2022

Forecasting the term structure of interest rates in Kenya using Bayesian models post 2007-2008 financial crisis.

Bosire, Luycer Nyanchama
Strathmore Institute of Mathematical Sciences
Strathmore University

Forecasting the Term Structure of Interest Rates in Kenya using Bayesian Models

Post 2007-2008 Financial Crisis.

By

Luycer Nyanchama Bosire

1234384

Master of Science in Statistical Science

2022

Forecasting the Term Structure of Interest Rates in Kenya using Bayesian Models

Post 2007-2008 Financial Crisis.

By

Luycer Nyanchama Bosire

124384

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science in Statistical Science at Strathmore University.

Institute of Mathematical Sciences

Strathmore University
Nairobi, Kenya

October, 2022

Declaration and Approval

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made in the thesis itself.

Bosire Luycer Nyanchama

Signature: ______________________ Date: ______________________

Approval

The thesis of Luycer Nyanchama Bosire was reviewed and approved for examination by the following:

Prof. Samuel Mwalili

Senior Lecturer, School of Humanities and Social Sciences

Strathmore University

Dr. Godfrey Achono Madigu,

Dean Institute of Mathematical Sciences,

Strathmore University

Dr. Bernard Shibwabo,

Director of Graduate Studies,

Strathmore University

Abstract

Despite the growing significant advances in the modelling of the term structure of interest rates after the great recession of 2008, little attention has been paid to the problem of forecasting the term structure which has proven to be an important rate in several products and instruments offered by financial institutions. This dissertation makes use of a Dynamic Nelson-Siegel model with a Time-Varying Vector Auto-Regressive component to fit a model and forecast the h-step ahead expected yield. The model makes use of four parameters representing a decay factor, level, slope and curvature latent factors estimated with high efficiency. We propose to use our DNS-TV-VAR model to estimate our factors and demonstrate the model consistency to a range of stylized yield curve initial data. We apply the model in forecasting a term structure for short and long horizons and conclude that the forecasts appear more accurate for long horizons.

Keywords: *Bayesian Statistical modelling,Dynamic Nelson-Siegel, TV-VAR models, Term Structure of Interest rate*

Table of Contents

List of Figures

List of Tables

List of Abbreviations

AFNS: Arbitrage-Free Nelson-Siegel

AIC: Akaike Information Criterion

ARMA-GARCH: Auto-Regressive Moving Average-Generalized AutoRegressive Conditional Heteroskedasticity

ATSM: Affine Term Structure Models

BIC: Bayesian Information Criterion

CBK: Central Bank of Kenya

CDS: Central Depository System

CIR: Cox-Ingersoll-Ross (Model)

DNS: Dynamic Nelson-Siegel

DCOSB: Dynamic Constrained Smoothing B-Splines

DDNM: Dynamic Dependence Network Models

HMM: Hidden Markov Model

MCMC: Markov Chain Monte Carlo

PDE: Partial Differential Equation

RMSE: Root Mean Squared Error

SDE: Stochastic Differential Equation

T-Bill: Treasury Bill

TV-VAR: Time-Varying Vector Auto Regression

Acknowledgement

I thank the Almighty God for the gift of life and for making this course a success thus far. I acknowledge the support I have received from both the lecturers and my fellow students at the Strathmore Institute of Mathematical Sciences during my study. I wish to extend my sincere gratitude to my supervisor, Prof Samuel Mwalili, for sparing time to guide, direct and assist me whenever I encountered challenges. His support made this work a reality. Lastly, I thank my parents, siblings and friends for their moral support and encouragement during my studies.

God bless you all.

Dedication

To my beloved parents and family, classmates and friends.

Chapter 1

INTRODUCTION

1.1. Background of the Study

The African economy has undergone many structural changes with enhanced moderation of business cycle fluctuations. The monetary policy conduct has become more transparent and aggressive, with the relation between the real and nominal side of the economy changing more drastically. Macroeconomic factors are constantly evolving. Macroeconomic forecasting is a vital component of monetary policy formulation and credit modelling by central banks and commercial banks. This is pivotal to the imposition of certain policies by central banks on commercial banks where they wish to anchor certain macroeconomic variables as their policy targets. Examples include the use of inflation to stabilize the economy or nominal interest rate targets as monetary policy instruments. It is therefore almost compulsory to build accurate forecasting models that encompass how the change of a policy control variable could impact policy targets over a defined period of time.

The term structure of interest rates, also known as the yield curve, describes the relationship between Treasury yields of various maturities. It is normally a representation of the state of the bond market. It is critical because spreads between short-term and long-term rates are a particularly potent predictor of future economic contraction. Long term rates normally fluctuate with interest rate changes. Producing accurate term structure estimates over a range of forecast horizons is crucial for bond portfolio managers as well as monetary policymakers. Forecasting the term structure of interest rates involves fitting and predicting the evolution of a cross-section of Treasury bills and bonds with varying maturities across time. This is a very complex dimension issue, seeing that a comprehensive term structure of interest rates does not exist in the real world. Observable market data generate discrete points that are utilized to establish a relationship between interest rates and maturities.

Within the framework of decision theory, the Bayesian methods offer a consistent way of fusing the prior knowledge with the data. Forming a prior distribution for a future analysis allows us to take into account historical data regarding a parameter or hypothesis. The prior posterior distribution can be used as the prior distribution when new observations become available. The Bayes theorem logically supports these inferences. Without relying on asymptotic approximation, the Bayesian analysis presents precise inferences that are dependent on the data.

Bayesian approaches are preferred in financial and econometric problem resolutions as they address and define a structural dynamic model for multivariate time series while utilizing hierarchical and contemporaneous structure across the time series. Bayesian methods are especially appropriate in studies where subjectivity might lead to misinterpreted findings or preconceived inferences. Financial research, especially those centered on investment vehicles such as stocks and bonds, are particularly prone to subjectivity due to the behavioral and psychological aspects of financial decisions. Furthermore, finance stocks and bonds parameters such as stock returns are highly erratic and are heavily influenced by system shocks and noises.

Bayesian modelling is increasingly becoming popular in macroeconomic studies with common applications in dynamic factor models (Stock & Watson, 2002), vector auto-regressive models (Sims, 1992) and Time-Varying Auto-Regressive Models (Koop & Korobilis, 2010). Dynamic Factor Models (DFMs) received great attention over their ability to model data where the number of series exceeds the number of observations, for example ten years of monthly data (Stock & Watson, 2002). According to (Press & Tanur, 2012), Bayesian methods could replace some of the classical frequentists' statistical methodology. Bayesian analysis can be distinguished from classical statistics through the concept of inverse or prior probability and by the premise that deductive logic alone is not sufficient for inference (Beck, Niendorf, & Peterson, 2012). The Bayes' theorem, where posterior probability depends both on prior and sample information, is defined by:

$$\text{Posterior probability} = \frac{(Likelihood)(PriorProbability)}{MarginalLikelihood}$$

The majority of mainstream macroeconomic models have attempted to explain critical aspects of interest rate term structure, particularly in terms of rationalizing the average term spread. This study focuses on the dynamic link between bond and bill yields and the term structure of interest rates in Kenya.

1.2. Statement of the Problem

Numerous mainstream macroeconomic models struggle to explain the primary characteristics, keys, and attributes of the term structure of interest rates. This is exacerbated by the fluidity of macroeconomic circumstances and the subjectivity of financial data around investment vehicles. The average term spread of bond yields and the joint structure of bond yields and macroeconomic factors may not be explicable by standard models. Additionally, majority of studies that fit predictive models for the term structure of interest rates have focused on developed markets, with little research conducted on developing economies and the African market in particular. Due to their effectiveness in coping with the volatility of financial data, the models of a vast majority of concluded research are founded on financial mathematics theories and techniques. This creates a statistical gap for further research into how to best incorporate statistical models into existing financial models or create a hybrid model that incorporates both Bayesian features and financial model strengths around volatility. This could lead to scalable and more accurate models used to capture the dynamics of the interest rate term structure. To this end, the success of this study hinges on two crucial aspects: applying a Bayesian approach to the analysis of financial data time series and evaluating both time series and Bayesian model properties by predicting the future term structure of interest rates for an African and specifically Kenyan market context following the 2007-2008 financial crisis.

