risk faced by traditional private equity.

Chapter 3: Methodology

3.1 Introduction

This chapter aims to outline the methodology that will be used for this study.

3.2 Research design

The study used an experimental design to investigate the impact of alternative assets on the pension fund portfolio. The control group used was the portfolio with the traditional asset classes of equity and fixed income assets. In contrast, the experimental group was the portfolio with either private equity, real estate, or a combination of both as additional asset classes to the original portfolio. The risk and return characteristics of the experimental group will be compared to the traditional-asset portfolio to determine the profitability of investment in alternative asset classes.

3.3 Data sources

Information on asset class investment limits for pension fund portfolios was sourced from the Retirement Benefits Authority website.

Secondary time-series data on historical prices of stocks and fixed-income assets were sourced from Thomson Reuters terminal to use as asset inputs to the original portfolio. The nature of the data will allow for monitoring of portfolio performance over the years 2013 to 2019.

Data on interest rates were obtained from the Central Bank of Kenya's website, also from 2013 to 2019.

The housing consumer price index data were obtained from Kenya National Bureau of Statistics from 2013 to 2019. In this study, it will serve as a proxy for housing price returns. For private equity, the proxy used in this study will be the stocks of listed vehicles Centum and TransCentury. It is assumed that stock holdings of the investment firms will indirectly expose the portfolio to private equity investments.

3.4 Analysis procedure

3.4.1 Estimation of affordable housing deficit

Since the study wishes to investigate the ability to use pension funds in providing affordable housing, the calculation of the amount required to purchase affordable housing is required. This is done by taking the deficit number of housing units and multiplying it by the cost per unit of affordable housing. For this study, the cost per unit of affordable housing is assumed to be equal to KES 1 million (Noppen, 2013). The tax levy and budget allocation towards housing will be deducted from the initial total amount to get the housing finance gap. With the use of the market returns of the pension fund portfolio, it is possible to obtain the amount that can purchase affordable housing and hence the number of units too.

The tax levy component will require some calculation to estimate the amount of tax deducted from salaries. Hence average salary of income brackets as specified in the KNBS statistical report will be utilised. In addition, the data on number of salaried workers per specified income bracket will be taken into account.

Hence the total deficit is calculated as:

$$(deficit\ units\ \times cost\ per\ unit\ of\ housing) - tax\ levy - budget\ allocation \quad (3.1)$$

The tax levy is calculated as:

$$tax\ rate \times total\ average\ salary \quad (3.2)$$

Where the total average salary is the sum product of the vector of average salary per income bracket and number of salaried workers per income bracket.

3.4.2 Determination of the nature of the asset class data

The Mean-CVaR method of portfolio optimization will be used for this study, where the conditional value at risk will replace variance as the measure of risk. The input to the portfolio model will be monthly historical asset returns; therefore, it will be a necessity to calculate them from the historical asset price data sample. The logarithmic returns will be used because of their preferable qualities over arithmetic returns in financial applications:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (3.2)$$

Where

R_t is the return of the asset at time t

P_t is the price of the asset at time t

P_{t-1} is the price of the asset at time t-1

To answer the first objective of the study, the descriptive statistics of the returns data will be obtained using the statistical software R. These include the mean, median, skewness, and kurtosis of returns, among others such as the standard deviation, minimum and maximum values of the data. Then plots such as histograms and Q-Q plots will be used to visualize the data distribution using the fitdistrplus package. The Shapiro-Wilk normality test will be used to test if the data follows a normal distribution.

3.4.3 Asset classes in the portfolios

The aim of designing multi-class portfolios instead of single class portfolios is to benefit from the effects of diversification. Investment in asset classes that are negatively correlated (move in different directions at points of the economic cycle) with each other will significantly reduce the risk in the portfolio, which is essential for pension funds.

3.4.3.1 Equity

In this asset class, the stocks will be selected from the NSE 20 Index. These stocks will be analysed for their risk-adjusted returns using the conditional Sharpe ratio. The stocks with the highest 5 conditional Sharpe ratios will make up the equity asset class.

The conditional Sharpe ratio is calculated as:

$$CSR = \frac{E(R_i) - r_f}{CVaR_\alpha(x)} \tag{3.3}$$

Where:

$E(R_i)$ = the expected returns of the i^{th} security

r_f = the risk-free rate in the market

$CVaR_\alpha(x)$ = the conditional value at risk of the asset at the α^{th} percentile.

The 95th percentile will be used to estimate the CVaR from the historical asset losses (negative of the returns). To obtain the empirical CVaR of the assets let $X_1, X_2, \ldots, X_T$

be a set of random variables representing the asset's losses over time. Let $X_{(1)}, X_{(2)}, \ldots, X_{(T)}$ represent the asset losses in ascending order from $i = (1), (2), \ldots, (T)$. The CVaR of the assets would then be:

$$CVaR_{\alpha}(x) = \frac{1}{T - T\alpha} \sum_{i=T\alpha}^{T} X_{(i)} \tag{3.4}$$

3.4.3.2 Fixed-income government securities

The second asset class to be considered for the portfolio is fixed income assets consisting of T-bonds. T-bonds issued before 2013 and maturing after 2019 will be included in the portfolio since the investment horizon has been limited to 6 years. This limits the selection to T-bonds of 10-year, 15-year, 20-year, and 25-year duration.

This asset class will also consist of treasury bills. They are included into the portfolio for their liquidity benefits and low risk levels.

3.4.3.3 Real estate

The traditional asset portfolio will consist of equity and fixed income security asset classes. Real estate will be included as the third asset class to the portfolio, and a comparison will be made on the risk-return profile of the portfolio with and without this asset class.

3.4.3.5 Private equity

Private equity will be included as the fourth asset to the traditional portfolio, if including real estate as the third asset class. The study will compare the risk-return profile of the portfolio with private equity as an asset class, with and without real estate as the third asset, to that of the traditional asset class portfolio.

3.4.4 Portfolio Optimisation

For this study, the CVaR will be the risk constraint of the portfolio optimization problem, with maximization of the portfolio's returns as the objective. The ROI and Performance Analytics package will be used to carry out the optimization of the portfolio in R.

3.4.4.1 Optimisation of sub-portfolios

Each asset class is treated as a sub-portfolio of securities, which will need to be optimised to get the optimal returns of the asset classes. Assume n securities in each asset class, where the initial holdings of securities in each asset class is denoted as $\boldsymbol{x}^0 = (x_1^0, x_2^0, \dots, x_n^0)^T$. The return matrix, in which returns will be calculated using equation (3.1), is denoted by $\boldsymbol{R}$. Denote the expected returns of each security as $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_n)^T$. The expected return of each asset class will be obtained by the equation:

$$R(x) = \boldsymbol{\mu}^T \boldsymbol{x}^0 \tag{3.5}$$

The CVaR of the asset class is the average of the loss amounts that exceed the α-VaR of the loss distribution. The CVaR of the sub-portfolios, $CVaR_\alpha(\boldsymbol{x})$, (as linear constraints) can be formulated as:

$$\theta + (1-\alpha)^{-1} \sum_{i=1}^{J} p_i z_i \leq \rho \tag{3.6}$$

$$z_i \geq -\boldsymbol{R}_i^T \boldsymbol{x} - \theta, \qquad z_i \geq 0 \tag{3.7}$$

Where p_i is the probability associated with the i^{th} scenario and ρ is the value that the CVaR cannot exceed at the α confidence level. We also assume no short selling of securities is allowed in the portfolio; hence we constrain the weights of the assets such that they cannot fall below zero. Hence a mathematical representation of the portfolio problem is as follows:

$$\max_{x \in \mathbb{R}^n} R(\boldsymbol{x}) \tag{3.8}$$

Subject to:

$$CVaR(\alpha, \boldsymbol{x}) \leq \rho \tag{3.9}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3.10}$$

$$x_i \geq 0 \tag{3.11}$$

The solution to this problem will provide the optimal allocation of securities within the asset classes, which we will multiply with the returns of the securities to get the optimal returns of the asset classes. The VaR of the portfolio will be obtained as a consequence of solving the optimization problem.

