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Valuing Inpatient Cover as a Put Option

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DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due-reference is made in the Research Proposal itself.

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CHAPTER 1: INTRODUCTION

1.1. BACKGROUND INFORMATION

Kenya has witnessed a steep rise in the cost of healthcare since January 2008 (Nduati, 2010) says Mr. Peter Nduati, the chief executive officer of Resolution Health East Africa, a medical insurance provider. The cost of healthcare in the country has been increasing by 10 to 20 per cent annually in the last 10 years, as per the research by Smart Company. Over the period, the consultancy fee in various hospitals has risen from Sh800 to Sh2,000, which is a 150 per cent increase. The cost of simple painkillers have gone up by up to 28 per cent as of recently. In the three years preceding September 2010, medical costs in Kenya were reported to have risen by an average of 20% per annum, more than any other country on the continent. This was largely attributed to the steep rise in doctor's fees, thanks to the collapse of the pricing guidelines fronted by the Medical Practitioners and Dentists Board. Consultation fees for general practitioners tripled within the same period while specialists such as gynecologists and oncologists were charging up to Ksh. 10,000 before diagnosis (Yumbya, 2010). For a country grappling with high inflation and perennial increase in prices of medicines, the need for health insurance needs little emphasis. (Liss, 2014)

Although the nation leads the Eastern Africa region by a substantial distance, insurance penetration values suggest that the industry is still in its infant/young stages by any standards with currently ranging at a penetration rate of 4% (Silver, 2010). Indeed, as of 2009, just 5 out of the 16 medical insurers in the nation were entailing an underwriting profit from their medical business. In the following year, operational hindrances led to an industry high 81.5% loss ratio. So where do the solutions to these problems lie?

Amongst the biggest setbacks to the highly competitive industry has been undercutting by (especially) smaller insurers in the bid to stay and sustain in business. Due to this reason, the Insurance Regulatory Authority (IRA) has been emphasizing on the implementation of an act to compel insurers to set up actuarial functions so as to accurately price their products (IRA, 2014). In addition, the regulator has also implemented risk-based supervision in a bid to raise compliance. Only 50% of the health facilities in the nation are run by the government. As a result, private health insurance has become a major area of growth, especially with the advent of micro-insurance products but with non-life premiums representing 2% of the GDP, there is still adequate room for

improvement. (Aetna, 2011). If the growth is to be maintained, then insurers would have to find a way in order to reach the millions of Kenyans who are living without insurance cover. In addition, the insurers must charge an accurate premium which not only makes them competitive but also keeps them in business.

The health insurance industry in the country is composed of two major players: insurance companies and the providing firms (OBG, 2016). While these companies provide a range of custom health insurance to both groups and individuals, these providing firms – hospitals, clinics and physicians – engage in the actual act of delivery of health services. The nation's health system simulates a very competitive market in that the price (premium) is fixed for the product depending on the beneficiary's age, medical status and the type and extent of cover (Battacharya, 2016). Small-sized companies mostly insure out-patient health services, on the contrary medium and large companies extend their cover to in-patients.

Our work focuses on a financial approach considering that the coverage pattern may easily be replicated by the use of option contracts. Through the illustrated equivalence, we suggest and propose an insurance premium valuation framework which is revolving around option pricing theory. Traditionally, the premiums have been determined purely via actuarial methodology, known as credibility theory (Behan, 2009). We therefore hope to indicate that Black-Scholes model will be able to provide a premium that falls precisely well within the bounds fronted by actuarial techniques. (Merton, 1973)

1.2. PROBLEM STATEMENT

Health insurance is without doubt one of the most crucial and viable divisions of major insurers in Kenya, only second to life insurance in terms of gross annual premiums which have been collected (Chan, 2008). Due to that, the success or failures of the insurance companies in Kenya hugely attributable to the success of their health business. Unfortunately, mispricing of insurance products has therefore resulted on several companies shutting down or going into receivership. (Parkway & Hancock, 2016)

Mispricing of insurance products is as a result of the use of incorrect valuation methods as well as faulty interpretation of model results. This in turn increases the probability of ruin – the probability whereby the claims incurred in a year exceed the overall premiums collected during the same

period. On the other hand, over-pricing premiums leaves the policyholders with a bigger burden than most can carry. (Stienmetz, Muller , & Emanuel, 2013) The company could also end up losing clients to other companies charging much better premiums in terms of fair pricing and hence remaining with a small un-insurable pool. This perhaps elaborates why insurance penetration still stands at a pitiable 2-3% in Kenya.

Actuarial tables that are relating to the mortality and the morbidity have, for quite a long while, been used in pricing of the health insurance products (Benett & Ezatti, 2015). Unfortunately, most third world nations such as Kenya have done a little when it comes to the collecting of vital statistics (Martin & Kinsella, 2009). In spite of the Association of Kenyan Insurers (AKI) launching the Kenyan Tables, most of the companies utilize tables from developed countries such as U.S.A and U.K. But how easy is it to translate these tables to the Kenyan case?

Lastly, major insurance companies in the Kenya insurance industry have asset management divisions which invest in various sectors all across the region. A major tenet of the actuarial valuation is that the same types of models should be utilized to value both liabilities and assets (Lasalle, 2013). If the assets are being value by the use of financial models, how prudent is it to value their liabilities by the use of non-financial approaches?

1.3. OBJECTIVES OF THE STUDY

This study aims at replicating a health insurance cover as a put option contract.

1.4. RESEARCH HYPOTHESIS

Null Hypothesis (H_0): Option pricing model can be used to valuate price of inpatient cover.

Alternative Hypothesis (H_1): Option pricing model cannot be used to valuate price of an inpatient cover.

1.4. SIGNIFICANCE OF THE STUDY

Methods of actuarial valuation have recorded a huge success globally when it comes to the most efficient and accurate pricing of insurance products, and by extension most of the financial contracts. The quantifiable success has therefore led to the spread of the profession to third world countries such as Ghana, India and Kenya. These have come to be known as emerging actuarial markets. However, the success of techniques of actuarial valuation does not lie with the nature of the models. An important factor for their success is the presence of large vital statistics on matters relating to mortality, morbidity, and fertility. For example, insurers in the U.K. have been hugely relying on the data provided by the Continuous Mortality Investigations (CMI) bureau, which has consolidated data for close to two centuries. (IFA, 2011)

In a nation such as Kenya, the collection of important statistics for the purposes of pricing of insurance is a relatively new concept. Most insurance companies have had to depend on actuarial tables from the United Kingdom. (Nsubuga, 2006) Needless to say, the factors which are affecting mortality in Europe differ largely with those in Kenya. A good example, epidemics and tropical diseases are something rare in Europe but very common in Africa. With healthcare facilities and services improving greatly in the nation since the turn of the millennium, underestimating longevity of risk is already hurting the annuity market. It will be disastrous if the same translates to the health insurance market. (Mccarthy, 2016)

Via this study, we thus propose the use of a well-defined financial model in the pricing of health insurance covers. The model purely depends on historical data obtained from existing insurers, which is without doubt, more reliable and relevant than any projection that can be made (Getzen, 2007). Instead of replacing existing models, this will also present another option for the insurers looking for a reasonable check for their premiums.