1.3. Objectives

1.3.1. General Objective

To forecast the term structure of interest rates in Kenya using Bayesian models post the financial crisis of 2007-2008.

1.3.2. Specific Objectives

1. To develop a Bayesian model for forecasting the term structure of interest rates.
2. To evaluate the Bayesian model properties through a simulation study.
3. To apply the Bayesian model in predicting the term structure of interest rates post the financial crisis of 2007-2008.

1.3. Rationale of the Study

The purpose of this study is to utilize the Bayesian framework to model and forecast the term structure of Kenyan interest rates. The research study period focuses on the years following the 2007-2008 financial crisis. The research localizes significant statistical and empirical forecasting approaches that have been successful in industrialized/developed economies to the Kenyan economy and other frontier markets.

This research is aimed at banks and financial institutions that have only used the average weekly T-bill rate or monthly averages for the 91-day T-bill rate as a basis for decision making on lending rates and inter-bank rates. The study is additionally a theoretical or practical option for predicting future rates, as well as the modelling the impact on their customers, lending models, and loan book performance.

Furthermore, the findings of this study will benefit issuers of government bills, as well as banks and financial institutions who base their rates on treasury bills, such as inter-bank rates, lending/loan rates, and money market fund interest rates. The Kenyan government, as the issuer of T-bills, will have a new practical viewpoint on forecasting the term structures of treasury bills, taking into account previous regimes' behaviors and trends.

The study results are indicative of the behavior of interest rates over time. This could serve as an investor guide in deciding what type of financial asset to hold and over what investment horizon. Additionally, this study provides researchers and academics with fresh models and perspectives on the junction of mathematics and statistical finance as a foundation for future research.

The remainder of this paper is organized as follows: Chapter 2 assesses existing literature around this study, Chapter 3 looks into the methodology we aim to employ in our study and data analysis, Chapter 4 discusses results based on data analysis and Chapter 5 offers the principal investigator’s concluding remarks and recommendations.

Chapter 2

LITERATURE REVIEW

2.1. Introduction

The term structure of interest rates is closely related to the macroeconomic activities of its host country (R.-R. Chen, Cheng, & Wu, 2013). The monetary policy, according to financial analysts and traders, is a major determinant of the volatility of the term structure. On the other hand, the treasury market is sensitive and responds quickly to macroeconomic news and variables. As an example, premiums tend to increase with an increase in the bond term where the macroeconomic environment remains favorable, while the inverse applies during recession (Fama & French, 1989).

2.2. Macroeconomics and the Term Structure

The term structure of interest rates, with a focus on the junction of macroeconomics and finance, remains essential for investors and policymakers who desire to model macroeconomic expectations based on long-term interest rates and take action to impact these rates (Gürkaynak & Wright, 2012). The expected returns on long-term investments include a premium for term, maturity and risk that relates to the longer-term component of business conditions. According to Fama and French (1989), despite widespread belief that expected returns are lower under stronger economic conditions and higher under weaker economic conditions, standard macroeconomic models have struggled to rationalize the average term spread and the failure of the expected hypothesis when attempting to explain key characteristics of interest rate term structure.

Numerous macroeconomic and financial elements that affect the stock market have been investigated and documented. Variables that are frequently studied include, but are not limited to, the gross domestic product, price levels, industrial production rates, interest rates, exchange rates, bank rates, current account balances, unemployment rates, and fiscal balances, among others. Few studies have examined the direct effect of some of these variables and their dynamic characteristics on Kenyan stock market results. A stochastic endogenous growth model with imperfect price adjustment and monetary policy shocks predicts the existence of an equilibrium term structure of nominal and real interest rates and time-varying bond risk premia. For the majority of mainstream macroeconomic models, it has been difficult to explain the different key aspects of interest rates, particularly in terms of justifying the average term spread and expectation hypothesis.

The correlation between bond yields and macroeconomic fluctuations is supported by empirical research (Ang, Piazzesi, & Wei, 2006). In addition, monetary policies serve as a conduit between interest rates and aggregate factors. Several studies on the modeling of interest rates have utilized stochastic differential equilibrium or equation models for their final models. This model has numerous distinguishing characteristics. Means, volatility, and auto-correlations of nominal bond yields are accounted for using a model calibrated to match time series features. Also captured by the model are the empirical failures of the expectation hypothesis of excess bond returns anticipated using forward spreads and linear combinations of future rates (Piazzesi, 2005).

The effect of interest rates on the stock market has significant consequences for monetary policy, risk management methods, the value of financial securities, and government financial market policies. The specific explanation for interest rates and stock prices is their negative correlation. High interest rates typically have a negative impact on stock market returns, as they reduce the value of equity, make fixed income securities more attractive as a factor of stock holding, may reduce the likelihood of investors borrowing and investing stock, and increase operational costs, thereby impacting profit margins.

Since the 1970s, numerous research have been conducted on the relationship between stock prices and interest rates. Fama (1981) found a substantial positive correlation between stock returns and real economic factors such as capital expenditure, gross domestic product, money supply, lagged inflation, and interest rates. N.-F. Chen, Roll, and Ross (1986) established that variations in interest rates, maturity risk premium, and default risk premium are economic variables that explain stock price fluctuations. Smirlock (1985) discovered that changes in interest rates can have an effect on stock prices by either influencing the rate at which a company's estimated future cash flow will be capitalized or by altering cash flow expectations.

In 1987, Hardouvelis observed an inverse link between stock prices and fluctuations in interest rates, which may be explained by unexpected money supply growth. He stated this in terms of the following premise regarding the predicted real interest rate and inflation.

i. Stock prices decline since the real component of nominal interest rates is expected to increase hence rising the discount rate at which future cash flows are capitalised and further since higher interest rates affect output adversely as seen in prior research thereby reducing future operating cash flows.

ii. Stock prices decline as a result of an increase in the inflation premium resulting in a decrease in real dividends after tax.

Wu (2001) grouped macroeconomic variables into either money supply or interest rate elements and determined that money supply did not suggest patterns of influence on stock prices, however interest rates played a crucial role in establishing the investment horizon of a stock. The conclusion of a study by Wong, Khan, and Du (2006) was that there were opposite connections between interest rates and stock prices in various nations - the United States and Singapore - emphasizing the necessity to fit country- and economy-specific models to interest rate forecasting.

2.3. Statistical Modelling of the Term Structure of Interest Rates

Traditional modeling of term structure decomposes yields into latent variables with broadly classified specification and evolution of two sets of models. The first group consists of statistical models that are constructed by modeling the terms structure using a set of smooth exponential basis functions (Nelson & Siegel, 1987). Diebold and

Li (2006) extended this paradigm to dynamic three-factor models demonstrating that the latent factors correlate to the yield curve's level, slope, and curvature. This has been expanded to investigate the interplay between the yield curve and macroeconomic variables (Diebold & Li, 2006; Mönch, 2012).

Although these models are manageable and generate relatively accurate forecasts, they lack a relationship to economic and financial theories concerning the term structure of interest rates. As a result, a second class of models called no-arbitrage Affine Term Structure Models (ATSM) was developed to incorporate economic theory into the approach by enforcing no-arbitrage rules in bond markets (Ang & Piazzesi, 2003). Years of innovation in the modeling of the term structure of interest rates have led to the development of dynamic models that account for the temporal variations in the dynamic interplay of economic variables.

Yield curve dynamics became a prominent issue in 1971. Diebold and Li published the Dynamic-Nelson-Siegel models for the first time in 2003. They contended that forecasting of term structures received scant consideration, whereas forecasting of yields was given considerable weight. The fact that no-arbitrage models have little to offer about term structure dynamics and the belief that existing affine equilibrium models provided poor forecasts were identified as obstacles to term structure forecasting (Duffee, 2002).