3.4.4.2 Optimisation of the pension fund portfolio

Assume a portfolio of N asset classes in a portfolio P, where $\boldsymbol{p^0} = (p_1^0, p_2^0, \dots, p_N^0)^T$ are the initial respective asset allocations for the portfolio. Let the vector of expected returns of the portfolio's asset class be denoted by $\boldsymbol{r} = (r_1, r_2, \dots, r_N)^T$, which is estimated using the return matrix $\boldsymbol{R_p}$ obtained from the optimisation of the asset classes. The vector $\boldsymbol{p} = (p_1, p_2, \dots, p_N)^T$ is the optimal allocation of the portfolio in each asset class. We would like to optimise the portfolio using the mean-CVaR framework. The return on the portfolio to be maximised would be obtained through the following calculation:

$$R(x) = \boldsymbol{r}^T \boldsymbol{p^0} \tag{3.12}$$

The CVaR of the portfolio will be calculated in a similar manner to that of the asset classes/sub-portfolios.

Since pension funds have regulatory constraints on the amounts investable in each asset class, box constraints will be a part of the optimization problem. They allow for the implementation of lower and upper bounds of weights allocated to the portfolio's various asset classes. In addition to the constraints, the weights of the assets should sum up to one, and none should fall below zero. The latter constraint implies that the portfolio will be long-only; that is, no short-selling of any assets is allowed.

A mathematical representation of the optimization problem is as follows:

$$\max_{\boldsymbol{p} \in \mathbb{R}^N} R(\boldsymbol{p}) \tag{3.13}$$

Subject to:

$$CVaR(\alpha, \boldsymbol{p}) \leq \rho \tag{3.14}$$

$$\sum_{i=1}^{N} p_i = 1 \tag{3.15}$$

$$p_i \geq 0 \tag{3.16}$$

Where

$R(\boldsymbol{p})$ is the return of the portfolio

$\boldsymbol{p}$ is the weight vector of the portfolio where each p_i will be constrained as per the regulatory requirements given by the RBA.

α is the confidence level. Values of 0.90, 0.95, and 0.99 will be used in the analysis.

ρ is the constraint value of the expected shortfall of the portfolio

The following optimisation problem should provide the solutions to the optimal asset allocation $\boldsymbol{p}$ and the VaR of the pension fund portfolio as a by-product.

With the returns obtained from the designed optimal portfolios, the return amount from investment will be compared with the total housing deficit amount to determine the number of units and the total amount of permissible pension fund income that can be allocated towards affordable housing. For example, if the portfolios created in this study produce returns of 5% and KES 1 billion was invested into the portfolio at the outset, then KES 50 million can be allocated towards affordable housing projects. The allocated amount would then be divided by the cost per unit of affordable housing to get the number of units that can be purchased.

Chapter 4: Analysis

4.1 Introduction

This chapter displays the analysis of the results. The following subheadings detail the analysis as per the objectives of this study.

4.2 Nature of distribution of asset class and security data

Since the asset classes are sub-portfolios of securities, we analyse the distribution of the securities data. First we take each asset class defined in the previous chapter and obtain the descriptive statistics of the individual securities. Then we plot CDFs and histograms for a visual display of the data distribution, followed by a Shapiro-Wilk test to test the normality of the data.

4.2.1 Descriptive statistics

The tables below detail the summary statistics of the securities forming the asset classes. They are visually captured in the histograms and Q-Q plots in the appendices of this report.

	Centum	*TransCentury Ltd.*
Mean	0.0104	-0.0267
Median	0.0000	-0.0362
Standard Deviation	0.0887	0.1117
Minimum	-0.2308	-0.3072
Maximum	0.3269	0.2855
Skewness	0.1445	0.2259
Kurtosis	4.7005	3.2450

Table 1: Descriptive statistics for the private equity asset class

The mean return of Centum's stock is positive at 1% in contrast to TransCentury Ltd. Stock with a negative 2.7% percent. This is indicative of better returns expected from an investment in CTUM stock compared to TCL. The standard deviation of returns for TCL is higher than CTUM making it a much riskier investment.

	91 day T-bill	*182 day T-bill*	*364 day T-bill*	*10 yr. T-bond*	*15 yr. T-bond*	*20 yr. T-bond*	*25 yr. T-bond*
Mean	0.0861	0.0937	0.1018	-0.0004	-0.0006	-0.0004	-0.0004
Median	0.0834	0.1029	0.1088	0.0000	0.0000	0.0000	0.0000
Standard Deviation	0.0201	0.0333	0.0347	0.0417	0.0305	0.0290	0.0335
Minimum	0.0592	0.0007	0.0009	-0.1409	-0.0867	-0.1003	-0.1175
Maximum	0.2165	0.2152	0.2161	0.1886	0.0943	0.0850	0.1332
Skewness	3.7245	-1.1731	-1.5436	0.3400	-0.0351	-0.2692	0.1993
Kurtosis	24.603	7.8893	8.0849	9.3503	4.5404	5.5018	7.7709

Table 2: Descriptive statistics for the fixed income asset class

Looking at the table above, the fixed income assets have relatively lower volatility of returns about their mean, making them less riskier of an investment compared to the other asset classes. The table also reveals increasing standard deviation with increasing T-bill maturity, which is expected due to increasing risk of longer term investment. However, this is not the case with T-bonds whereby the volatility of returns lowers with the 15-year bond and further to 2.9% with the 20-year bond, increasing significantly with the 25-year bond to 3.4%. The mean returns of the T-bonds are significantly poorer than that of the T-bills, yet both have similar volatility magnitudes, making T-bonds a worse investment choice to T-bills in Kenya.

	Returns
Mean	0.0041
Median	0.0029
Standard Deviation	0.0076
Minimum	-0.0181
Max	0.0378
Skewness	1.6952
Kurtosis	9.8440

Table 3: Descriptive statistics for the housing asset class

Returns in the real estate class are relatively low compared to other securities at 0.4%, which would not be beneficial for the portfolio from a returns perspective. However, it has the lowest standard deviation among the other asset classes, which is likely to give it an advantage in terms of weight allocation in the portfolios to be constructed.

	BRIT	*COOP*	*EQTY*	*KCB*	*SCOM*
Mean	0.0048	0.0071	0.0096	0.0071	0.0218
Median	-0.0010	0.0030	0.0000	0.0088	0.0228
Standard Deviation	0.1110	0.0848	0.0926	0.8863	0.0605
Minimum	-0.2924	-0.2756	-0.3234	-0.2231	-0.1424
Maximum	0.3709	0.2011	0.2288	0.3194	0.1448
Skewness	0.6683	-0.2267	-0.3412	0.4008	-0.4045
Kurtosis	4.5862	3.5491	4.7371	4.4224	3.0581

Table 4: Descriptive statistics for the equity asset class

The best performing stock among the selected is Safaricom's with an average return of 2% and the lowest standard deviation at 6.1%. Generally, the stocks selected are high risk as can be seen with their standard deviations. The returns of the stocks are rather sub-optimal with all other than Safaricom's being less than 1%. The stocks have seen significant highs and lows in terms of returns. The skewness and kurtosis of the securities in this asset class are closest to representing figures that would be seen in a normal distribution compared to the other asset classes.

	Equity	*Fixed Income*	*Private Equity*	*Housing*
Mean	0.0130	0.0562	-0.0018	0.0041
Median	0.0198	0.0563	-0.0013	0.0029
Standard Deviation	0.0609	0.0210	0.0719	0.0076
Minimum	-0.1308	0.0048	-0.1444	-0.0181
Maximum	0.1778	0.1612	0.2806	0.0378
Skewness	-0.1843	1.1404	0.6047	1.6952
Kurtosis	3.1452	10.6894	4.6776	9.8440

Table 5: Descriptive statistics of the optimised asset classes

The table displays the summary statistics of the optimised asset class returns. Among the asset classes, fixed income government securities offer the highest returns, with the lowest volatility in returns seen in the housing asset class. Private equity as a class performs poorly, with low negative returns and relatively higher volatility of returns around the mean compared to the other asset classes. Equity as an asset class does not provide higher returns than the fixed income asset class, which is rather atypical and is likely to lead most portfolios to favour investment in fixed income instead of equity.