Years after the Insurance Regulatory Act was passed, very minimal insurers had set up actuarial departments. Most of them had to outsource actuarial services from other financial services companies and institutions such as ActServ and Alexander Forbes. With this financial modeling, even the small health insurers with no resident actuaries can thus accurately price their insurance products

CHAPTER 2: LITERATURE REVIEW

2.1 USE OF DERIVATIVES IN RISK HEDGING

In most recent times, with both individuals and organizations seeking to mitigate their financial positions against uncertainty and risk, derivatives have become the most appropriate tool for risk hedging (Lopez, 2003). This has thus led to more studies on derivatives valuation methods as an alternative valuation methods and their effective application to other contracts. Among these derivatives, options are the most favored tool as they only provide a right and not an obligation to buy or sell the underlying asset on a specified future date, unlike other derivatives that are obligatory in nature (Shirreff, 2004).

Options have also been used in transactions involving many countries. (Geczy, 1997) , provides a case for its application in currency options. However, this may have contributed to increased financial instability in the world money markets due to the lag in the creation of institutions to regulate such transactions (Black & Scholes, 1996). Brendon Williamson used the Black-Scholes model and the Skewness and Kurtosis amended Black-Scholes model in order to estimate the value of exotic options contracts on water. In the Australian water market, this was in turn applied in improving the efficiency of resource application across the dry country.

(Nakaoka, 2006) introduced the 'Futures term structure approach', which applied the basic theory of term structures of a futures contract. From the financial model of a project, one can track the underlying asset value and its volatility using real options. This proved successful in the valuation of oil and gas exploration as well as production projects. With the electricity market restructuring (Deng & Oren, 1998) showed that electricity derivatives play an important role in establishing price signals and facilitating effective risk management. Electricity financial instruments and structured transactions can not only provide energy price certainty but also hedge volumetric risk. In addition, they can synthesize generation and transmission capacity as well as implement uninterruptible service contracts. Moreover, Hamada and van der Hoek used forwards contracts to price electricity in the highly volatile Australian national electricity market. (Hamada & Van, 2007)

Vividly, derivatives have served to minimize credit default risk as well as providing an alternative and practical valuation methodology (Young & Ernst, 2014). However, just like any other great invention, they also have their problems. First and foremost, the derivatives' returns have not been

very consistent with the market returns. Secondly, the risk of using these derivatives has not yet been quantified as almost all projects are in implementation phases. Moreover, increased speculation on these financial instruments by unsupervised traders has added to the risk of relying on the derivatives. In light of these set-backs and limitations, a various number of mechanisms have been designed in order to confront these issues. Variance or volatility swaps, which focus on future volatility levels, are a good example of such. Although these swaps are forwards or futures contracts on volatility, they can be replicated theoretically by a covered portfolio of appropriately selected standard options (Asensio, 2013).

The use of derivatives has had an effect on areas such as insurance, whereby risk has traditionally been managed by other instruments. In spite of lowering the cost of hedging against risk, this trend has raised a large number of questions. Most pressingly, will the risk associated with derivative management be shared in the market? Can government policies or market solutions lead to more efficient outcomes?

2.2 Fair or Risk Neutral Valuation of Insurance Contracts

The trend emerging in the insurance sector is the use of concepts from financial mathematics to value insurance contracts. This is so, because the cash flows from an insurance contract can be replicated using those of a derivative contract. Consequently, methods of derivative valuation s can be used to price insurance contracts (Wang, 2002). The hard work in turn lies in translating the assumptions of the original model to suit the insurance contract valuation model.

In the application of the Option Market Paradigm to the Solution of Insurance Problems, (Mildenhall, 2006) points out the similarities between insurance contracts and options, the trivial similarity is that they are both derivatives; an insurance payment is a function of or is derived from the insured's actual loss while the terminal value of an option is a function of the value of some underlying security. The put option is equivalent to the insurance savings function and the call option to the insurance charge function. Midenhall then shows that insurance can be regarded as a swap transaction, where future losses are swapped for premium payments. Finally, he shows that the Black-Scholes model and actuarial pricing models produce almost similar results. The slight differences arise because the interest rate assumed in actuarial pricing is often different from the

risk-free rate assumed in the Black-Scholes model. He concludes however that without making the assumption for a risk free rate of return, the market prices are consistently closer to the price obtained from the Black-Scholes model.

(Chicaiza & Cabedo, 2009) Cite a case where the Black-Scholes model has been used to value high cost insurance provided in the Colombian health care system. By making the appropriate assumptions to portray the equivalence between insurance contracts and option contracts, they managed to determine the price of an insurance contract and test this against that obtained from the actuarial valuation. Their end conclusion was that that; both models can be used to estimate premiums. In "Risk Neutral Valuation of With-Profits Life Insurance Contracts", Daniel Bauer, present a framework in which options are considered and priced separately. Their cash flow model takes into account special circumstances of the valuation of German insurance contracts. (Bauer, 2005) This framework allowed for the separate valuation and analysis of embedded options and other components of the contract. An efficient Monte Carlo algorithm was used to allow the contracts components to be considered separately while using the Black-Scholes for the surrender option.

2.3 The Black Scholes Model

In their 1973 paper "The pricing of options and corporate liabilities", Fischer Black and Myron Scholes put up a theorem for valuing option contracts under ideal conditions in the market for the stock and for the option. (Black & Scholes, 1973) This theorem came to be known as The Black Scholes Model, and has since been used in valuing option contracts. Over time, the model has received criticism from various and different quotas. Key to the Black-Scholes model is the assumption that the underlying stock price moves randomly following a geometric Brownian motion. However, the stock price distribution does not conform strictly to this normality assumption. In addition, the assumption of a constant and known short term interest rate is also adopted for convenience and not strictly true. The focus on European contracts was designed to allow us ignore the potential influence of early exercise.