The Nelson-Siegel yield curve used in the DNS model was more stable and simple, and it imposed critical and fundamental economic qualities, such as a discount function that approaches zero as maturity evolves with variables expressing short-, medium-, and long-term behaviors. The one notable disadvantage of DNS models is that they do not restrict arbitrage opportunities. As a result, practitioners will continue to be exposed to important financial risks, as asset pricing depends on interest rates that are based on the arbitrage-free hypothesis.

To tackle all of these hazards, a class of Arbitrage-Free Nelson Siegel models with DNS structure preservation and no-arbitrage limitations was developed (Christensen, Diebold, & Rudebusch, 2011). Through the inclusion of a yield adjustment term to the Nelson-Siegel yield curve given by ordinary differential equations, arbitrage opportunities are eliminated. This analysis was repeated for larger predicting horizons in which no-arbitrage proved most beneficial (Caldeira, Moura, Santos, & Tourrucôo, 2016).

For further smoothing of financial data in the AFNS model, restricted smoothing B-splines were used to construct the yield curve as a projection of smooth functions into the spaces of the splines. This was accomplished by calculating the conditional median function according to the theory of quantile regression (Koenker & Bassett Jr, 1978). The model benefited greatly from the inclusion of the conditional median function, as it was resistant to outliers, and the construction of the linear programming model allowed for the implementation of constraints without substantially increasing computational costs.

Later in 2020, a study validated a more advanced model based on the Dynamic Constrained Smoothing B-Splines (DCOSB) model and includes constrained smoothing B-Splines to anticipate the term structure of interest rates (Laurini & Moura, 2010). This model, the Dynamic Constrained, described the yield curve model's coefficients as developing processes across time. To construct a mutual grounding and curve-shaped evolution across time, equally distributed knots were established in the data set to represent the short-, medium-, and long-term aspects of the observed data. The DCOSB model demonstrated excellent short-term predictability and long-term stability.

The DNS model has proven to be simple and effective for predicting a term structure, and various studies have confirmed its effectiveness in forecasting yield curves or other term structures for investment instruments. Shaw, Murphy, and O'Brien (2014) predicted CDS using the Nelson–Siegel model to fit the CDS curve, which is a classic example of the use of DNS models in predicting term structures. The Nelson–Siegel model was utilized by Guo, Han, and Zhao (2014) to model the term structure of implied volatility. GrØnborg and Lunde (2016) used it to model the term structure of future oil contracts and anticipate their prices, whereas West [60] used it to forecast agricultural commodity prices. Studies using longer-term maturity vehicles have been effective, and other macroeconomic variables that frequently impact the performance of these vehicles have been integrated into the models. Given that minimum studies employ treasury bills as a platform for their investigations, this serves as a foundation for additional studies and research. Treasury bill rates have a long-term impact and should be modelled with clarity and effectiveness.

Dynamic Dependence Network Models (DDNMs) permit univariate series to be detached for rapid and parallel processing, then re-coupled for forecasting purposes. In forecasting time series, TV-VAR models are typically combined with time-varying auto-regressive models because they capture variations and permit model dynamics to alter over time for multivariate data.

The parental set for each univariate dynamic linear model containing contemporaneous values of other univariate series is a defining characteristic of DDNMs. Using lagged predictors, the contemporaneous values can be forecast and the findings incorporated into the primary predictive model.

DDNM models enables for both contemporaneous and lagged dynamic linkages across series (Lazzaro, 2019). It is a multi-regression dynamic model by extension which impose a hierarchical conditional dependence structure across series allowing for a triangular/Cholesky-style specification of the resulting dynamic graphical model. This particular feature facilitates the modelling of higher numbers in a single time series given univariate series can be decouples for sequence analysis and re-coupled for multivariate forecasting and analysis.

DDNM models have been applied to developed countries market contexts and have proven superior to TV-VAR models over longer horizons. Over short prediction horizons the models forecasting results are not significantly different. Ideally, the Cholesky-style specification can be introduced during model specification as expected in this study from the DDNM models to support increased model flexibility given we will be able to enumerate the series' specific lagged predictors, state-space evolution equations and discount factors for the state space evolution, faster computation and increased scalability to high dimension problems.

Cox-Ingresoll-Ross (CIR) models, which are short-rate models, are also popular in financial institutions owing to their relatively simplistic and superior performance to Vasicek framework. The models was first introduced in 1985 to study the term structure of interest rates while aiming to describe the evolution of interest rates as a diffusion process and based of the SDE:

$$drt = k(\vartheta - r(t))dt + \sigma\sqrt{r(t)}\,dW(t), \qquad r(0) = r_o > 0$$

where k, ϑ and σ are positive constant parameters.

The CIR model has become one of the most extensively used term structure models in finance due to its relatively simple implementation and tractability, as well as the special feature of excluding negative interest rates, which was an undesired feature under the 2008 pre-crisis assumptions. However, the necessity for more sophisticated frameworks that could handle numerous sources of risk, as well as shocks and/or structural changes in the market, led to the development of a number of papers for pricing interest-rate-dependent financial derivatives a few years later. The approach has proven insufficient, given the loss of computational tractability offered by the usage of a Gaussian distribution, in detailing the term structure for interest rates (Orlando, Mininni, & Bufalo, 2019), providing leeway to assess better models that promote the study, modelling and forecasting the term structure of interest rates.

Stochastic Differential Equation models (SDE) Given a continuous time short rate model, the evolution of the short-rate is modelled with a continuous-time Stochastic Differential Equation of the form:

$$dr_t = \mu(r_t, \vartheta)dt + \sigma(r_t, \vartheta)dW_t$$

W_t is a standard univariate Brownian motion. The general functions $\mu(r_t, \vartheta)$ and $\sigma(r_t, \vartheta)$ can encompass linear and non-linear transformations of r_t and the parameter vector ϑ (Gray, 2005).

A Euler discretization of the process is utilized to approximate the likelihood function, while Markov chain Monte Carlo simulations with Gibbs sampling and Metropolis-Hastings stages are employed to estimate the joint posterior density of the model. The sample of observed interest rates is augmented by simulating values for each pair of observations. An increase in data frequency has the potential to lessen discretization bias (Gray, 2005).

2.4. Forecasting Models applied in the Kenyan market

Interest rate modelling in Kenya so far has focused on more descriptive approaches with the majority of the models focusing on the incorporation of the dynamics of interest rates and most importantly the term structure of interest rates.

In analysing interest rate dynamics in Kenya, Caporale and Gil-Alana (2016) examine the stochastic properties of the interest rate spread to provide useful information about the effects of shocks and appropriate policy responses. They test the stationarity of the interest rate series. Treasury bill rate spread under the assumption of autocorrelated errors.

Olweny and Omondi (2011) focuses on the link between short-term volatility of the interest rate and the level of interest rates in Kenya using the Treasury bill rates from August 1991 to December 2007. His findings indicate that the volatility is positively correlated with the level of the short-term interest rate. He also finds that the GARCH model is better suited for modeling volatility of short rates in Kenya, compared to ARCH models. This supports the requirement of this study to ensure a time-varying component is incorporated in the model to cater for volatility of the interest rates.

Chelimo (2017) attempts to calibrate a Vasicek term structure model to the evolution of intrest ratae dynamics in Kenya for both single state and multi-stage models and using a Hidden Markov Model (HMM). His findings begin the studies for the management of risk posed by interest rate dependent instruments. They illustrate that risk volatility fluctuates independent of interest rates, hence risk is not necessarily a function of interest rates but rather dependent on on the inherent variability of rates in a particular state. The study provides support for incorporation of regime switches while neglecting the term structure dynamic of interest rates owing to the utilisation only of the 91-day T-bill for data analysis.

