Strathmore Institute of Mathematical Sciences(SIMS)
Bachelor of Business Science (Financial Engineering)
End of Semester Examination
BSM 3220 - Optimization Methods in Finance

Date: 5th December 2022 Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

a) List the three essential components of an optimization problem.
. [3 Marks]

b) Consider the optimization problem $Min_x C^T x$ subject to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \geq 0$.

  i. When do we say that the problem is feasible? [1 Mark]
  ii. When does it have an optimal solution? [1 Mark]
  iii. When do we say that it is unbounded? [1 Mark]

c) A company makes two products A and B. The relevant production data is as follows: Profit per unit: $2K.Sh$ and $5K.Sh$ respectively, labour time per unit: $2hrs$ and $1hr$ respectively. The available labour and machine time: $80hrs$ and $65hrs$ respectively. Formulate this as a linear programming problem in standard form. [3 Marks]

d) Write the dual of the following problem. [3 Marks]

$$\begin{aligned} Minimize : Z &= 20x_1 + 40x_2 \\ 36x_1 + 6x_2 &\geq 108 \\ 3x_1 + 12x_2 &\geq 36 \\ 20x_1 + 10x_2 &\geq 100 \\ x_1, x_2 &\geq 0 \end{aligned}$$

e) Daisy eats a mix of cereal A and cereal B for breakfast. The amount of calories, sodium and protein per scoop of some special spoon is: $100mg$ of calories, $150mg$ of sodium and $9mg$ of protein for cereal A and $140mg$ of calories, $190mg$ of sodium and $10mg$ of protein for cereal B. Her breakfast should give at least $480mg$ of calories but less than or equal to $700mg$ of sodium. She would like to maximize her protein intake.

i. What is the objective function? [1 Mark]

ii. List all the necessary (in)equalities. [3 Marks]

iii. Graph the solution. [3 Marks]

iv. Find the corners of the feasible region and the maximum of the objective function. [3 Marks]

v. Make a conclusion based on the values in (iv) above. [1 Mark]

f) Consider the quadratic programming problem:

$$Min_x f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + x_1x_3 + 2.5_2^2 + 2x_2x_3 + 2x_3^2 - 8x_1 - 3x_2 - 3x_3$$

subject to $x_1 + x_3 = 3$ and $x_2 + x_3 = 0$. Express the problem in matrix form. [2 Marks]

g) Tecno mobile company produces two new products P_1 and P_2. Products are produced and sold on a weekly basis. The weekly production can not exceed 25 for product P_1 and 35 for P_2 because of facility limitations. The company has some 60 casuals. Product P_1 needs $2-$ man-weeks of labour, while P_2 needs $1-$ man-week of labour. Profit margin on P_1 is $60K.Sh$ while that on P_2 is $40K.Sh$. Formulate this problem as a LPP. [2 Marks]

h) Use graphical method to solve (if possible): [3 Marks]

$$Max : Z = x_1 + \frac{x_2}{2}$$
$$3x_1 + 2x_2 \leq 12$$
$$x_1 + x_2 \geq 8$$
$$-x_1 + x_2 \geq 4$$
$$5x_1 = 10$$
$$x_1, x_2 \geq 0$$

QUESTION TWO [20 Marks]

a) *RUBIS* petroleum company has two refineries in Mombasa. Refinery 1 costs $20,000 K.Sh$ per day to run and it can produce 400 barrels of high grade oil, 300 barrels of medium grade oil and 200 barrels of low grade oil. Refinery 2 is newer and modern. It costs $25,000 K.Shs$ to run daily and produces 300 barrels of high grade oil, 400 barrels of medium grade oil and 500 barrels of low grade oil daily. The company has orders totalling $25,0000$ barrels of high grade oil, $27,000$ barrels of medium grade oil and $30,000$ barrels of low grade oil. How many days should it run each refinery to minimize the cost and still meet the orders? Use simplex method. [10 Marks]

b) Give the dual of the following $LPP's$:

i. The variable x_3 is unrestricted in sign. [5 Marks]

$$\begin{aligned} Minimize: Z = x_1 + x_2 + x_3 \\ x_1 - 3x_2 + 4x_3 = 5 \\ x_1 - 2x_2 \leq 3 \\ 2x_2 - x_3 \geq 4 \\ x_1, x_2, \geq 0 \end{aligned}$$

c) Are the following statements true or false? [2 Marks]

i. The value of the objective function for any feasible solution of the primal is less than the value of the objective function for any feasible solution of the dual.

ii. If either the primal or the dual has an unbounded solution, then the solution to other problem is feasible.

d) Give three advantages and/or applications of duality. [3 Marks]

QUESTION THREE [20 Marks]

a) Consider some portfolio-selection problem where the decision variables are the amounts to invest in each security type:

$$Max(Z) = 0.043X_A + 0.027X_B + 0.025X_C + 0.022X_D + 0.045X_E$$

subject to

$$\begin{aligned} Cash: X_A + X_B + X_C + X_D + X_E &\leq 10 \\ Goverments: X_B + X_C + X_D &\geq 4 \\ Quality: 0.6X_A + 0.6X_B - 0.4X_C - 0.4X_D + 3.6X_E &\leq 0 \\ Maturity: 4X_A + 10X_B - X_C - 2X_D - 3X_E &\leq 0 \\ X_A, X_B, X_C, X_D, X_E &\geq 0 \end{aligned}$$

i. Solve the problem using simplex method. [10 Marks]

ii. Formulate its dual and solve it as well using simplex method. Compare the optimal solutions to both primal and dual.
. [10 Marks]

QUESTION FOUR [20 Marks]

a) Consider the quadratic optimization problem:

$$\begin{aligned} Minimize: f(x,y) &= 4x^2 + 5y^2 \\ s.t, 0 &= 2x + 3y - 6 \end{aligned}$$

Solve using the method of Lagrange multipliers. [7 Marks]

b) Use Lagrange multipliers to find the maximum and minimum values of $f(a,b,c) = 3a - b - 3c$ subject to $a + b - c = 0$ and $a^2 + 2c^2 - 1 = 0$.
. [13 Marks]

QUESTION FIVE [20 Marks]

a) Distinguish between integer/discrete optimization problems and continuous optimization problems. [2 Marks]

b) Nigel is due to take an aptitude exam in order to become a trainee in one of the banks. The exam has 10 essay questions and 50 short questions. He has 90 minutes to take the exam. The essay questions are worth 20 points each and the short questions are worth 5 points each. An essay question should take 10 minutes to answer while a short question should take 2 minutes. Nigel must do at least 3 essay questions and at least 10 short questions to get full points on all questions he attempts and wants to maximize the number of points he will get. Help him make the best decision. [8 Marks]

c) A leather shop makes custom designed, hand tooled briefcases and luggage. The shop makes $400K.Sh$ from each briefcase and $200K.Sh$ from each piece of luggage. The shop has a contract to provide NAIVAS supermarket with exactly 30 items per month. A tannery provides the shop with at least $80m$ of leather per month. The shop must use at least this amount but can order no more. Each briefcase needs $2m$ of leather and each luggage needs $8m$ of leather. From records, they can not make more than 20 briefcases per month. They want to know the number of briefcases and pieces ofluggage to produce in order to maximize profit. Use simplex method. [10 marks]