Merton however shows that if there are no additional payments made during the lifetime of the option then it would be irrational for an investor to exercise an American option before the maturity date (Merton, 1973). The Black-Scholes model can therefore be used to evaluate

American options based on non-dividend paying common stocks. He further modified the equations to account for both American and European style options as well as stochastic interest rates. In "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing," Ball, performed tests on the Merton's Jump- diffusion model and the Black - Scholes model. Interestingly, they concluded that there were no operationally significant differences in the models. Merton and Cox and Ross further modified the model to allow for discontinuous stock price movements. (Recenzovany, 2014)

Merton remarked that the influence of the Black-Scholes option theory in finance isn't limited to financial options traded in markets or even to derivatives in general. It can also be used to price and evaluate risk in a wide array of applications, both financial and non-financial (Hull, 1997). (Kyun, 2004) applied the Black-Scholes warrant pricing model to the stock exchange of Malaysia He concluded that despite the existence of strike price, time to maturity and variance biases in the model there were no significant differences between the market value of warrants and the Black-Scholes value of warrants.

2.4 Credibility Theory

In define credibility theory as a technique in general insurance for predicting the aggregate claim or number of claims for the following year Credibility theory revolves around the computation of a weighting factor known as the credibility factor, which is a measure of how much emphasis can be placed on data from the specific risk itself when making the projection. The desired feature of the credibility premium formula proposed by the two is that the formula is simple and can easily be explained to a layman.

Dickson and Walters use the Empirical Bayes' Credibility Theory Model I to explain the complex computation of such models. They then proceed to introduce model II, (Donald, 2009) which is more practical since it considers the effect of the volume of risk written in each of the past years in predicting the following year's numbers.

CHAPTER 3: METHODOLOGY

3.1. The Black-Scholes Model

The Model

In their 1973 paper "The pricing of options and corporate liabilities" Fischer Black and Myron Scholes put forward a theorem for valuing European call option contracts (Black & Scholes, 1996). Let $f(t, S_t)$ be the price of a call option at time t , given:

The model proposes that:

$$f(t, S_t) = S_t \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)$$

Where

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\sqrt{T-t},$$

$\Phi(z)$ is the cumulative distribution function of the standard normal distribution.

- i. **The current share price, S_t** ; Also known as the market value. It is the price at which goods are currently being sold in the market. Similar to market price, which is the price determined by buyers and sellers in an open market, the current price of a security is the price at which a security last traded
- ii. **The time to maturity, $T > t$** ; The remaining life of a financial instrument (option)
- iii. **The exercise price, K** ; Refers to the price at which an underlying security can be purchased
- iv. **The risk free rate, r** ; is the theoretical rate of return of an investment with zero risk
- v. **The volatility of volatility of the share price, σ** ; is the degree of variation of a trading price series over time as measured by the standard deviation of returns.

Shortcomings of the Black-Scholes model

The Black-Scholes model has been widely used in the pricing of option contracts. However, like any other financial model, it is not immune to shortcomings (Haug & Taleb, 2011).

There are various drawbacks that the Black Scholes Model faces such as: volatility is constant over time, people can't consistently predict the direction of the market, the returns of log-normally distributed stock prices follow a normal distribution, interest rates are known and constant, the underlying stock doesn't pay dividend during the life of the option and that option can be exercised at expiration date.

3.2. Adaptation of the Black-Scholes Model to Health Insurance

Equivalence between option contracts and health insurance contracts

A major part of our study revolves around how the Black-Scholes model that has been widely used in pricing derivatives can be applied in the health insurance market. Before we delve into the translation of parameters for the model, it is important to appreciate the similarities between options and health insurance.

To begin with, option contracts and health insurance contracts are both hedging operations. While options cover agents against unexpected price movements, insurance covers policyholders against unexpected illnesses or accidents. In both cases, the purchaser of the contract has to part with a

premium. The buyer of an option pays a premium at the time of purchase to obtain the desired hedge while policyholders pay premiums at the start of the period they intend to be covered. In the event that the unexpected situation arises, compensation must be paid in either case. When unexpected price changes occur, the buyer of an option will execute it, receiving a compensation equivalent to the difference between the market price and the strike price (Bowers, Stephenson, & Storero, 2006). In case of an illness whose treatment costs exceed the set minimum the buyer of health insurance cover will be compensated. In case the covered event does not materialize, the buyer forfeits the premium in both cases. The timing of both insurance and option contracts is usually short. In the current financial market, options are rarely contracted for periods longer than one year. Similarly, most health insurance products offer one year coverage.

Parameter Translation

So how can the parameters of a health insurance cover be adjusted to fit those of the Black-Scholes model?

Suppose a hypothetical individual intends to purchase health insurance cover. We also assume the individual's disease will generate 3 payments in the course of the year, at times 1, 2, 3, and that the accumulated cost will exceed the deductible. Without insurance, the individual will have to foot the entire annual treatment cost. However, with insurance, there will be a maximum cost for the individual.

In nominal terms, the total cost paid out by this individual will be the deductible plus the premium amount paid at the start of the year. In this case, the cost minus insurance will be more than when the individual is insured. So how does this resemble the purchase of an option?

When purchasing a European call option, the buyer guarantees a maximum price for buying the underlying asset when the contract matures. In case the market price is lower than the strike price, the buyer is under no obligation to exercise the call option. If he/she wishes to purchase the asset, the market price has to be paid (Economic, 2016). The total cost of the asset will therefore be sum of the market price and the premium paid at the time of purchasing the option.

On the contrary, if the market price exceeds the strike price, the buyer of the option will exercise the right and only pay the strike price. The total cost paid will now be the strike price and the initial premium. In both hedge schemes, the buyer is guaranteed a maximum cost to be incurred when an unexpected event occurs, and has to pay a premium for that benefit.

In the case of a health insurance contract, the buyer is the prospective policyholder while the writer of the contract is the insurance company providing the cover. The strike price here will be the amount

of the deductible. This can be defined as the lower limit beyond which the insurance company will foot the treatment cost.

The most important part of an option, and a derivative by extension, is the underlying asset. In this valuation model, we define the underlying asset as the accumulated treatment cost in the year.

The expiration date will be defined as the end of the year and the premium for the “option” is the premium paid to the insurance company at the policy’s inception. At expiration date, the contract will be settled by differences.

In this coverage pattern, in case the accumulated cost of treatment at the end of the year (expiration date) are lower than the amount of the deductible, the individual will not have exercised their right and will therefore assume all the payments. The total cost for the patient will be the sum of the annual treatment cost and the premium paid at the start of the year.