Mukono (2019) attempts to forecast the term structure of Kenyan government bond yields using a Dynamic Nelson-Siegel with time varying components incorporated in the model. He ascertains that a Dynamic Nelson-Siegel with an ARMA and ARMA-GARCH time component perform the poorest based on their RMSE while the Dynamic Nelson Siegel models with a Random Walk, AR and TV-VAR component perform the best in fitting and forecasting the term structure of the bond interest rates. The study proves further the power of a dynamic Nelson-Siegel as a forecasting model, term structure modelling is possible in third world countries and the importance of using blended models to cater for the varying maturities for treasury bills and bonds as opposed to focusing studies on an instrument with a common maturity over time.

Chapter 3

METHODOLOGY

3.1. The Bayesian Model

Bayesian inference refers to the process of learning by updating prior probabilistic beliefs in light of new information. We wish to estimate a parameter $\vartheta \epsilon \Theta$ from a dataset $y \epsilon Y$ given:

- $p(\vartheta)$ defined for all $\vartheta \epsilon \Theta$ is our prior distribution about the space of all parameters.
- Bayesian methods require a sampling model $P(y/\vartheta)$ describes the probability of a specific data set given a parameter.

We wish to update our belief distribution about ϑ given the *posterior distribution* is defined as :

$$p(\vartheta/y) = \frac{p(y/\vartheta)p(\vartheta)}{\int_{\Theta} p(y/\vartheta)p(\vartheta)d(\vartheta)}.$$

This can also be expressed as:

$$p(\vartheta/y) \ a \ p(y/\vartheta)p(\vartheta).$$

There are three stylized facts of time series from macroeconomics in the construction of priors according to Ciccarelli and Rebucci (2003):

(i.) A majority of time series are characterized by a trend component

(ii.) Despite the fact that most macroeconomic data is persistent, the lags that matter the most are the most recent ones

(iii.) A variable's lags influence the variable more than the lags of other variables

3.2. Dynamic Nelson Siegel Model

Bayesian extensions in the Dynamic Nelson Siegel model involve the use of more flexible parametric forms of the yield curve and allows the parameters to vary in time though the utilisation of a structure of latent factors. The inference further allows for the addition of a stochastic volatility structure component to control the presence of conditional heteroskedasticity observed in interest rates.

Assume that the spot interest rate on fixed income bonds considered to be risk-free is denoted by $s(\vartheta)$. We can obtain the present value of a financial instrument with a certain future value, assuming continuous compounding of its interest rate through the function:

$$d(\vartheta) = e^{-s(\vartheta)x\vartheta}.$$

We model the future forward rate as a function of maturity in a differential equation below:

$$r(\vartheta) = \beta_1 + \beta_2 exp(-\vartheta\lambda) + \beta_3[(\vartheta\lambda)exp(-\vartheta\lambda).$$

We obtain a yield curve from a forward curve through integration of $r(\vartheta)$ above and obtain the result below:

$$y_t(\vartheta) = \beta_{1t} + \beta_{2t}\left(\frac{1-exp^{\lambda t\vartheta}}{\lambda_t\vartheta}\right) + \beta_{3t}\left(\frac{1-exp^{-\lambda t\vartheta}}{\lambda_t\vartheta} - exp^{-\lambda_t\vartheta}\right),$$

with the parameters described as:

Table 1: Model Parameters

Parameter	Parameter Description
β_{1t}	level factor/long-run interest rate
β_{2t}	slope factor/short-term effect
β_{3t}	curvature factor/midterm effect
m	time to maturity
λ_t	decay factor

3.2.1. Estimation of the parameter λ

λgovernshowmuchtheslopeandcurvaturef actorscontributetotheyieldcurverelativetothelevelfactor.

We evaluate the maxima on β_{3t} loading, $\frac{1-exp^{-\lambda m}}{\lambda m} - exp^{-\lambda m}$, to estimate the parameter λ, then replace ϑ with the preferred forecast period to obtain the value of λ. Small values of λ indicate a slow decay which is associated with long-term maturities. Several literature report that this model explains more than 90% variations in the yield curve. Large values of λ indicate faster decay hence associated with short maturities.

$$y_t(m) = \beta_{1t} + \beta_{2t}\left(\frac{1 - exp^{\lambda\vartheta}}{\lambda\vartheta}\right) + \beta_{3t}\left(\frac{1 - exp^{-\lambda\vartheta}}{\lambda\vartheta} - exp^{-\lambda\vartheta}\right)$$

$y_t(m)$ defines the interest rate at time t for maturity m.

We differentiate the parameter loading on β_{3t} with respect to ϑ, which can also be expressed under a common denominator as:

$$\frac{1 - exp^{-\lambda\vartheta} - exp^{-\lambda\vartheta}\lambda\vartheta}{\lambda\vartheta}.$$

Employing the quotient rule,

$$u = 1 - exp^{-\lambda\vartheta} - exp^{-\lambda\vartheta}\lambda\vartheta,$$

and,

$$v = \lambda\vartheta.$$

Thus,

$$u^{/} = \vartheta e^{-\lambda\vartheta} - [-\vartheta e^{-\lambda\vartheta} * \lambda\vartheta + \vartheta e^{-\lambda\vartheta}],$$

$$= \vartheta e^{-\lambda\vartheta} + \vartheta e^{-\lambda\vartheta} * \lambda\vartheta - e^{-\lambda\vartheta} = \vartheta e^{-\lambda\vartheta} * \lambda\vartheta,$$

$$= \lambda\vartheta^{-\lambda\vartheta},$$

$$= \lambda^2\lambda e^{-\lambda\vartheta}.$$

and,

$$v^{/} = \vartheta.$$

$$\frac{\partial L_3}{\partial\lambda} = \frac{\vartheta^2\lambda e^{-\lambda\vartheta}\lambda\vartheta - \vartheta[1 - e^{-\lambda\vartheta} - e^{-\lambda\vartheta}\lambda\vartheta]}{\lambda^2\vartheta^2} = 0$$

$$\vartheta^3\lambda^2 e^{-\lambda\vartheta} - \vartheta[1 - e^{-\lambda\vartheta} - \lambda\vartheta e^{-\lambda\vartheta}] = 0$$

$$\vartheta^2\lambda^2 e^{-\lambda\vartheta} - [1 - e^{-\lambda\vartheta} - \lambda\vartheta e^{-\lambda\vartheta}] = 0$$

$$\vartheta^2\lambda^2 e^{-\lambda\vartheta} - 1 + e^{-\lambda\vartheta} + \lambda\vartheta e^{-\lambda\vartheta} = 0$$

$$\vartheta^2\lambda^2 - e^{\lambda\vartheta} + 1 + \lambda\vartheta = 0$$

$$1 + \lambda\vartheta + \vartheta^2\lambda^2 - e^{\lambda\vartheta} = 0$$

A key factor in the modelling of the term structure in financial models is the addition of time-varying parameters for example the random walk, ARMA, AR, VAR and GARCH components. For this particular study, and owing to the limitations and recommendations by prior studies, we aim to utilize a Dynamic Nelson-Siegel model with a time varying vector auto-regressive component (DNS-TV-VAR). Extending the works of Diebold and Li (2006), we seek to find h- steps ahead forecast for each maturity ϑ_i at a given time t. The forecast model is therefore denoted by $\hat{y}_{t+h}(\vartheta_i)$

3.2.2. The Dynamic Nelson-Siegel model with a time varying vector auto-regressive component (DNS-TV-VAR) Model

Recall that from the Nelson-Siegel model, $y_t(m)$ defines the interest rate at time t for maturity m, and is defined by:

$$y_t(m) = \beta_{1t} + \beta_{2t}\left(\frac{1 - exp^{\lambda\vartheta}}{\lambda\vartheta}\right) + \beta_{3t}\left(\frac{1 - exp^{-\lambda\vartheta}}{\lambda\vartheta} - exp^{-\lambda\vartheta}\right) + \epsilon_t(\vartheta).$$

The yield forecasts based on a Dynamic Nelson-Siegel with Auto Regressive Moving Average specifications are:

$$\hat{y}_{t+h}(m) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1 - e^{-\lambda m}}{\lambda m}\right] + \hat{\beta}_{3,t+h}\left[\frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m}\right],$$

$$\beta_{i,t+h} = a_0 + \sum_{j=1}^{p} a_j \beta_{i,t-j+1} + \sum_{k=1}^{q} b_k \epsilon_{t-k+1}. \qquad i = 1, 2, \ldots$$

The β function for a Dynamic Nelson-Siegel with a Vector Analysis factor is given by:

$$\beta_{i,t+h} = a_0 + \sum_{i=1}^{p} a_j \left(\beta_{t-j+1} + b_j \beta_{2,t-j+1} + c_j \beta_{3,t-j+1}\right) + \epsilon_{t+h}, \qquad i = 1, 2, \ldots$$

where $\epsilon_{t+h} \sim N(0, \sigma^2)$, and can be summarised as:

$$\beta_{t+1} = (I + \Phi)\mu + \Phi\beta_t + \eta_t,$$

where,

$$\epsilon_t \sim N(0, \sigma^2 I_p), \epsilon_{t+h} \sim N(0, \sigma^2),$$

$$\eta_t \sim N(o, \Sigma_\eta),$$

$$\beta_1 \sim N(\mu, P_\beta)$$

.