4.2.2 Goodness of fit of Normal distribution

Part of determining the nature of asset class distribution includes testing the goodness of fit of the data. The Shapiro-Wilk normality test was used to test the normality assumption for distributions of data for all the securities within the asset class. The selected significance level was 5% for the test and the results obtained for each asset class from the analysis are as follows:

Asset Class	Private equity		Housing
	Centum	*TCL*	*Returns*
W	0.9726	0.98275	0.81674
p-value	0.07398	0.3231	7.965e-09

Table 6: W statistic and p-value of both the private equity and housing asset classes

Under the private equity asset class, Centum's stock has a p-value of about 0.07 which is greater than the 5% significance level, hence it can be concluded with 95% confidence that the stock data has a normal distribution. The same applies to TransCentury Ltd. Share which has a p-value much greater than the significance level hence we fail to reject the null hypothesis and conclude that it is possible with 95% confidence to model the stock with a normal distribution.

Housing returns data on the other hand have a p-value much less than 5% hence we reject the null hypothesis and conclude with 95% confidence that the data does not follow a normal distribution. This is visible in the histogram and CDF plot which show plot deviations from that of the normal distribution. The Q-Q plot also displays deviations at the tails which means that the housing returns data has fatter tails than that of the normal distribution.

	Fixed Income						
	10 Yr. T-bond	*15 Yr. T-bond*	*20 Yr. T-bond*	*25 Yr. T-bond*	*91 day T-bill*	*182 day T-bill*	*364 day T-bill*
W	0.861	0.959	0.934	0.88	0.7	0.713	0.674
p-value	2.2e-07	0.0088	0.00031	1.2e-06	8.3e-12	1.6e-11	2.3e-12

Table 7: W statistic and p-value of the fixed income asset class

Fixed income data also proves not to follow a normal distribution as the data p-values are all much lower than 5%. Hence we can conclude with 95% confidence that fixed income instrument data does not follow a normal distribution. The results were as expected, hence the need to optimise the portfolio using CVaR as an alternative measure of risk to variance.

	Equity				
	BRIT	*COOP*	*EQTY*	*KCB*	*SCOM*
W	0.953	0.989	0.963	0.977	0.984
p-value	0.0039	0.7135	0.0169	0.1371	0.4169

Table 8: W statistic and p-value of the equity asset class

In the equity class, we reject the notion that Britam and Equity Bank's stock data can be concluded with 95% confidence to follow a normal distribution. However, the other three stocks arguably can be modelled with a normal distribution.

	Optimised Asset Classes			
	Equity	*Fixed Income*	*Private Equity*	*Housing*
W	0.988	0.859	0.969	0.817
p-value	0.6217	1.95e-07	0.037	7.96e-09

Table 9: W statistic and p-value of the optimised asset classes

From the asset classes, we can see that only equity can be assumed to follow a normal distribution with 95% confidence. The results above display the need to model the data using a portfolio optimisation method other than MVO.

4.3 Optimal Portfolios

The asset classes were optimised using Mean-CVaR method to obtain the optimal weights of the securities in each asset class. The next step of the analysis included using matrix multiplication of the weights and monthly returns of each security to obtain each asset classes' optimal set of monthly returns. These returns made part of the larger pension fund portfolio returns.

Table 10 and figure 1 display the correlation of the asset classes:

	Equity	*Fixed Income*	*Private Equity*	*Housing*
Equity	1.0000	0.0772	0.0466	-0.0471
Fixed Income	0.0772	1.0000	-0.1387	0.1024
Private Equity	0.0466	-0.1387	1.0000	-0.0356
Housing	-0.0471	0.1024	-0.0356	1.0000

Table 10: Correlation matrix of the four asset classes of the portfolio

The correlation matrix reveals that the equity asset class is most negatively correlated with the housing asset class, fixed income negatively correlated with housing, private equity negatively correlated with fixed income and housing and housing negatively correlated with equity and private equity. The magnitude of correlation between the asset classes is relatively low, with the highest level of correlation being between housing and fixed income at approximately 10%. This implies diversity in the portfolios to be constructed.

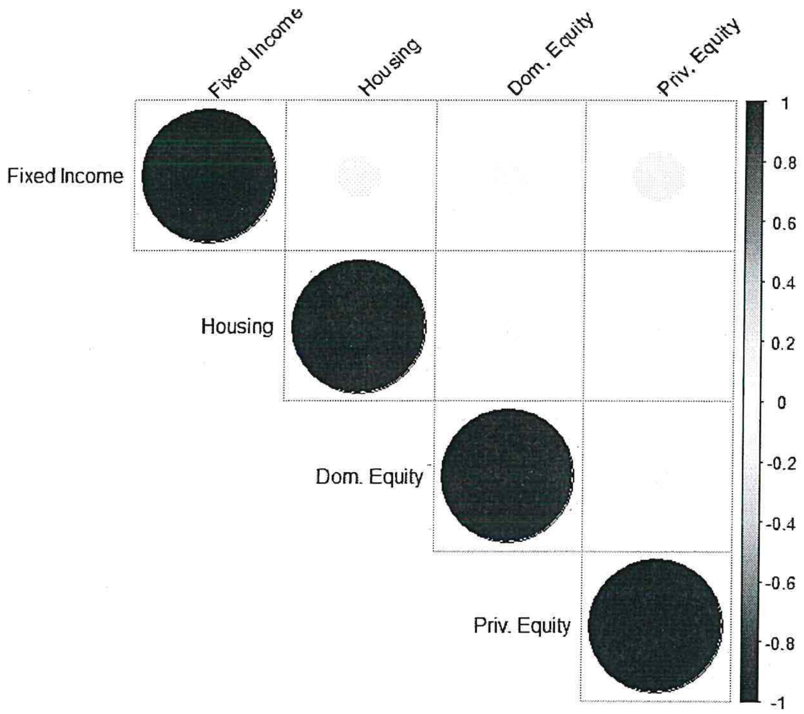

Figure 1: Correlation plot of the four asset classes

The portfolio of optimal asset class returns was optimised at 90%, 95% and 99% confidence levels. The portfolios at each confidence interval were constrained to long-only portfolios and each asset class constrained in weightings as per RBA regulations using box constraints. A comparison was made between the traditional asset portfolio and the portfolios including the other alternative asset classes.

The results of each optimisation are as summarised below in Table 11:

		Weights			
		Traditional asset portfolio	*Portfolio without Housing class*	*Portfolio without Private Equity class*	*Portfolio with all asset classes*
90% confidence level	*Equity*	10.00%	4.15%	2.94%	2.94%
	Fixed Income	90.00%	90.00%	90.00%	90.00%
	Private Equity	0.00%	5.85%	0.00%	0.00%
	Housing	0.00%	0.00%	7.06%	7.06%
Returns		5.19%	5.13%	5.12%	5.12%
CVaR		-1.4%	-1.50%	-1.56%	-1.56%
Performance Ratio		3.71	3.42	3.28	3.28
95% confidence level	*Equity*	10.00%	9.27%	3.97%	3.97%
	Fixed Income	90.00%	90.00%	90.00%	90.00%
	Private Equity	0.00%	0.73%	0.00%	0.00%
	Housing	0.00%	0.00%	6.03%	6.03%
Returns		5.19%	5.18%	5.13%	5.13%
CVaR		-0.90%	-0.91%	-0.99%	-0.99%
Performance ratio		5.77	5.69	5.18	5.18
99% confidence level	*Equity*	10.00%	10.00%	0.00%	0.00%
	Fixed Income	90.00%	90.00%	90.00%	90.00%
	Private Equity	0.00%	0.00%	0.00%	0.00%
	Housing	0.00%	0.00%	10.00%	10.00%
Returns		5.19%	5.19%	5.10%	5.10%
CVaR		-0.53%	-0.53%	-0.68%	-0.68%
Performance Ratio		9.79	9.79	7.50	7.50

Table 11: Summary of results from the portfolio optimisations

The best performing portfolio in all 3 confidence intervals is the traditional asset portfolio with equity and fixed income assets, with a return-risk ratio of 3.78, 5.77 and 9.79 in the 90, 95 and 99 percent confidence interval scenarios respectively. It is also worth noting that both the portfolio with all assets and the portfolio with all assets except private equity have the same portfolio performance. In addition to this, most optimal portfolios do not include private equity in the asset class. This is most likely due to the high risk levels of the private equity asset class.

4.4 Use of Pension Funds to provide housing

This study aimed at eventually determining the number of units that could be constructed with an optimal pension fund portfolio. This required inputs such as the initial value of the portfolio, which is the current value of portfolio assets under management, approximate number of deficit units of housing and the assumed cost per unit of housing. Other required puts include tax levy, average salary per income bracket, approximated number of salaried workers and the budget allocation. This resulted in an approximated deficit of about KES 2 trillion.