This means that the insured individual will only assume treatment costs lower than the deductible. The total cost on their part will be the sum of the premium paid at the outset and the deductible. Clearly, an option contracts and the insurance contract can achieve the same coverage pattern.

Table 1: Translation of the Black-Scholes parameters to a health insurance contract

	OPTIONS CONTRACT	HEALTH INSURANCE
Buyer of the contract	Buyer of an option	The insured
Writer of the contract	Writer of an option	Insurance company
Premium	Premium payable upfront	Premium payable by insured
Underlying asset	Security’s price	Accumulated treatment cost
Strike Price	Exercise price	Deductible
Time	Time to maturity	End of year

¹The following table summarizes the variables of health insurance policies that directly translate to my option pricing model which I will later on apply in pricing of the premium with the application of the Black-Scholes Model

Therefore, the price at the outset of a one-year health insurance cover, $P(S)$, given by:

¹ The following table summarizes the variables of health insurance policies that directly translate to my option pricing model which I will later on apply in pricing of the premium with the application of the Black-Scholes Model

$$P(S) = S\Phi(d_1) - Ke^{-r}\Phi(d_2)$$

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)}{\sigma}$$

$$d_2 = d_1 - \sigma$$

- i. the accumulated cost of treatment is S ;
- ii. the time to maturity is 1 year;
- iii. the amount of the deductible is K ;
- iv. the risk free rate is r ;
- v. the volatility of the cost of treatment is σ ;

$\Phi(z)$ is the cumulative distribution function of the standard normal distribution

Parameter Estimation

Volatility

Volatility is the measure by which a financial variable fluctuates during a period relative to a central trend or drift. Option pricing models require an estimate of expected volatility as an assumption because an option's value is dependent on potential underlying assets return over the period. The higher the volatility the higher the returns on the underlying asset are expected to vary either up or down.

From the Black-Scholes model

$$S_T = S_t e^{(r - \frac{\sigma^2}{2})(T-t) + \sigma(W_T - W_t)}$$

$$\frac{S_T}{S_t} = e^{(r - \frac{\sigma^2}{2})(T-t) + \sigma(W_T - W_t)}$$

$$\ln\left(\frac{S_T}{S_t}\right) = (r - \frac{\sigma^2}{2})(T-t) + \sigma\sqrt{T-t} * z_1$$

where; $(r - \frac{\sigma^2}{2})(T-t)$ is trend and $\sigma\sqrt{T-t} * z_1$ is white noise.

Letting $T-t$ be one day gives:

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = (r - \frac{\sigma^2}{2}) + \sigma * z_1$$

Define $R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$. Therefore:

$$\bar{R} = \frac{1}{n} \sum_{t=1}^n R_t = (r - \frac{\sigma^2}{2})$$

$$Var(R) = \frac{1}{n-1} \sum_{t=1}^n (R_t - \bar{R})^2 = \sigma^2$$

Therefore the volatility, σ , is the standard deviation of R

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (R_t - \bar{R})^2}$$

This standard deviation must however be annualized to make it suitable to use in valuation.

Let σ^* be the annualized volatility

$$\sigma^* = \sigma * \frac{1}{\sqrt{\text{sample frequency in years}}}$$

Deductible (Strike Price)

The deductible will be set as the lowest amount that an insurer will be willing to cover. This will always vary across different insurance policies.

Time

The time component of our model will be taken as the period over which the policyholder will be covered. Health insurance contracts in Kenya are usually 1 year contracts.

Assumptions of the model

For the above adaptation of the Black-Scholes model to be valid in pricing health insurance benefits, the following assumptions are made:

1. There are no costs associated with purchasing insurance cover and no taxes. In addition, the purchaser is free to choose the amount of the deductible.
2. There are no more additional costs apart from the accumulated treatment costs, which is our underlying asset.
3. All policies with the same extent of benefit have the same premium amount i.e. for the same premium; an individual can't obtain more cover from the same market.
4. There exists a risk-free rate of interest that is assumed to hold constant during the term of cover.
5. Policies can be purchased at any time in the course of the year.
6. There are no sudden increments in the amount of the deductible.
7. Individuals prefer more cover to less cover and agree on the variance of the deductible.

3.3 Sensitivity Analysis - The Greeks

Sensitivity analysis studies how any uncertainty in the output of a mathematical model can be apportioned to different sources of uncertainty in its inputs.

The Greek letters: Delta, Gamma, Rho, Theta and Vega are used in option pricing to show the sensitiveness of an option price relative to changes in value of either a state variable or a parameter (Banks & Siegel, 2006). Each of them shows the rate of change of the option price with respect to different parameters or state variables allowing financial institutions that sell option products to manage their risk they are likely to encounter as option contracts do not usually correspond to standardized products traded in the exchanges (Macmillian, 1993).

In my study i aim to find out the sensitiveness of the premium obtained using the Black-Scholes Model due to variations in different parameters that we will estimate.

3.4 Credibility Theory

Introduction

After valuing health insurance benefits using our adaptation of the Black-Scholes model, we intend to demonstrate that this falls within the bounds provided by actuarial methods. In this section, we examine one of the most common techniques used in the setting of premium levels by insurers – credibility theory (Venter, 2003).

Credibility Theory is a technique in actuarial science that is used in the estimation of next year's premium or claim frequency. Given the following information:

- \bar{X} – An estimate of the expected aggregate claim or number of claims for the coming year based solely on data from the risk itself.
- μ - An estimate of the expected aggregate claim or number of claims for the coming year based on collateral data, that is, data from risks similar (but not necessarily identical) to the risk being considered.

The credibility estimate for the aggregate claims or number of claims can be computed as:

$$P = Z \bar{X} + (1 - Z)\mu$$

Where Z is a figure between 0 and 1 referred to as the credibility factor. Clearly, it is a measure of how much trust can be placed on data from the risk itself as an estimate next year's expected aggregate claims or number of claims.