β is the unobserved vector of our latent factors, μ is a vector containing the factor means, Φ is a vector auto-regressive coefficient matrix corresponding to a stationary process. In this case, the β factor loadings can be interpreted as follows:

1. β_1 is the level factor for all interest rates
2. β_2 is the slope of the yield curve
3. β_3 is the curvature or shape of the yield curve

3.2.3. Bayesian estimation of the Dynamic Nelson Siegel Model

The Dynamic Nelson-Siegel Model with Stochastic Volatility, which models the conditional heteroscadasticity, is subjected to Bayesian inference.

Prior

We can determine prior distribution, from past data. For interest rate data, it is rare to find a negative interest rate, the presence of which translates to a collapsing financial environment or systems. We can therefore assume that interest rates are all positive and consequently priors that support the positive side of the real line.

We assume a hierarchical Bayes model where β_o, β_1, β_2 are assumed to have normal priors given that the betas take values in real line and the ψ >0 and is assumed to have a gamma prior, given it models the scaling effect of interest rate. λ>0 is assumed to have a uniform prior.

Likelihood

The likelihood of the Dynamic Nelson-Siegel model is constructed from the following model specification :

$$Y_t \sim (\mu, \psi),$$

$$\mu = \beta_0 + \beta_1\left(\frac{1 - exp^{\lambda\vartheta}}{\lambda\vartheta}\right) + \beta_2\left(\frac{1 - exp^{-\lambda\vartheta}}{\lambda\vartheta} - exp^{-\lambda\vartheta}\right),$$

$$\sigma = \sqrt{\hat{\psi}}.$$

ψ is a precision and can be expressed as the reciprocal of variance $\psi = \frac{1}{\sigma^2}$.

Chapter 4

Data Analysis and Results

4.1 Data Description

The dataset utilized for this study is comprised of weekly and monthly yields on Treasury Bills recorded by the Central Bank of Kenya from January 1997 through December 2021. Treasury bills in Kenya provide investors with risk-averse, competitive returns, making them an attractive investment option and are a way for the Kenyan government to raise money from the public. Prior to the Great Recession of 2008, macroeconomic policy regimes in different economies were consistent and stable. We utilize data from January 2011 to December 2021 specifically for our investigation. The data has been aggregated monthly given the 364 day T-bill is issued once monthly, while the 91- and 182-day T-bill is sold weekly. The maturities of treasury bills range from 91 days (3 months) through 182 days (6 months) to 364 days (12 months/1 year). This study period should be able to capture comprehensive data and the dynamic character of the Treasury bill maturity dates. Treasury bills' weekly recorded interest rates are depicted in Figure 1.

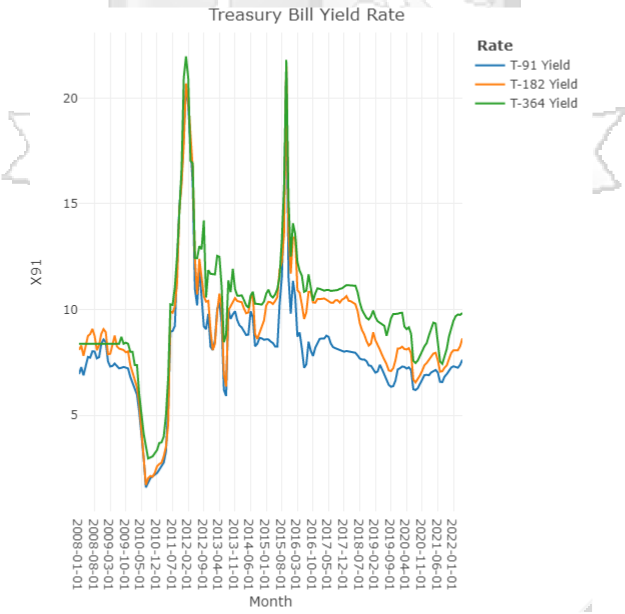

Figure 1: Treasury Bills weekly interest rates prices for the period January 1997 to December 2021

The volatility of the interest rates on treasury yields is informed by market conditions and controlled by the government that acts as the seller of the bills. The treasury bills have been known to have a higher rate whenever the government wishes to achieve limited borrowing, cash liquidity and pay its bills. The state limits borrowing through T-bill issuance to cover primarily maturities and liquidity management needs, rather than using them to fill a portion of the budget financing component from the domestic market. The aim of this is to lengthen the maturity profile of domestic debt and reduce short term refinancing pressure, which results in a T-Bills rates' rise as investors cash in on the government's desperate need for funds to roll over maturing commitments. Fluctuations in interest rates have an effect on inter-bank rates as well.

Assessing the statistics for the respective T-bills and summarised with respect to the three maturities:

Table 2: Descriptive statistics for the T-bills the period 2008 - 2022

	Mean	Std Dev	Min	Max	Median
T-91	7.55	5.24	0.83	27.15	8.11
T-182	8.49	3.07	1.35	21.63	8.49
T-364	10.44	3.03	2.96	21.96	10.44

The mean values for the different maturities depict a characteristic upward sloping yield curve, originating from the 3-month maturity rate of 9.37 to a maximum 10.44 yield rate at 364-month. The standard deviation for the data reveals that volatility decreases with an increase in maturity period that is, short maturity rates are more volatile than the longer maturity rates. The volatility for the 91-day period stands at 5.24 while that of the 364-day period stands at 3.03. Therefore, it is peremptory that shorter rates are riskier than longer rates. Investors should therefore stand advised to consider longer-maturities for their risk-free assets.

4.2. Parameter estimation

Fitting the model has incorporated the below assumptions for the prior distributions:

- $\beta_0 \sim Normal(y(\vartheta_m ax), 0.01)$
- $\beta_1 \sim Normal(y(\vartheta_m ax), 0.01)$
- $\beta_2 \sim Normal(y(\vartheta_m ax), 0.01)$
- $\lambda \sim Unif(0.02, 0.15)$
- $\psi \sim Gamma(0.1, 0.001)$

4.2.1. Estimation and interpretation of The parameter λ

This parameter governs the decay rate of the yield curve and the behavioral aspect relating to the fluctuation (how it achieves its maximum) of the peak for the latent factorβ_{3t} representing the curvature of the yield curve. Small values of the parameter λ produce slow decay rates and can therefore be used to fit the yield curve for long maturities, while large values of the parameter produce fast decays and are therefore more suitable for fitting yield curves for short maturities. We compute the value of λ that maximizes the loading on the factor β_{3t} at 364-day maturity to achieve a value of 0.969.

4.2.2. Estimation and interpretation of The Latent Dynamic Factors

The latent dynamic factors are β_{1t}, β_{2t} and β_{3t}. Table 7 below provides a summary of the parameter estimates of these factors.