The return of the portfolio with the highest performance ratio at 95% CVaR was used to determine the value of the portfolio at the end of the investment horizon, which equated to about KES 1.18 trillion. The number of units that could be financed by the pension fund portfolio was calculated by dividing the portfolio value by the cost per unit which resulted in a value of 1,189 units.

The result is not very impressive over 5 years of investment, considering the size of the deficit units and the investment horizon. This could be attributed to the performance of the hypothetical portfolios in general, which tended to prefer maximum possible investment in fixed income securities compared to the other asset classes.

Chapter 5: Conclusion

5.1 Conclusion of the study

The study aimed at determining the effectiveness of pension funds in addressing the affordable housing finance gap. This objective wad further explored by addressing the research questions below:

- If asset class data could be assumed to follow a normal distribution, requiring the need to utilize an alternative optimization method to MVO
- How profitable it would be for pension funds to invest in alternative asset classes such as real estate and private equity
- How pension funds could reduce the affordable housing finance gap through portfolio returns

The greatest challenge faced while approaching the objectives and questions of the study included the lack of access to private equity and real estate index data. The lack of access to private equity fund data was addressed through the use of proxy data (listed stock data of Centum and TransCentury Ltd.) which mimicked listed private equity investment vehicles in the NSE. The housing CPI served as a proxy for real estate asset class data.

Visualizations such as histograms and Q-Q plots with Shapiro-Wilk normality tests were carried out on the asset classes to justify the need to use the Mean-CVaR method of optimization, which is more robust compared to MVO with regards to the distribution of asset data provided. The tests proved that all other asset classes other than equity data could not be assumed to follow a normal distribution, which is similar to the justifications for the Mean-CVaR model as seen in (Krokhmal et al., 2001).

The asset classes were optimized as sub-portfolios of the larger pension fund portfolio. Before determining the optimal weights of securities for the equity asset class, the stocks were selected through ranking NSE 20 stocks by highest Conditional Sharpe Ratio (CSR). The exercise proved that the ranking of the 5 best asset classes was very similar to the ranking provided by the original Sharpe Ratio in Sharpe's 1966 paper. This was very likely the case since the normality tests proved that the equity class follows a normal distribution, meaning that CVaR would provide similar results to standard deviation, a point noted in (Krokhmal et al., 2001).

Sequentially, the study analyses the best combinations of asset classes an optimal pension portfolio should hold while adhering to the regulations set by the RBA to determine the profitability of investment in asset classes other than equity and fixed income. Identifying the best optimal pension portfolio was made possible through the comparison of a traditional asset portfolio with portfolios inclusive of private equity and housing, individually and together. The analysis recommended a traditional asset portfolio for all CVaR confidence levels for having the best performance ratio (risk-adjusted returns) among all constructed portfolios, which contradicts the sentiment propagated by the CEO of RBA and other investors (EAVCA, 2019). The analysis also highlighted the ineffectiveness of investment in real estate in Kenyan pension funds, which parallels the conclusion in (Kisia & Muthoni, 2019).

In all constructed portfolios, the optimization recommended maximum allocation of 90% of pension asset value towards fixed income government securities. This was very likely considering that a review of descriptive statistics of the asset classes displayed fixed income government securities having higher expected returns compared to the equities asset class. This results of this study illuminate the general poor performance of stocks in the NSE 20, which propagates the need to extend research to expand the selection of asset classes in the portfolio within and outside the domestic financial market and evaluate the effectiveness of these asset classes within the portfolios constructed.

The analysis is inclusive of a model used to estimate the units of housing that could be purchased using returns from an optimal pension fund portfolio. The calculation led to an estimation of a financial deficit of KES 2 trillion and with the projected KES 1.18 trillion return in the five-year investment horizon, an expected 1,189 units could be constructed using the funds. Compared to the estimated deficit units of 2 million, it is a small number of units. In addition, due to the estimated urbanization rate of 4.4% the deficit units are expected to grow further during the investment horizon. Hence with the best performing 95% CVaR portfolio constructed, pension funds are useful in providing majority of the financing required to reduce the finance gap. However, the pension fund cannot be used on its own to provide units of housing to cover the deficit units.

5.2 limitations of the study

Some limitations include the choice of proxies for the asset classes. For example, the choice of representatives for the private equity asset class were stocks of Centum and TransCentury Ltd., which are companies known to make significant investment in pension funds. The performance of the private equity class as a whole and within portfolio investments was affected by the poor performance of the stocks in general, particularly that of TransCentury Ltd. In addition, the choice of proxy for real estate investment was Kenya's housing CPI with base year of 2009 which was noted in previous research as not the most ideal proxy.

Moreover, the model is a very simplistic representation of the method of optimisation implemented by pension funds. First, the study utilises Mean-CVaR which is an asset-based style of optimisation. Since most pension funds have an objective of meeting obligations when due as a result of heavy statutory regulations and expectations imposed on them, a liability-driven optimisation method would be more appropriate. Therefore, Mean-CVaR optimisation may not be the most representative method of pension fund portfolio optimisation in practice.

The model used to determine the number of housing units that could be purchased using pension fund portfolio returns was also very simplistic in the assumptions used. For example, the assumption that some parameters such as the number of salaried workers (used in the calculation of tax levied) would grow at a constant rate equivalent to the rate of urbanisation. Hence, further research could be carried out in determining more accurate methods of estimating units that could be purchased to cover the housing deficit.

5.3 Recommendations for future research

To further enhance the effectiveness of this study, more research would need to be carried out in constructing well performing portfolios with relatively higher risk-adjusted returns than the portfolios in this study had generated. This would include evaluation of other asset classes that could be included within the portfolio as per investment regulations provided by the RBA. The derivatives market and assets (equities and bonds) outside the Kenyan market would be worth investigating for future work.

The study could also be further expanded by identifying better proxies for private equity and real estate asset classes. Centum and TransCentury Ltd. Stocks are listed on the exchange and have performed relatively poorly, which adversely affected the preference of investment in the asset class as a whole and within the larger pension fund portfolio. The housing CPI used in the study has a base year of 2009 which does not make it representative of real estate investment in the future. Hence, there is a need to look into proxies other than listed investment vehicles and housing CPI with base year of 2009 as representations of private equity and real estate respectively.

Stronger assumptions would need to be made to construct the model that estimates the number of units that could be purchased using pension funds assuming direct investment in affordable housing. Better assumptions to estimate the tax levied in particular would be useful for future research.