This formula has come to be known as the credibility premium formula

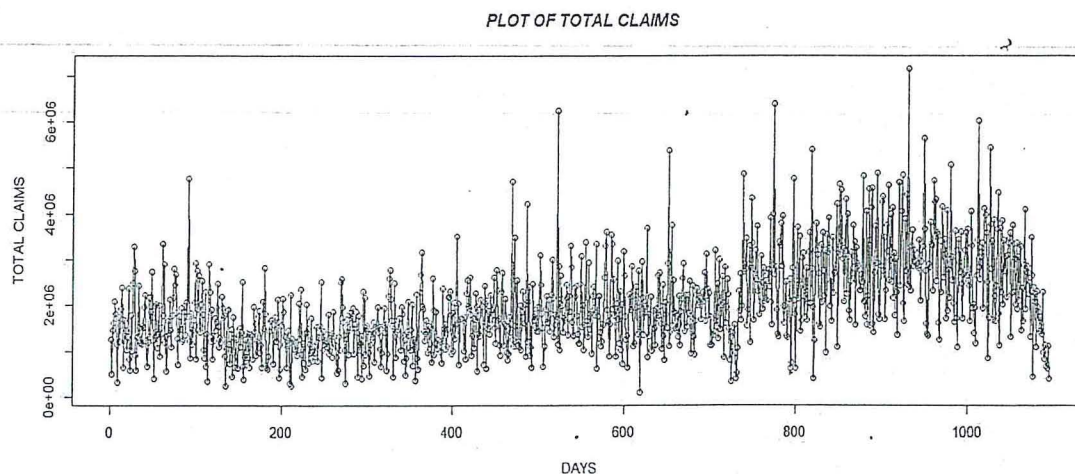
CHAPTER 4: DATA ANALYSIS

4.1. Description of the Data

To the application of the proposed valuation model, I used medical bills of UAP Insurance Company Limited. I decided to settle for this particular Insurance firm because, by the fact that UAP Insurance has a history that dates back over 80 years with a functional Health Segment operating since 1994. The firm also entails robust data recording system that has won them the award for the best presented accounts for seven years.in a row, these factors together with the fact that they have elaborate health products made UAP Insurance health data an unmatched choice in my study.

The data I obtained represented medical bills corresponding to the period between 1st January 2013 and 31st December 2015. In total, there were more than 300,000 medical bills whose payments occurred on a daily basis. The claims were classified as per various factors such as member number, policy code, service date, total amount paid and the date settled.

Figure 1: A time plot of total daily claim amounts



² A graphical representation of the variation in total claims with time, which indicates resemblance with a martingale. As seen from the graph above, the data has no indication of any trends in the level of claims for the three years.

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4.2. Data filtration

The Insurance Company offers mandatory outpatient cover with the option for an inpatient cover for some of their health products. The data i obtained comprises of 7 different types of cover, from which I streamlined and settled on 3 types of products that were more aligned with the objectives of my model. These were the Maximed, the Afya Imara and the Afya Imara Senior. The two Afya Imara products were mainly included in my research for purposes of estimating the pure premiums using the Empirical Bayes' Credibility Theorem (EBCT).

For the purposes of the empirical application of my proposed model, i decided to settle on medical bills under the MaxiMed product which offers corporate and individual cover with an annual inpatient cover totaling to KShs. 10,000,000 per member. These data presented me with a Brownian motion kind of process which was in tandem with the assumptions of the Black Scholes model.

The data was divided into 3 sub-periods representing the years in which the bills fell due. The sub-divided yearly data were then tested for any trends or correlations among the three years. The scatter plots for all the three years indicated that there were no trends or correlations among the data points since the plots were randomized with respect to the year and amounts of claims.

The data was truncated to exclude outpatient claims and bills for which payment from the insurer was zero (this included data recorded by the insurer as negative liabilities).

At the end of the filtration process, I was thus left with around 75,600 data points from which all further analysis and estimations in my research work are based. The final data also covered the three year period: 2013 (16887), 2014 (22643) and 2015 (36063).

A further analysis of the truncated and filtered data was also subjected to tests of correlation and/or trend. The scatter plot were still indicating that the medical bills were independent of each other, both with respect to the year in which they fell due and the period within the said year that they fell due. As a result, these data were still justifiably

accurate for the empirical application of my proposed premium valuation model (Black Scholes Model).

4.3 Price Variation (Volatility) of the Underlying Asset

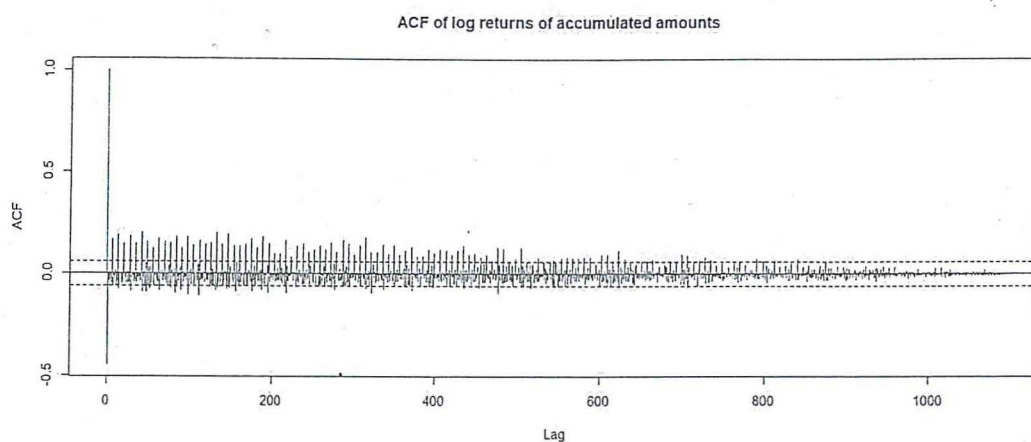
The key variable during valuation of these premiums is the volatility of the yield of the underlying asset. Since this is not an observable variable, I was thus assuming a hypothesis in order to estimate volatility from the information available at the moment of the valuation. To achieve this aim, first I began by constructing a time series for the yield of the underlying asset and then estimated the volatility from this time series. To construct the time series, I calculated the daily mean value of the medical bills recorded by UAP Insurance for all the days covered by the calculation period (January 2013 to December 2015).

Before estimating the daily variation in these accumulated claims, I tested the possibility of an autoregressive conditionally heteroscedastic (ARCH) pattern in the accumulated daily claims data.

For this purpose, I used the log of the daily accumulated claims data to obtain a three-year time series of the daily returns on the underlying asset. An examination of both the autocorrelation and partial autocorrelation functions on this time series did not suggest the existence of an autoregressive or moving average pattern for the daily variation of accumulated medical bills.

The results of these analyses are shown in the graphs below:

Figure 2: The auto-correlation function for accumulated treatment costs



³ The auto-correlation function was meant to measure the degree of similarity between the given time series and a lagged version of itself over given successful time intervals hence measuring

³ The auto-correlation function was meant to measure the degree of similarity between the given time series and a lagged version of itself over given successful time intervals hence measuring the relationship between variables current value and past values of the accumulated treatment costs.