Table 3: Parameter estimates for the fitted DNS model

Parameter	Parameter Estimated Value	Mean	Standard Error
β_{1t}	0.3253	0.03568	9301
β_{2t}	0.1715	0.00031	2290
β_{3t}	0.0860	0.02407	105
Total (n)	2273		
Time Elapsed (in seconds)	6.8390		

From the summary table above, β_{1t} has yielded a positive mean. Additionally, the loading on this factor is one (1), a constant. As such, this factor does not decay to zero in its limits and is therefore held as a long term factor. This output is aligned to the works of Diebold and Li on the parameter. β_{2t} has a non-negative mean explained further by the dataset yield curve which bears a positive slope from the sampled period. β_{3t} equally has a non-negative mean yielding a humped shape yield curve. This factor illustrates the curvature of the yield curve. The loading on the curvature and slope factors decays quickly into zero and is suitable to represent short term maturity.

We analyze the time series plots for the factor loadings, provided below.

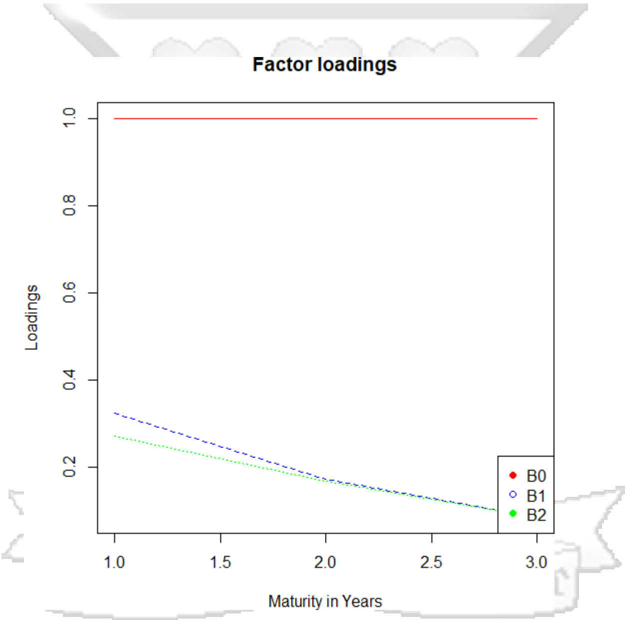

Figure 2: Plots of the factors of the DNS model

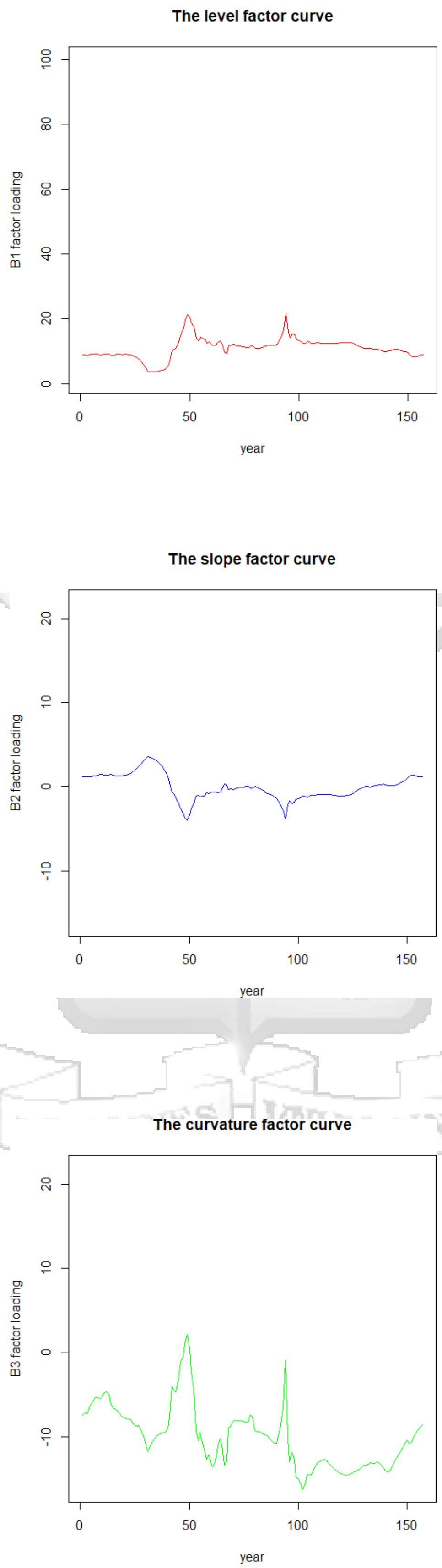

Figure 3: Time Series Plots of the factor loadings of $\beta_{1t}, \beta_{2t}, \beta_{3t}$ of the DNS model

4.3. Fitting the Yield Curve using the dynamic Nelson-Siegel model

We fit a yield curve based on an estimated output from the DNS model:

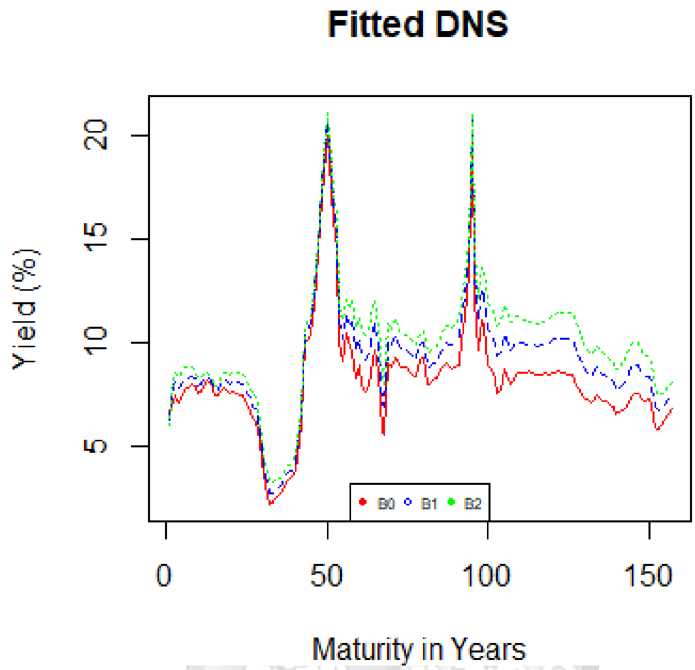

Figure 4: Plot of fitted plot based on the DNS model to come up with average yield curves

Note that the fitted yield curve almost mimics the curve for the actual values as discussed and shown at the beginning of this section. It is clear that the Dynamic Nelson-Siegel model adequately captures and reflects the actual yields of the Treasury Bills as well as fitting the yield curves appropriately.

Below is a summary of the fitted Yield Curve residual values using the dynamic Nelson-Siegel model

Table 4: Residual Statistics for the fitted DNS model

Tbill Maturity	Mean	Standard Deviation	Maximum	Minimum
91	0.01157	0.1327	8.06073	-8.43873
182	0.02804	0.2742	7.33192	-6.9815
364	0.08508	1.7346	6.80679	-5.8944

4.4. Forecasting the Term Structure of Interest Rates and Factor Loadings

We provide forecasting results for the three maturities with the fitted DNS model in this section. The data for the out-of-sample forecasting is for the period January 2019 to December 2021. We forecast over various horizons. The maturities for the data to be used remain at 91-days, 182-days and 364-days representing the short term, medium term and long term maturities respectively.

Having asserted that the fitted yield curve mimics the curvature behaviour of the actual values from the dataset, we can ascertain that the DNS model is best fitted to predict future period expected yields. We use the AIC and BIC diagnostics from the forecasted model to measure the performance of our predictive model and assess for a parsimonious model.