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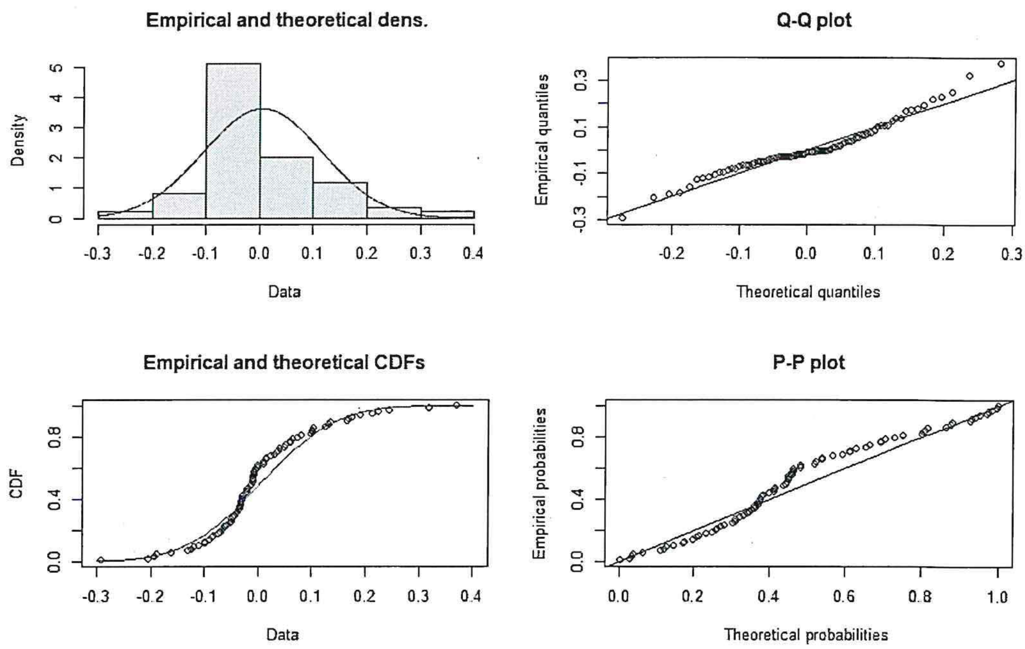
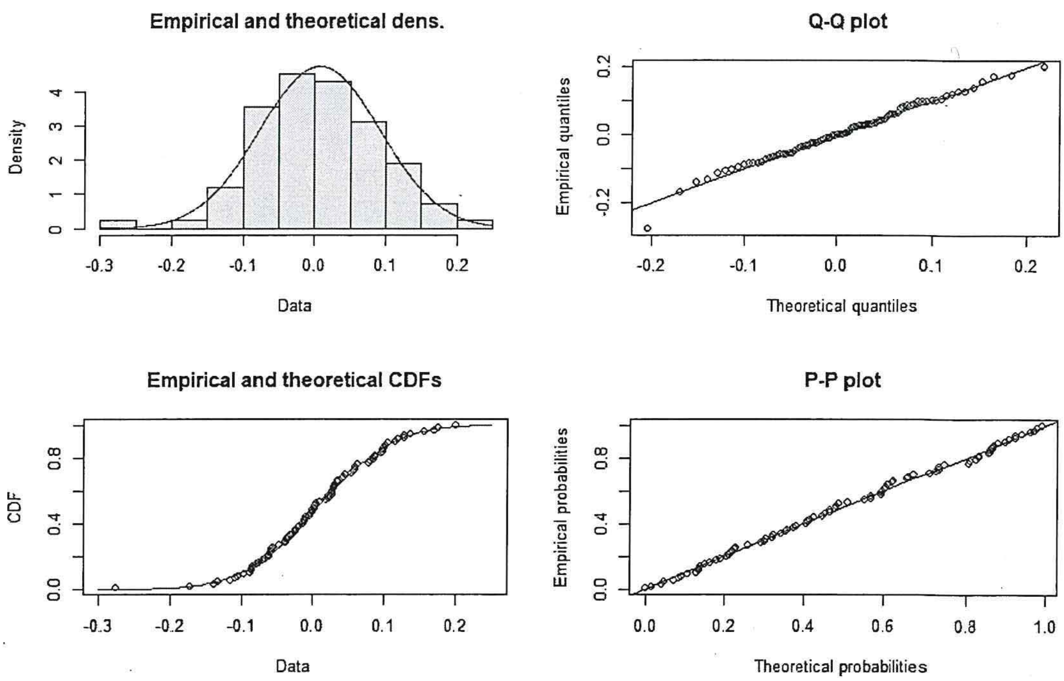
Appendices

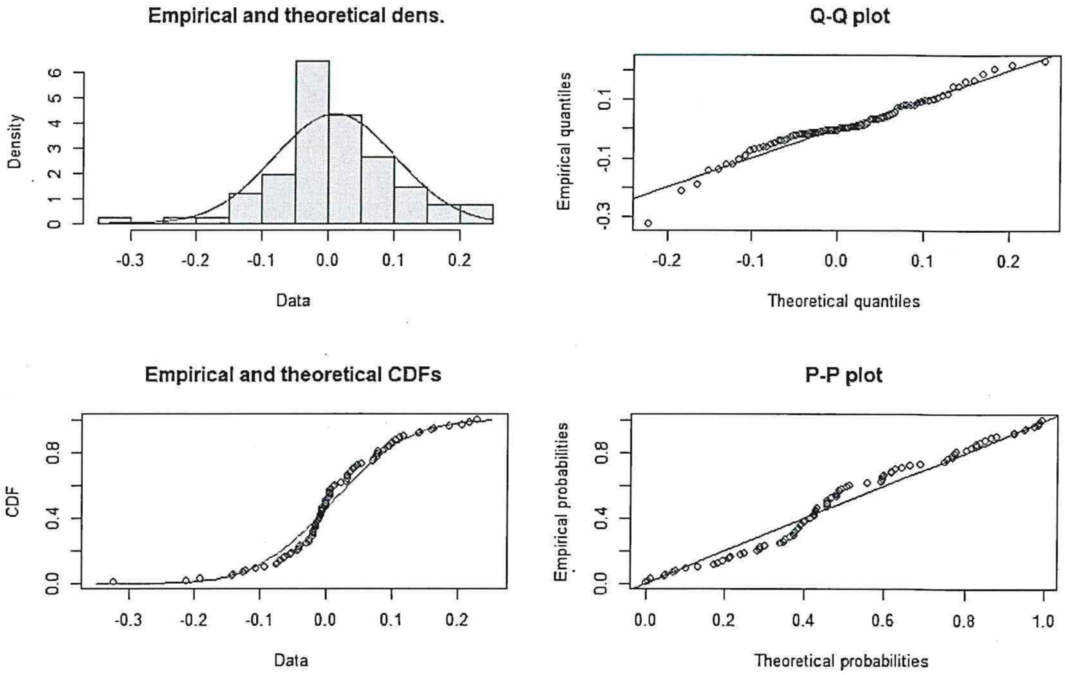
Appendix 1: Histogram, Q-Q plot, P-P plot and CDF of BRIT stock data



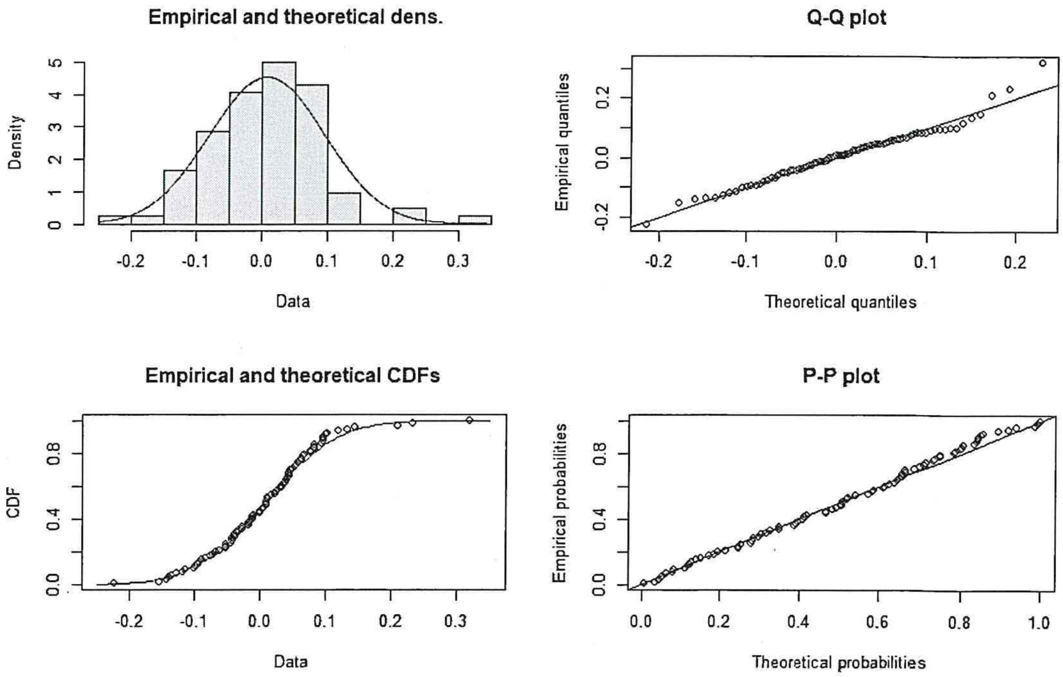
Appendix 2: Histogram, Q-Q plot, P-P plot and CDF of COOP stock data