However, the relevant value for the deviation is that expressed in annual terms. Multiplying the daily standard deviation by the square root of the number of days on which bills were recorded (1096 days in this case) – as in formula

$$\sigma^* = \sigma * \frac{1}{\sqrt{\text{sample frequency in years}}} - \text{gives a result of } 8.530504767.$$

This is the value corresponding to the standard deviation for the period January 2013 – December 2016, on an annualized basis. I used this value as a forecast for the volatility in the validation period, which is January 2016 – December 2016.

4.4 Direct Application of the Option Pricing Model

Incorporating my set assumptions in the context of health insurance, I modified the original Black Scholes model and simplified it so that I can suitably apply it in context.

After determining the volatility, the other parameters of the model must be defined in order to calculate the premium. At this point, the market price of the underlying asset becomes especially important.

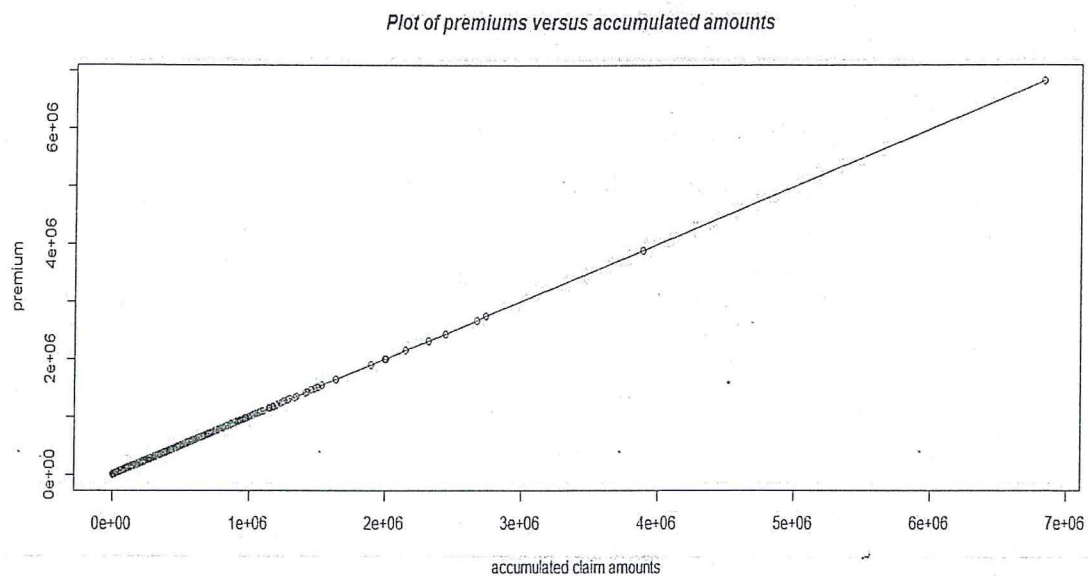
As stated before, I defined the underlying asset as the accumulated cost over the previous 12 months. If I were to estimate a premium for each of the claimants, there will be as many different prices as there are claimants. However, the numbers of claimants at the moment is known and hence value the premium, but not the number of claimants in the coming year.

For this reason I assumed one of the aforementioned hypotheses: The distribution of number of claimants in the coming year will be the same as that of the current year.

Using the aforementioned intrinsic and translational assumptions of my model, the premium was estimated for each of the claimants using their respective accumulated annual claims for 2015. For example, for the affiliate with the highest accumulated cost (KShs. 6,828,412.0) the premium is KShs. 6,828,411.99. I repeated this calculation for each of the 9,023 claimants considered in 2015.

Below is a graph showing the Relation between the price of the underlying asset (accumulated cost of claims) and the premium:

Figure 5: A plot of premium against accumulated treatment costs



⁶The graph plotting of accumulated claim amounts against the premium payable result to a positive gradient. This shows that, an increase in the accumulated claim amounts inversely results to an increase in the premiums payable by the policy holder

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The sum of all premiums for all the claimants with accumulated bills equal to or higher than KShs. 1.00 equaled **KShs. 1,008 million** and the total number of policyholders under the Maximed health insurance product at the time of valuation was **103 thousand**. Hence, the mean premium per policyholder would be **KShs. 9,744.39** for a whole year's coverage.

This premium is assumed to be paid immediately at the beginning of the coverage period. Therefore this is the annual average premium payable by each policyholder under the Maximed product for inpatient cover.

4.5. Parameter Estimation and Premium Calculation under EBCT II

Under the Empirical Bayes' Credibility Theory model II, I first analyzed the data from the three classes of health insurance products as follows:

Table 2: Aggregate claims and risk volumes for 3 products from 2013-2015

		YEAR, <i>j</i>		
		1	2	n = 3
RISK, <i>i</i>	1			
	Y _{1j}	291,723,897.60	516,993,687.05	851,494,099.05
	P _{1j}	75,512	82,937	103,454
	2			
	Y _{2j}	8,936,179.07	16,936,114.65	31,549,953.67
	P _{2j}	1,445	3,629	6,212
	N = 3			
	Y _{3j}	716,056.20	76,469.04	1,078,714.87
	P _{3j}	12	15	108

⁷ The data was analyzed from the three types of health insurances classes on the three consecutive years (2013-2015) by equating the number of claims on that year against the risk volume which is the new business written.

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Computing the X_{ij} values of aggregate claims for the three products:

Table 3: Aggregate claims for the 3 products from 2013-2015 weighted by risk volume

		YEAR, j			
		1	2	3	
RISK, i	1	X_{1j}	3,863.28	6,233.57	8,230.65
	2	X_{2j}	6,184.21	4,666.88	5,078.87
	3	X_{3j}	59,671.35	5,097.94	9,988.10

⁸¹ The aggregate claims of the three products was computed as $X_j = \frac{I_j}{P_i}$, which is the accumulated number of claims against the risk volume which gives us the claim frequency

Then found the values of the risk volume statistics:

Table 4: Computation of the risk volume statistics

⁸ The aggregate claims of the three products was computed as $X_j = \frac{I_j}{P_i}$, which is the accumulated number of claims against the risk volume which gives us the claim frequency

