Strathmore
UNIVERSITY

SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE INFORMATICS AND COMPUTER SCIENCE
END OF SEMESTER EXAMINATION

CNS 1204: Calculus II

Date: 15th March, 2022 **Time: 2 Hours**

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

(a) Integrate the following:

(i) $\int_0^9 (2\sqrt{x} - 4)\, dx$ [3 Marks]

(ii) $\int \left(\frac{1}{7-5x}\right)\, dx$ [3 Marks]

(b) The GoldPlus Company determines that the marginal cost for their *Aiibe 64GB flash drive* is given by the differential rate equation

$$\frac{dC}{dx} = -0.02x + 6$$

where x is the number of flash drives produced and C is the cost in dollars.

(i) Determine the general solution of the cost function C [3 Marks]

(ii) Find the particular cost function C, knowing that the cost of producing 10 *Aiibe 64GB flash drive* is $400. [3 Marks]

(c) If the population of a country is 40 million people and the population is increasing exponentially with a growth constant $k = 0.69$, calculate precisely the population after 7 years. [Hint: $y(t) = y_0 e^{kt}$] [3 Marks]

(d) Determine the exact area of the shaded region in the figure 1 below. [5 Marks]

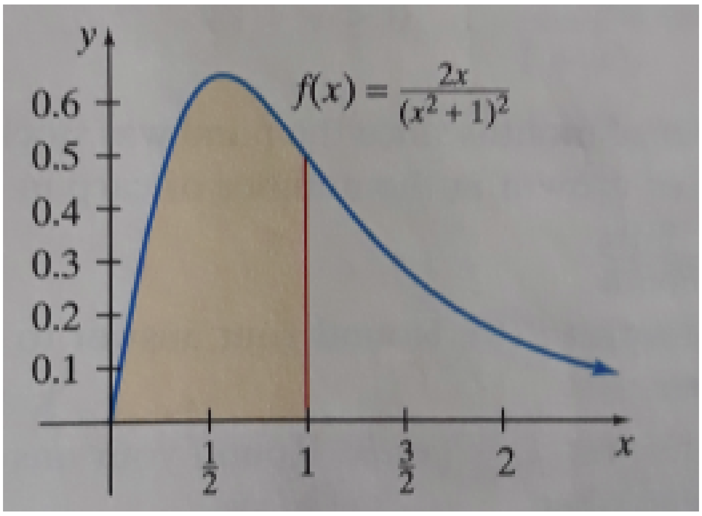

Figure 1

(e) Determine the particular solution for the following initial value problem [5 Marks]

$$\frac{dy}{dx} = xy - 3x, \quad y(0) = 4$$

(f) Use the Midpoint Rule with $n = 5$ to estimate $\int_0^1 e^{x^2}\,dx$ [5 Marks]

QUESTION TWO (20 MARKS)

(a) Media consultants for the new local magazine *ITRave!* have projected that the number of subscriptions will grow during the first five years at a rate given by

$$S'(t) = \frac{1000}{(1+0.3t)^{3/2}}, \qquad 0 \le t \le 60$$

where t is the number of months since the magazine's first issue and $S'(t)$ is the rate of change in the number of subscriptions measured in *subscriptions/month*.

(i) Evaluate $S'(12)$ [3 Marks]

(ii) Evaluate and interpret $\int_0^6 S'(t)\,dt$ [7 Marks]

(b) Determine the area between the x-axis and $f(x) = x^2 - 7x + 6$, on the interval $[2, 6]$. [3 Marks]

(c) Given that $x = 5$ when $y = 0$, solve the following initial value problem (IVP): [7 Marks]

$$e^y \frac{dy}{dx} = (2x - 4)$$

QUESTION THREE (20 MARKS)

(a) Since running a series of first-come, first-served promotions, the FineHomes Furniture Store has found that its sales rate during its three-month sales drive is given by the function

$$s(t) = \frac{10}{t} + 2, \qquad 1 \leq x \leq 12$$

where t represents the number of weeks that the promotion has been running and $s(t)$ is the sales rate measured in $\frac{thousands\,of\,dollars}{week}$.

(i) Determine the total increase in sales generated from the first to the fifth week. [4 Marks]

(ii) Knowing that \$6000 was made during the first week, recover $R(t)$, the revenue generated after t weeks. [5 Marks]

(b) Evaluate the integral [5 Marks]

$$\int \sin^2 x \cos^3 x \, dx$$

(c) The Current (year 2020) population of the Earth is 7.5 billion people, and the yearly birth and death rates are $\beta = 0.021$ and $\delta = 0.009$ respectively. Assuming the birth and death rates remain constant, find the population of the Earth in the year 2120. [6 Marks]

QUESTION FOUR (20 MARKS)

(a) A manager at the Black Box microprocessor manufacturing company finds through data gathered in research that the marginal cost function for a certain type of automobile computer chip made at the facility is given by

$$MC(x) = C'(x) = 6x\sqrt{x^2 + 11}$$

where x represents the number of auto computer chips produced each hour and MC(x) represents the marginal cost. The manager also knows that it costs \$1932 to manufacture 5 chips.

(i) Recover the cost function C [7 Marks]

(ii) Find the fixed costs. [To avoid confusion, call the arbitrary constant d.] [3 Marks]

(b) Use integration by parts method to determine

$$\int xe^{0.2x} \, dx$$

[6 Marks]

(c) Given that the half-life T of radium is 1690 years, how much will remain of **five** grams of radium after 2000 years? [4 Marks]

QUESTION FIVE (20 MARKS)

(a) The marginal profit $MP(x)$ for producing x units of an electrical appliance is given by the linear function

$$MP(x) = 80 + 0.03x$$

where $MP(x)$ is in dollars per unit. Knowing that $P = 0$ when $x = 0$, recover the profit function P. [Hint: $P(x) = \int MP(x)\,dx$] [5 Marks]

(b) Evaluate the following antiderivative.

$$\int x \ln x \, dx$$

. [5 Mark]

(c) Solve the given differential equation

$$x\frac{dy}{dx} = 2(y-3)$$

[5 Marks]

(d) Find the area A of the region $\mathcal{R}$ under the line $y = \frac{1}{2}x + 2$, above the parabola $y = x^2$ and between the y-axis and $x = 1$. [5 Marks]

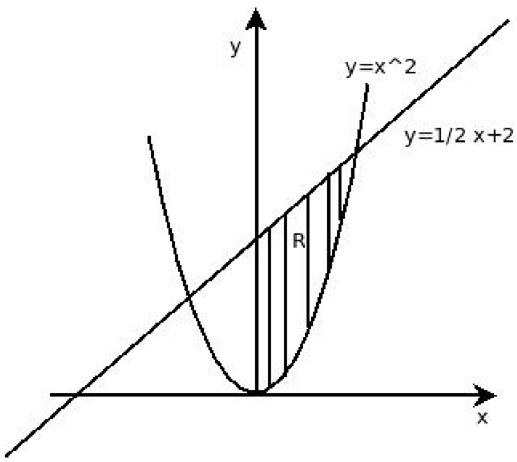

Figure 2

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