Table 5: Predictive analysis for a parsimonious model

Forecast horizon	AIC	BIC	LogLik
1-step	0.5729	1.4526	4.2894
3-step	1.1954	1.7525	5.2894
6-step	1.228	1.857	7.702
12-step	1.281	2.0869	8.276
60-step	1.945	2.1525	9.6879
120-step	0.8728	2.7525	11.2894

The AIC and BIC values above indicate a parsimonious model given they lie within the best fit limit range of 2 - 4 for the BIC (Raftery, 1995). We also analyse the log-likelihood value of the models and select the best model based on the highest log-likelihood value achieved by the 12-step model. The model seems to be performing well over longer horizons given the steady rise in forecast trajectories. This support the level factor loading that does not decay into zero supporting the notion that the DNS model is fairly stable for longer horizon forecasts. We note from the graphical presentation below that the graph steadily rises with minimal fluctuations and close to none after the 25th year.

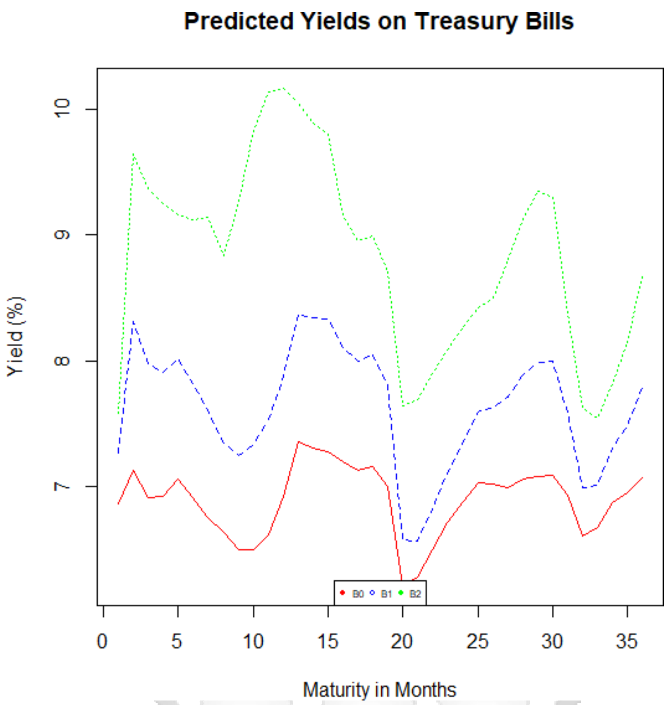

Figure 5: Yield Forecast

Chapter 5

Conclusions and Recommendations

The purpose of this study is to confirm whether there are any Bayesian techniques to modeling and predicting the term structure of interest rates, as well as whether theoretic approaches may be used to anticipate the term structure of Treasury Bills yield rates over a long time horizon. Based on similarities between actual and fitted data values, the availability of an ideal predicted values BIC, and a positive correlation between loglik metric and forecast horizon, we confirm that the DNS model with a TV-VAR component may be utilized to predict the term structure of interest rates. The studies support the use of factor loadings and, most notably, the level factor B1, which is a constant (1), demonstrating that the DNS-TV-VAR model is applicable for predicting over longer horizons.

It is vital to note that the inclusion of a Cholesky specification in fitting our model to support model flexibility, faster computation time and increased scalability. The DNS model has proven to be simple and effective for predicting a term structure, confirming prior studies that have suggested and applied the model's effectiveness in forecasting yield curves or other term structures for investment instruments. It is further observed that the assessment of the prior distribution owing to Bayesian approaches provide a better framework for modelling the term structure of interest rates and should therefore be considered for the Kenyan market, whilst incorporating superior aspects borrowed from other models that have been applied to the market.

Future opportunities for research lie in evaluating comparative model behaviour between other models like the DDNM models as well as DNS models with GARCH, ARCH, ARMA time components for short rate forecasting. Additionally, examining the evolution of interest rate term structure dynamics and volatility when limits are introduced to the interest rate process, such as with politically or economically motivated interest rate controls from the government, would be of major importance.

Furthermore, this effort concentrated on fitting and forecasting, leaving room for future work to incorporate model calibrating and diffusion with other models that assume parameters relating to some economic phenomena.

Additionally, while researching impulse political or economic responses is a more established component of macroeconomic time-series analysis, a number of modeling concerns must be addressed before doing impulse response analysis in this context. These include, but are not limited to, whether to include specific macroeconomic predictors in the model rather than a group of parameters, and whether or not to allow macroeconomic variables to respond to yield curve shocks concurrently. Future research could look into these issues in tandem with the impact of exogenous shocks to macroeconomic data on treasury bill yields, and vice versa. Additional industry context can be incorporated for the exogenous variables, for example bank base rates applied as a minimum for customer rates as well as customer behavioral analysis models.

Future research can also try to add regime changes, which are appealing in models because they incorporate stochastic interest rate behavior within a stationary model. A better model can be developed by combining the advantages of a dynamic Nelson-Siegel model with a TV-VAR component and regime switching models to capture the mean reversion features of short term rates and the leptokurtic unconditional distributions of short term rate changes.

References

Ang, A., & Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary economics*, *50*(4), 745–787.

Ang, A., Piazzesi, M., & Wei, M. (2006). What does the yield curve tell us about gdp growth? *Journal of econometrics*, *131*(1-2), 359–403.

Beck, K., Niendorf, B., & Peterson, P. (2012). The use of bayesian methods in financial research. *Investment Management and Financial Innovations*, *9*(3), 68–75.

Caldeira, J. F., Moura, G. V., Santos, A. A., & Tourrucôo, F. (2016). Forecasting the yield curve with the arbitrage-free dynamic nelson–siegel model: Brazilian evidence. *EconomiA*, *17*(2), 221–237.

Caporale, G. M., & Gil-Alana, L. A. (2016). Interest rate dynamics in kenya: Commercial banks' rates and the 91-day treasury bill rate. *Journal of International Development*, *28*(2), 214–232.

Chelimo, J. K. (2017). *Calibration of vasicek model in a hidden markov context: the case of kenya* (Unpublished doctoral dissertation). Strathmore University.

Chen, N.-F., Roll, R., & Ross, S. A. (1986). Economic forces and the stock market. *Journal of business*, 383–403.

Chen, R.-R., Cheng, X., & Wu, L. (2013). Dynamic interactions between interest-rate and credit risk: Theory and evidence on the credit default swap term structure. *Review of Finance*, *17*(1), 403–441.

Christensen, J. H., Diebold, F. X., & Rudebusch, G. D. (2011). The affine arbitrage-free class of nelson–siegel term structure models. *Journal of Econometrics*, *164*(1), 4–20.

Ciccarelli, M. M., & Rebucci, M. A. (2003). *Measuring contagion with a bayesian time-varying coefficient model*. International Monetary Fund.

Diebold, F. X., & Li, C. (2006). Forecasting the term structure of government bond yields. *Journal of econometrics*, *130*(2), 337–364.

Duffee, G. R. (2002). Term premia and interest rate forecasts in affine models. *The*

Journal of Finance, *57* (1), 405–443.

Fama, E. F. (1981). Stock returns, real activity, inflation, and money. *The American economic review*, *71* (4), 545–565.

Fama, E. F., & French, K. R. (1989). Business conditions and expected returns on stocks and bonds. *Journal of financial economics*, *25* (1), 23–49.

Gray, P. (2005). Bayesian estimation of short-rate models. *Australian Journal of Management*, *30* (1), 1–22.

GrØnborg, N. S., & Lunde, A. (2016). Analyzing oil futures with a dynamic nelson-siegel model. *Journal of Futures Markets*, *36* (2), 153–173.

Guo, B., Han, Q., & Zhao, B. (2014). The nelson–siegel model of the term structure of option implied volatility and volatility components. *Journal of Futures Markets*, *34* (8), 788–806.

Gürkaynak, R. S., & Wright, J. H. (2012). Macroeconomics and the term structure. *Journal of Economic Literature*, *50* (2), 331–67.

Hardouvelis, G. A. (1987). Macroeconomic information and stock prices. *Journal of Economics and Business*, *39* (2), 131–140.

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, 33–50.

Koop, G., & Korobilis, D. (2010). *Bayesian multivariate time series methods for empirical macroeconomics*. Now Publishers Inc.