		$\bar{P}_i = \sum_{j=1}^3 P_{ij}$	$\bar{P}_i \left(1 - \frac{\bar{P}_i}{\bar{P}}\right)$
RISK, i	1	261,903	10,943.77
	2	11,286	10,819.98
	3	135	134.93
	$\bar{P} = \sum_{i=1}^3 \bar{P}_i$	273,324	
	$P^* = \frac{1}{9-1} \sum_{i=1}^3 \bar{P}_i \left(1 - \frac{\bar{P}_i}{\bar{P}}\right)$		2,737.34

⁹The risk volume statistics, which is also known as new business written statistics is computed using the values of the previous tables using the formulae indicated on the tables as a procedural process under the EBCT Model II

Also found the weighed aggregate claim statistics:

Table 5: Computation of the weighted aggregate claim statistics

		YEAR, j			$\bar{X}_i = \sum_{j=1}^3 \frac{P_{ij} X_{ij}}{\bar{P}_i}$
		1	2	3	
RISK, i	1	1,113.86	1,973.99	3,251.18	6,339.03
	2	791.79	1,500.63	2,795.49	5,087.92
	3	5,304.12	566.44	7,990.48	13,861.04
	$\bar{X} = \sum_{i=1}^3 \sum_{j=1}^3 \frac{P_{ij} X_{ij}}{\bar{P}}$				25,287.99

¹⁰ After computing the risk volume statistics, the table above shows the values of the computation of the weighted aggregate claim statistics as per under the procedural process of the EBCT Model II

⁹ The risk volume statistics, which is also known as new business written statistics is computed using the values of the previous tables using the formulae indicated on the tables as a procedural process under the EBCT Model II

¹⁰ After computing the risk volume statistics, the table above shows the values of the computation of the weighted aggregate claim statistics as per under the procedural process of the EBCT Model II

Computing the estimate for weighted variance within each risk:

Table 6: Computation of the estimate for weighted variance within each risk

		YEAR, j			
		1	2	3	$\frac{1}{n-1} \sum_{j=1}^3 P_{ij} (X_{ij} - \bar{X}_i)^2$
RISK, i	1	462840093235.	922437730.87	370182398269.24	416972464617.86
	6				
	2	1736671172.91	643319014.79	508362.32	1190249275.01
	3	25183016389.3	1151879310.99	1619961399.05	13977428549.69
	3				
					$\frac{1}{3} \sum_{i=1}^N \frac{1}{3-1} \sum_{j=1}^3 P_{ij} (X_{ij} - \bar{X}_i)^2$
					144046714147.52

¹¹The table above shows the computation of the weighted variance within each risks. They were computed using the formulae indicated in the table. The variance will be needed in the eventual computation of the risk premium

My final linear estimate was then obtained by calculating the estimate for weighted variance across the 3 risks:

¹¹ The table above shows the computation of the weighted variance within each risks. They were computed using the formulae indicated in the table. The variance will be needed in the eventual computation of the risk premium

Table 7: Computation of the estimate for weighted variance across 3 risks

		YEAR, j				
		1	2	3	$\sum_{j=1}^3 P_j (X_j - \bar{X})^2$	
RISK, i	1	346611383309550.90	30112006331291.00	30100217546094.30	94873607186936.20	
	2	527359234536.69	1543159869629.82	2537033007710.06	4607552111876.58	
	3	14186586170.89	6114573596.99	25281350998.46	45582510766.34	
					$\sum_{i=1}^N \sum_{j=1}^3 P_{ij} (X_{ij} - \bar{X})^2$	99526741809579.20
					$\text{var}[m(\hat{\theta})]$	4544873463.14

¹² The table portrays the workings of the estimated weighted variance, whereby the parameters will then be used in obtaining the credibility factor which will then be used in estimating the risk premium for the next year.

The credibility factor for risk 1 was then computed as:

$$Z_1 = 0.99878999$$

This shows that with the huge volume of data, the estimate of next year's average claim for this risk will be close to the estimate provided by the data from the risk itself.

The weighted average claim for 2016 can then be obtained using the EBCT II formula as:

$$\hat{X}_{1,4} = 6,341.33$$

To get the average claim for 2016, we multiply this estimate by the ratio of the volume of risk in 2016 to that in 2015. However, since we assumed that the distribution of the

¹² The table portrays the workings of the estimated weighted variance, whereby the parameters will then be used in obtaining the credibility factor which will then be used in estimating the risk premium for the next year.

number of claimants in 2016 and the volume of risk will resemble that in 2015, this ratio is just 1. Therefore, the estimate for next year's average claim amount is:

$$\hat{Y}_{1,2} = 6,341.33$$

UAP insurance provides that their pure premium only represents 65% of the total premium, with the remaining 35% catering for expenses and the profit margin. Therefore, the credibility premium will be obtained by multiplying this amount by 100/65 to give:

$$P = \text{Ksh. } 9,755.89$$

This is the amount each policyholder under the MaxiMed portfolio will be required to pay on 1st January 2016 to enjoy a year of in-patient coverage.

4.6. Comparative Analysis between Option Pricing premium and Actuarial Premium

Under the proposed model, the 2016 premium for inpatient coverage under the MaxiMed policy was obtained as **Ksh. 9,744.39**. The actuarial premium obtained from credibility theory for the same cover comes to **Ksh. 9,755.89**.

This shows that the Black-Scholes premium is just **Ksh. 11.50** less than the actuarial premium, i.e. the Black-Scholes premium falls within an error margin of 0.12%, which is generally acceptable and adaptable.

4.7 Sensitivity Analysis

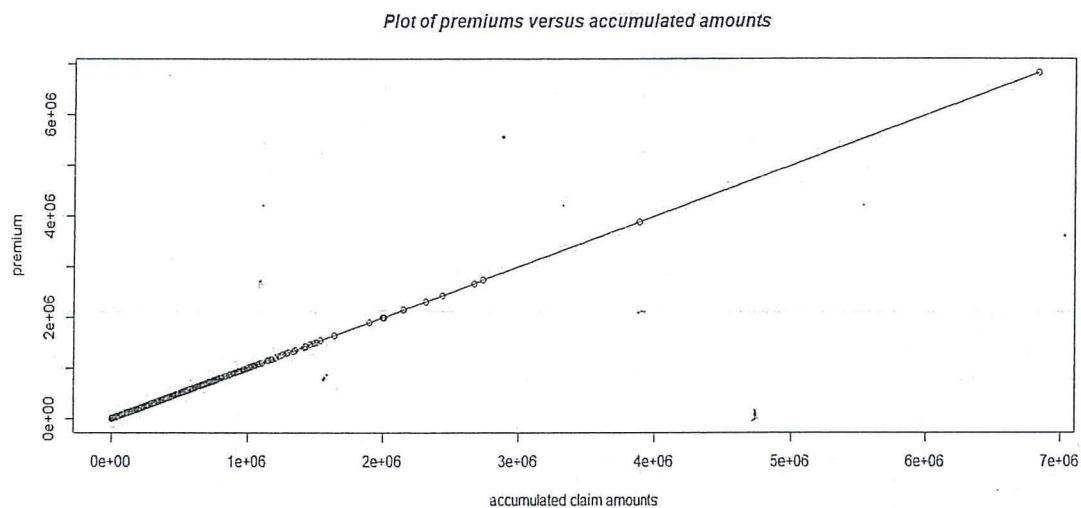
Conducted an analysis of the extent to which the uncertainties in the output of my proposed model could be explained by variations in the input parameter values. In the original Black Scholes model, the Greeks are the major method of measuring the sensitivity of an option price relative to changes in the values of the parameters. I

