SCHOOL OF COMPUTING & ENGINEERING SCIENCES

END-OF-SEMESTER EXAMINATION

BBT 1105; CNS 1104 & ICS 1104: DISCRETE MATHEMATICS

DATE: 13th OCTOBER 2023 Time: 2 Hours

Answer Question ONE and any other TWO questions in this Paper.

Question One (30 marks)

(a) A committee of six is to be formed from nine women and three men. In how many ways can the members be chosen so as to include:

- (i) at least one man (3 marks)
- (ii) at most four women (3 marks)

(b) Compute $(0.98)^6$ to two decimal places by use of a binomial formula. (4 marks)

(c) In a group of 100 people, 72 people can speak English and 43 can speak French.

- (i) How many can speak both English and French? (3 marks)
- (ii) How many can speak English only? (3 marks)
- (iii) How many can speak French only? (3 marks)

(d) Let $A = \{1,2,3,4,6\}$ and $R = \{(x, y): x|y, (x \text{ divides } y) \text{ for } x, y \in A\}$ be a relation on A

- (i) Write R as a set of ordered pairs. (2 marks)
- (ii) Show whether R is a partial order relation or a recurrence relation. (5 marks)

(e) Using the truth table, show that $q \longrightarrow p$ and $\sim p \rightarrow \sim q$ are logically equivalent. (4 marks)

Question Two (20 marks)

(a) Show that $[\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology. (5 marks)

(b) Evaluate each of the following bit expressions:

- (i) $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$ (2 marks)
- (ii) $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$ (3 marks)

(c) Prove by contradiction that $\sqrt{7}$ is irrational. (4 marks)

(d) In a survey of 260 college students, the following data was obtained: 64 are doing a Mathematics course, 94 are pursuing Computer Science course, 58 are doing Bachelor of Education course, 28 both Mathematics and Bachelor of Education, 26 both Mathematics and Computer Science course, 22 both Computer Science and Bachelor of Education while 14 are doing all the three courses.

(i) How many students were surveyed who had taken none of the three types of courses. (2 marks)

(ii) Of the students surveyed, how many had taken only one type of course. (4 marks)

Question Three (20 marks)

(a) Consider the relation R whose ordered pairs are $R = \{(1,b),(2,a),(2,c)\}$ and $S = \{(a,y),(b,x),(c,y),(c,z)\}$ on the sets A to B and B to C respectively where $A = \{1,2,3\}, B = \{a,b,c\}$ and $C = \{x,y,z\}$. Find:

(i) Matrix of R, M_R and hence matrix of $R^{-1}, M_{R^{-1}}$ (4 marks)

(ii) Matrix of S, M_S and hence matrix of $S^{-1}, M_{S^{-1}}$ (4 marks)

(iii) Matrix of $R \circ S$, $M_{(R \circ S)}$ (3 marks)

(iv) Multiply M_R by M_S and compare the product $M_R M_S$ with $M_{(R \circ S)}$ (3 marks)

(b) Construct a truth table for the compound proposition

$$[(p \vee q) \wedge (\neg r)] \leftrightarrow (q \rightarrow r)$$ (6 marks)

Question Four (20 marks)

(a) A computer company must hire 25 programmers to handle systems programming jobs and 40 programmers to handle applications programming. How many programmers will be expected to perform jobs on both types if a total of 55 programmers must be hired? (3 marks)

(b) Let $A = \{1,2,3,4,6\}$ and $B = \{3,4,5,6\}$. Find $(A \Delta B)$ (3 marks)

(c) Assume that the deeper population of a county is 1000 at a time $n = 0$, and that the increase from time $n - 1$ to time n is 10% of the size at time $n - 1$. Write a recurrence relation and an initial condition that defines the deeper population at time n and hence solve the recurrence relation. (5 marks)

(d) Show that $5 + 7 + 9 + \cdots + (2n + 3) = n(n + 4)$ whenever n is a positive integer. (5 marks)

(e) Given that $f(x) = x^2 + 2x + 2$ and $g(x) = x^3 - 1$. Find $f \circ g$. (4 marks)

Question Five (20 marks)

(a) Find the coefficient of x^5 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{10}$ (4 marks)

(b) Using binomial expansion, approximate $\sqrt{24}$ correct to 5 decimal places (4 marks)

(c) Find the inverse of each of the following functions, leaving your answer in terms of the independent variable:

(i) $f(x) = \frac{2}{x^2+1}$ (3 marks)

(ii) $g(t) = \frac{\sqrt{t^2+4}}{4t}$ (3 marks)

(d) Let $A = \{a, b, c, d\}$ and R be a relation on A with the adjacency matrix

$$\begin{matrix} a \\ b \\ c \\ d \end{matrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(i) Draw a Hasse graph represented by the adjacency matrix above (4 marks)

(ii) Write the incidence matrix of the graph in (i) above (2 marks)