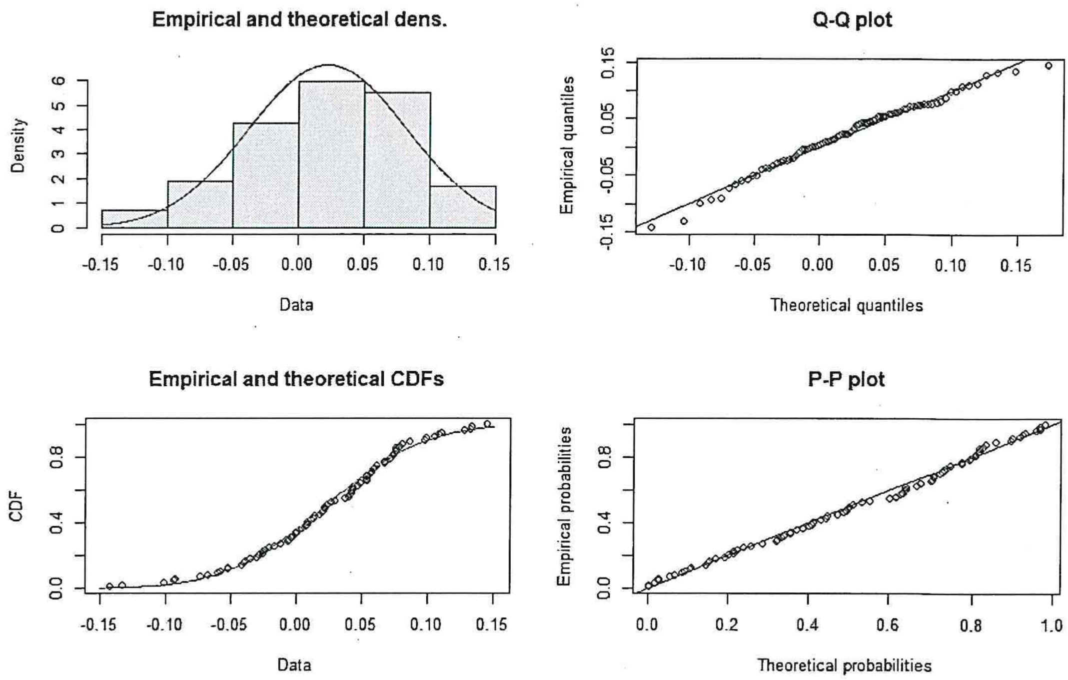
Appendix 3: Histogram, Q-Q plot, P-P plot and CDF of EQTY stock data



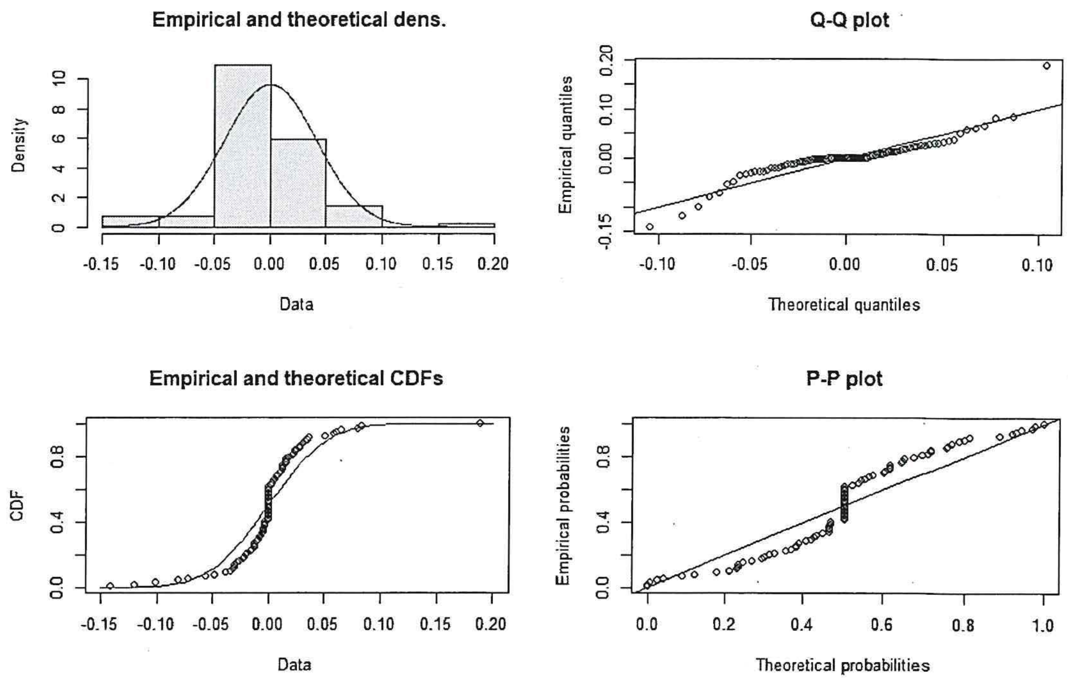
Appendix 4: Histogram, Q-Q plot, P-P plot and CDF of KCB stock data



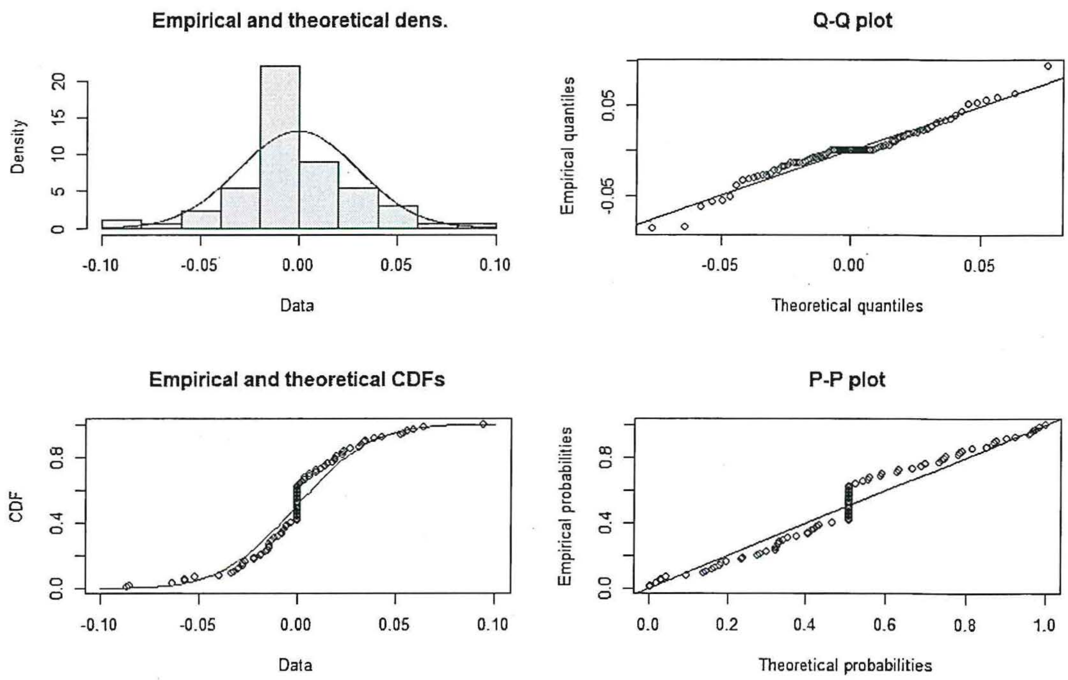
Appendix 5: Histogram, Q-Q plot, P-P plot and CDF of SCOM stock data



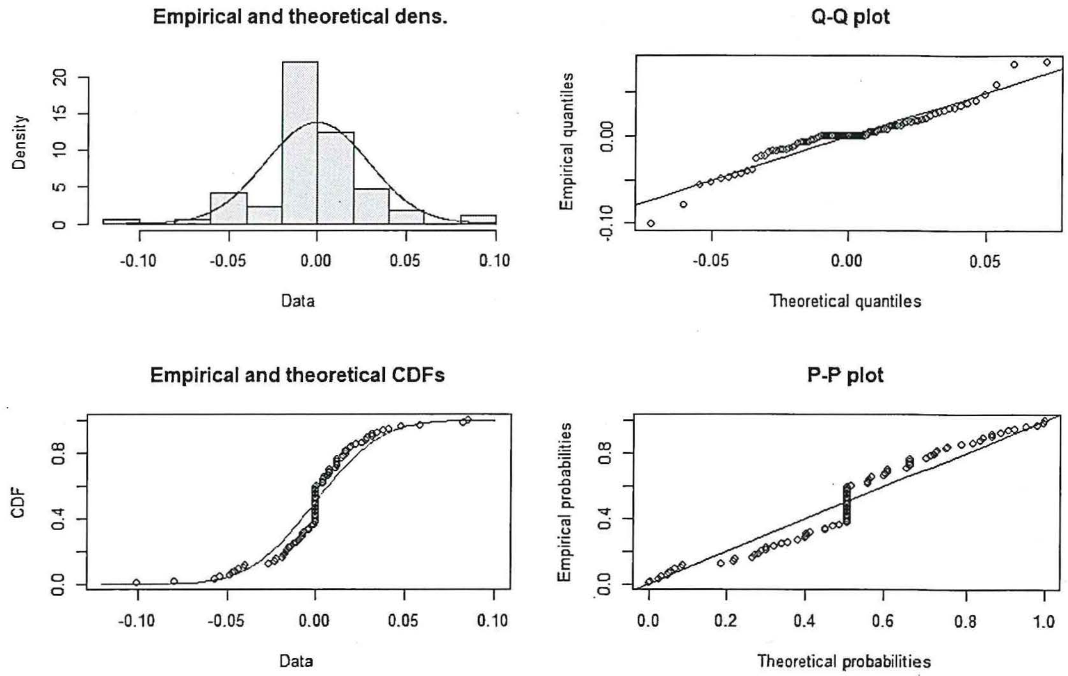
Appendix 6: Histogram, Q-Q plot, P-P plot and CDF of 10 yr. T-bond data