therefore analyzed the sensitivity of the model using the four Greeks: Delta, Gamma, Vega and Rho.

1) Delta (Δ)

This is the rate of change of the premium with respect to the changes in the accumulated cost of claims. It is the slope of the curve that relates the premium to the accumulated cost of claims. From the data on premium obtained, the Delta of the data fell within the range (0.9999995, 1.0000000) with a mean value of 1. This can be interpreted to mean that an extremely large proportion of the changes in the value of the premium could be attributed to changes in the accumulated cost of claims.

Figure 6: A plot of how premiums change with accumulated amount



¹³ The graph diagram above was meant to show the rate of change of the premium with respect to the changes in the accumulated cost of claims. The graph shows a positive gradient which implies increase in premium as a result of accumulated cost of claims

¹³ The graph diagram above was meant to show the rate of change of the premium with respect to the changes in the accumulated cost of claims. The graph shows a positive gradient which implies increase in premium as a result of accumulated cost of claims

Further analysis of the model indicated a linear relationship between the premium and the accumulated cost of claims whereby:

$$\text{Premium} = 2.263 + 1.00 * (\text{Accumulated cost of claims})$$

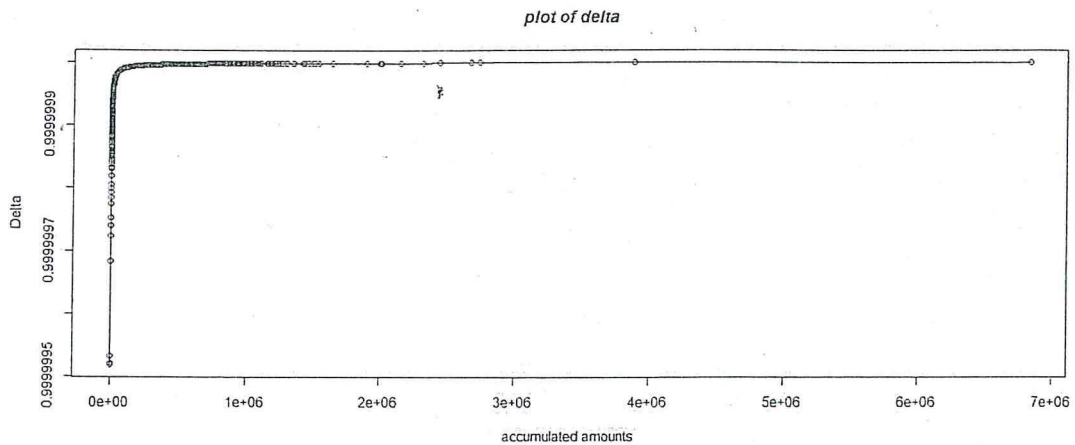
This confirms the value of delta to be 1 as indicated by the slope of the above linear equation.

2) Gamma (Γ)

This is the rate of change of the delta with respect to the accumulated cost of claims. It is the second partial derivative of the premiums with respect to the accumulated cost of claims.

From the data on premium obtained, the Gamma of the data fell within the range of $(4.952311 * 10^{-17}, 1.434817 * 10^{-9})$ with a mean value of $2.525852 * 10^{-12}$. This shows that the delta hardly changes with changes in the accumulated cost of claims as shown in the graph below:

Figure 7: A plot showing how delta changes with accumulated amount



¹⁴ The graph diagram above shows the rate of change of the delta with respect to the accumulated cost of claims. The result of it concludes that, the delta hardly changes with changes in the accumulated cost of claims

3) Vega (ν)

The Vega measures the rate of change of the premiums with respect to changes in the volatility of the accumulated cost of claims. From the data, the Vega lies in the range (0.0004895886, 0.0196979954) with an average of 0.004629788. This shows that on average, every unit change in volatility causes a 0.46% change in the premium. Hence the model is not affected by minor changes in volatility.

4) Rho (ρ)

The Rho measures the rate of change of the premiums with respect to changes in the risk free rate of interest. From our data, the Rho lies in the range (0.0001263333, 0.0071922929) with an average of 0.001447619. This shows that on average, every unit change in the risk free interest rate causes a 0.14% change in the premium. Thus the model is not affected by minor changes in the risk free rate of interest.

¹⁴ The graph diagram above shows the rate of change of the delta with respect to the accumulated cost of claims. The result of it concludes that, the delta hardly changes with changes in the accumulated cost of claims

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The main objective at the study was to replicate a health insurance contracts as an option contract. I find that; it is possible to replicate the coverage pattern of a health insurance product as an option contract. The study also revealed that it is possible to translate the major parameters of the Black-Scholes model to suit a health insurance context.

Consequently, it is also possible to price a health insurance product using the Black-Scholes model.

Most importantly, the premium obtained using the Black-Scholes model (**Ksh. 9,744.39**) was only **Ksh. 11.50** less than the calculated actuarial premium (**Ksh. 9,755.89**). It can therefore be concluded that the Black-Scholes model provides an accurate estimate actuarial valuations when it comes to the pricing health insurance products.

Lastly, the sensitivity analysis portrays that the premium calculated by the Black-Scholes model is robust to small changes in parameter values. The most significant parameter, as expected, is the value of the underlying asset. In this case, this is the annual accumulated cost of treatment for a policyholder.

5.2 Recommendations

In light of the revelations of the study and the conclusions above, i propose the following recommendations:

That the small section of insurers who have been undercutting, turn to the Black-Scholes model as a useful tool for valuation of their health insurance products. Secondly, insurers already using actuarial valuation methods use the Black-Scholes model as a reasonable check for their premium levels.

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