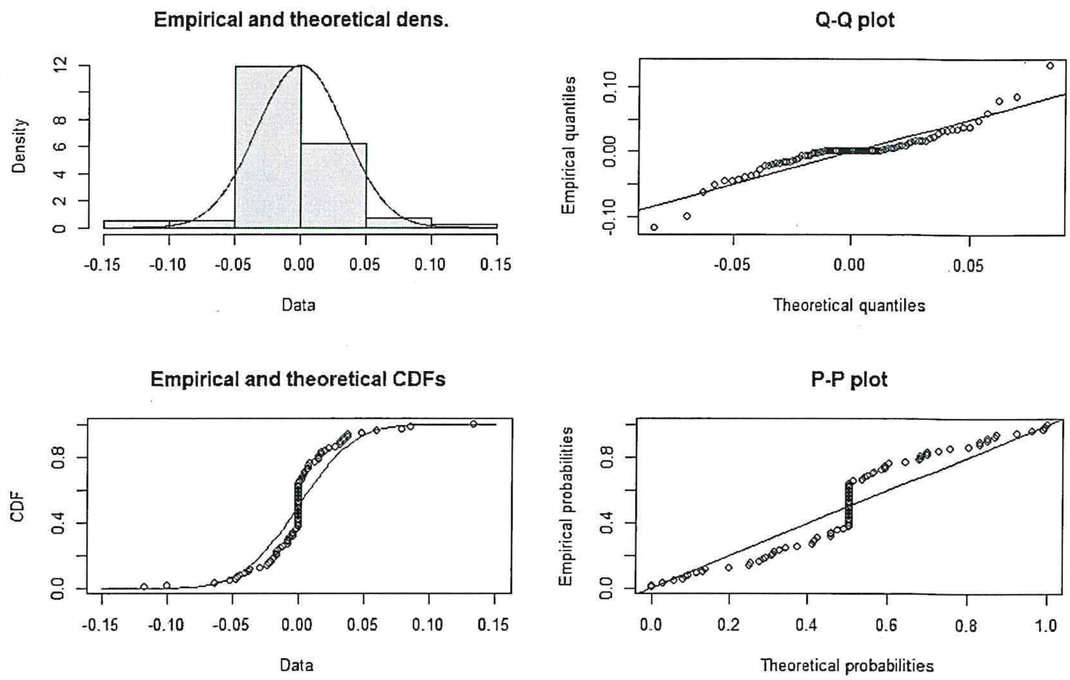
Appendix 7: Histogram, Q-Q plot, P-P plot and CDF of 15 yr. T-bond data



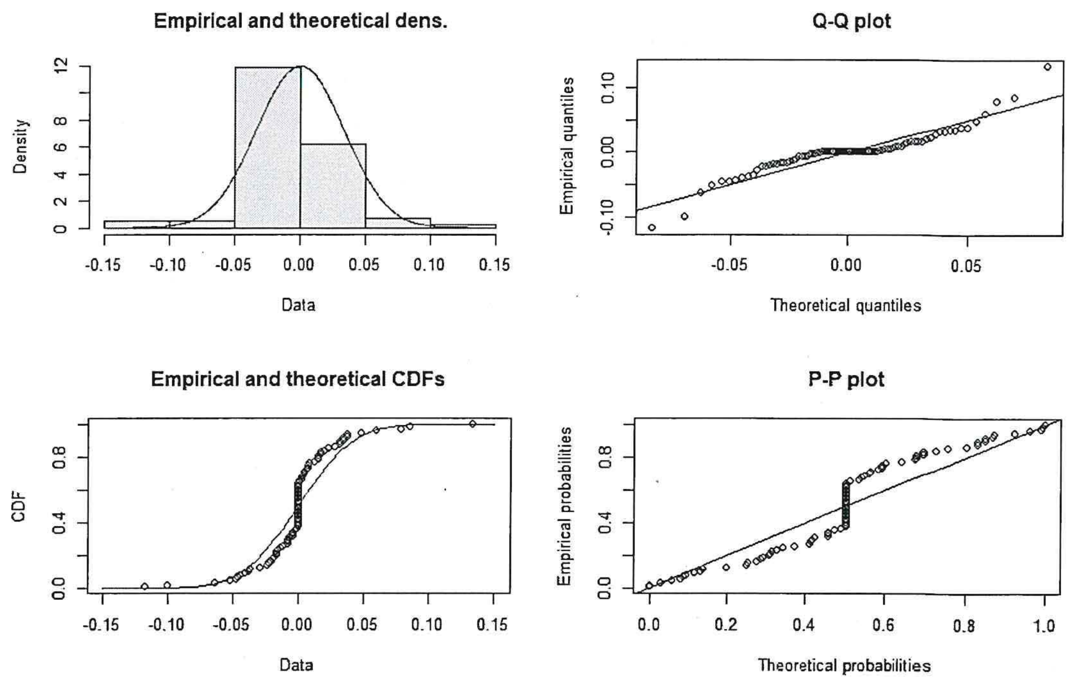
Appendix 8: Histogram, Q-Q plot, P-P plot and CDF of 20 yr. T-bond data



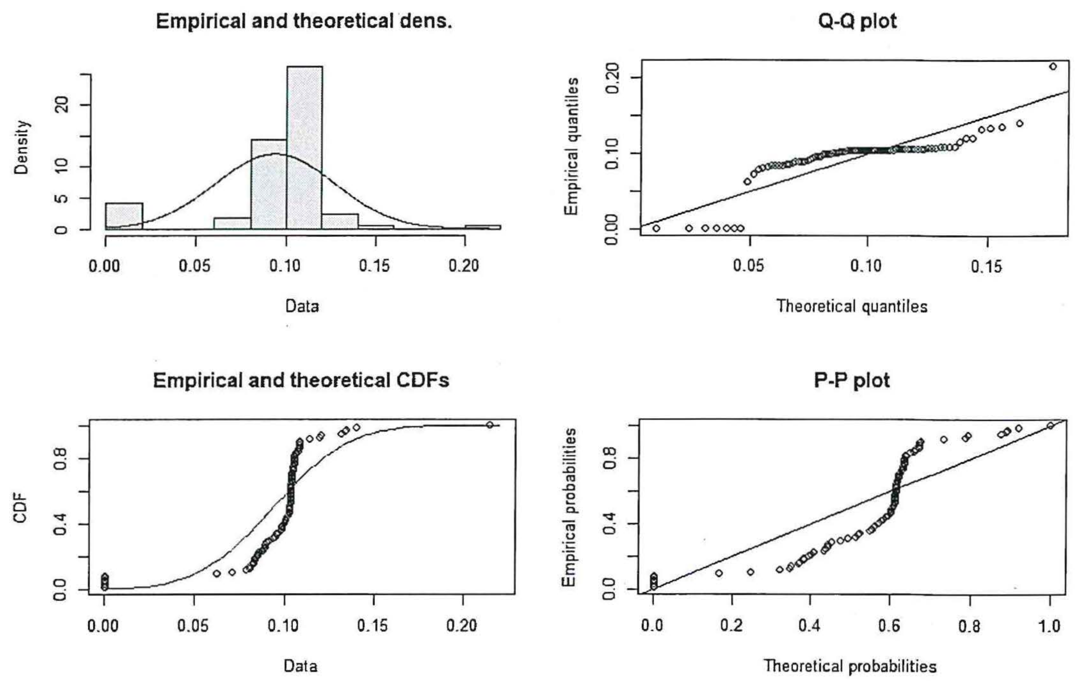
Appendix 9: Histogram, Q-Q plot, P-P plot and CDF of 25 yr. T-bond data



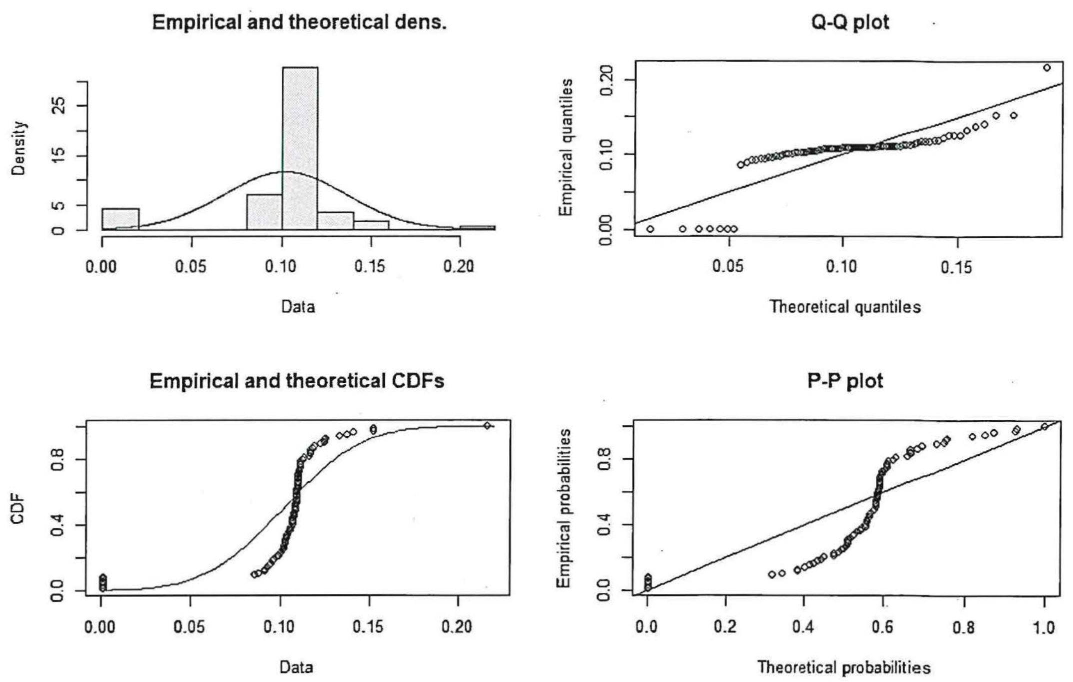
Appendix 10: Histogram, Q-Q plot, P-P plot and CDF of 91-day T-bill data



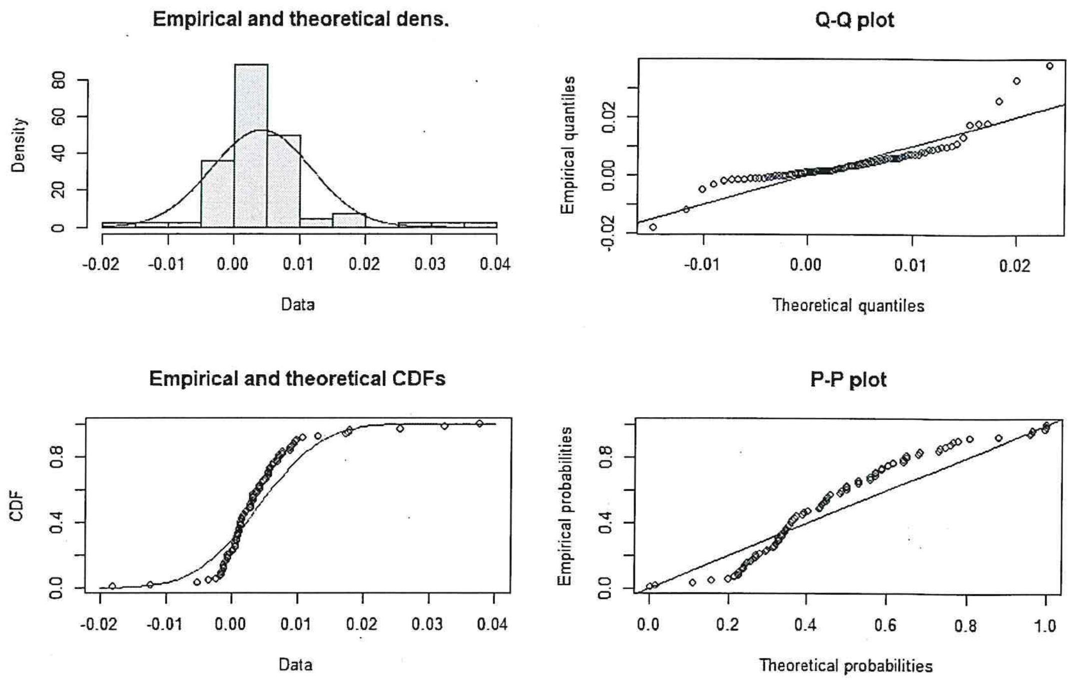
Appendix 11: Histogram, Q-Q plot, P-P plot and CDF of 182-day T-bill data



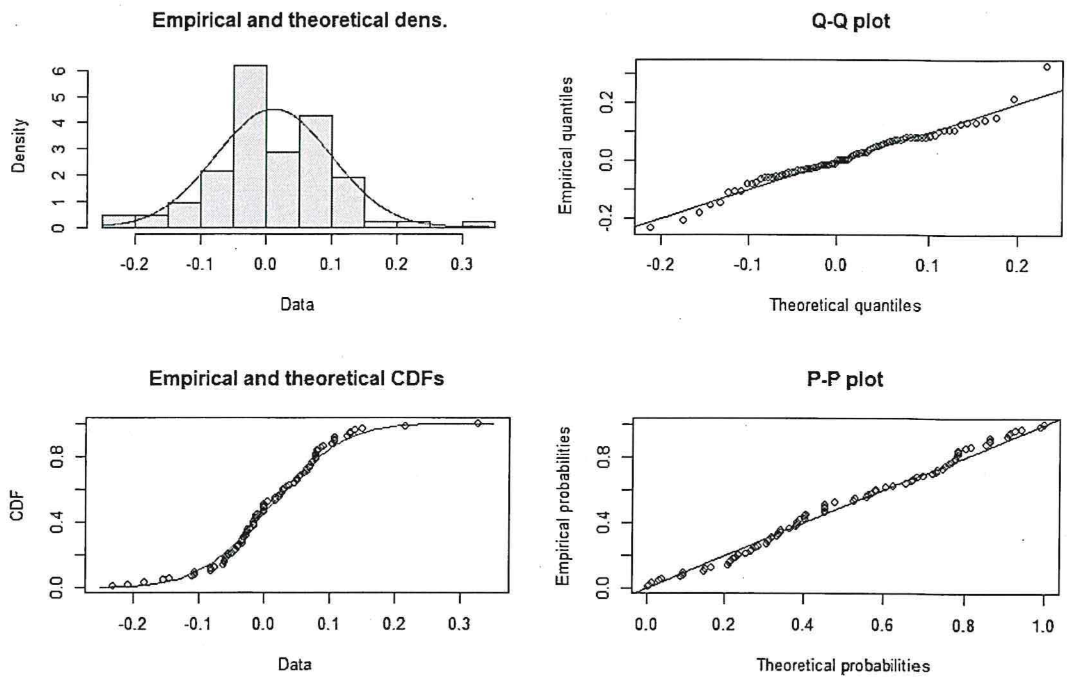
Appendix 12: Histogram, Q-Q plot, P-P plot and CDF of 364-day T-bill data



Appendix 13: Histogram, Q-Q plot, P-P plot and CDF of real estate data



Appendix 14: Histogram, Q-Q plot, P-P plot and CDF of CTUM stock data



Appendix 15: Histogram, Q-Q plot, P-P plot and CDF of TCL stock data