Laurini, M. P., & Moura, M. (2010). Constrained smoothing b-splines for the term structure of interest rates. *Insurance: Mathematics and Economics*, *46* (2), 339–350.

Lazzaro, J. C. (2019). Forecasting the term structure of interest rates: A bayesian dynamic graphical modeling approach.

Mönch, E. (2012). Term structure surprises: the predictive content of curvature, level, and slope. *Journal of Applied Econometrics*, *27* (4), 574–602.

Mukono, N. M. (2019). *Forecasting the term structure of government bonds in kenya* (Unpublished doctoral dissertation). UoN.

Nelson, C. R., & Siegel, A. F. (1987). Parsimonious modeling of yield curves. *Journal* *of business*, 473–489.

Olweny, T., & Omondi, K. (2011). The effect of macro-economic factors on stock return volatility in the nairobi stock exchange, kenya. *Economics and Finance review*, *1*(10), 34–48.

Orlando, G., Mininni, R. M., & Bufalo, M. (2019). Interest rates calibration with a cir model. *The Journal of Risk Finance*.

Piazzesi, M. (2005). Bond yields and the federal reserve. *Journal of Political Economy*, *113*(2), 311–344.

Press, S. J., & Tanur, J. M. (2012). *The subjectivity of scientists and the bayesian approach* (Vol. 775). John Wiley & Sons.

Shaw, F., Murphy, F., & O'Brien, F. (2014). The forecasting efficiency of the dynamic nelson siegel model on credit default swaps. *Research in International Business and Finance*, *30*, 348–368.

Sims, C. A. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy. *European economic review*, *36*(5), 975–1000.

Smirlock, M. (1985). Evidence on the (non) relationship between concentration and profitability in banking. *Journal of money, credit and Banking*, *17*(1), 69–83.

Stock, J. H., & Watson, M. W. (2002). Forecasting using principal components from a large number of predictors. *Journal of the American statistical association*, *97*(460), 1167–1179.

Wong, W.-K., Khan, H., & Du, J. (2006). Do money and interest rates matter for stock prices? an econometric study of singapore and usa. *The Singapore Economic Review*, *51*(01), 31–51.

Wu, Y. (2001). Exchange rates, stock prices, and money markets: evidence from singapore. *Journal of Asian Economics*, *12*(3), 445–458.

Appendix I

Forecasted Monthly T-bill rates

Forecasted Treasury Bills Interest Rates

Horizon_Month	T-91	T-92	T-364
1	6.553835	6.860941	7.012840
2	7.600120	8.640086	9.861442
3	7.329395	8.217766	9.437530
4	7.148261	7.952965	9.219068
5	7.103522	7.863399	9.086410
6	6.887666	7.596429	9.026897
7	6.687690	7.380088	9.051400
8	6.571721	7.227165	8.810016
9	6.445543	7.178369	9.208386
10	6.424117	7.288515	9.770824
11	6.558250	7.543723	10.132989
12	6.905579	7.979023	10.209371
13	7.378867	8.491132	10.081876
14	7.440981	8.488509	9.853862
15	7.397380	8.376725	9.703284
16	7.285609	8.108184	9.077905
17	7.183605	7.924116	8.840695
18	7.125135	7.838496	8.846342
19	6.945488	7.574350	8.581983
20	6.290854	6.665012	7.620661
21	6.227941	6.630484	7.663117
22	6.389781	6.877691	7.859377
23	6.605395	7.166205	8.023028
24	6.783628	7.396224	8.179688
25	6.931576	7.572921	8.290424
26	6.936488	7.571717	8.341978
27	6.902813	7.570696	8.615627
28	6.923483	7.647845	8.948023
29	6.928542	7.693532	9.185057
30	6.940071	7.706551	9.183557
31	6.854895	7.466217	8.381471
32	6.633060	7.079607	7.631328
33	6.655227	7.083017	7.495731
34	6.772242	7.246086	7.695419
35	6.795183	7.310014	7.967116
36	6.853471	7.456363	8.459252

Figure 6: Forecasted Treasury Bill Interest Rate

Appendix II

Time Series Analysis

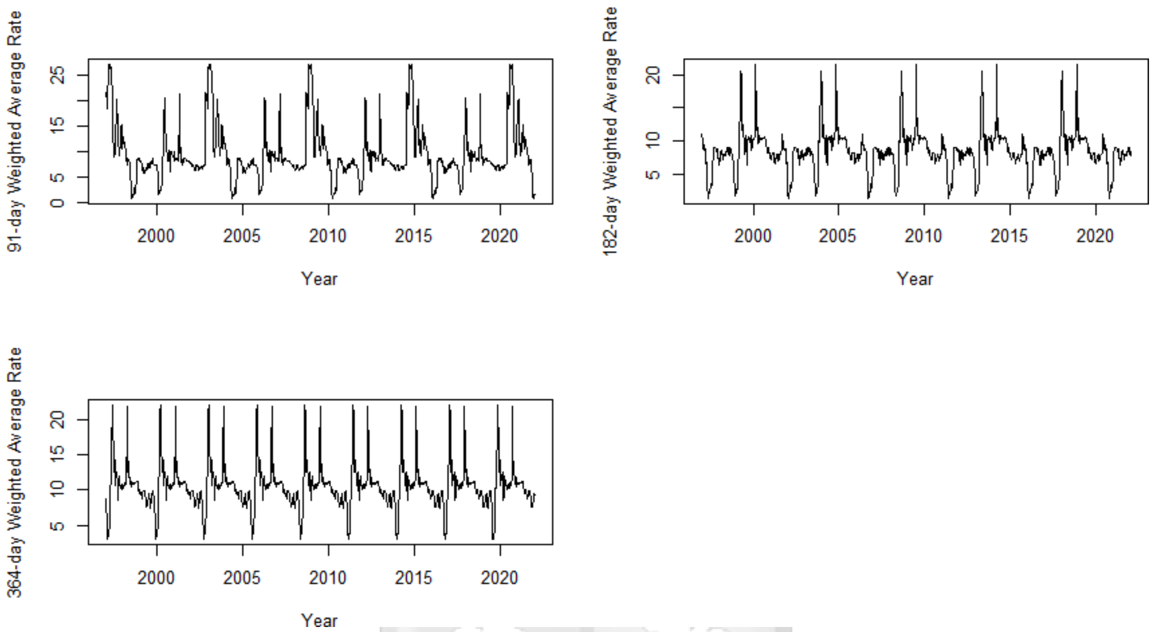

Figure 7: Time Series Analysis of Interest Rate

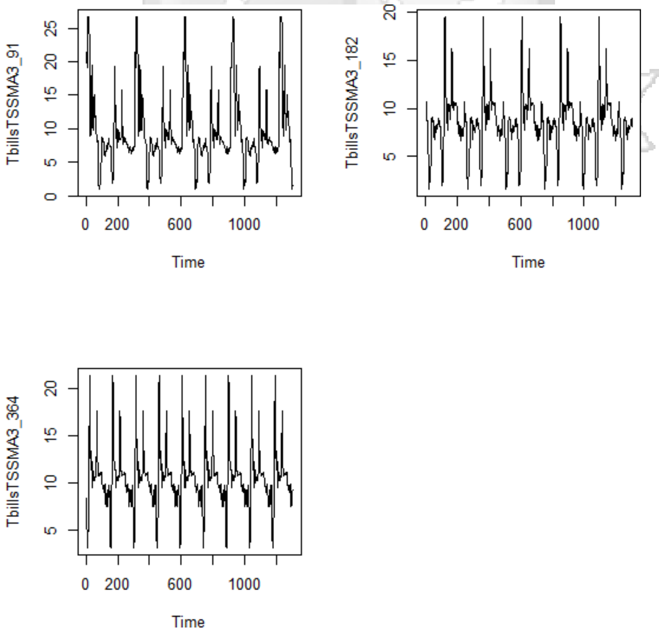

Figure 8: Decomposing Non-Seasonal Data composing of trend and irregular component

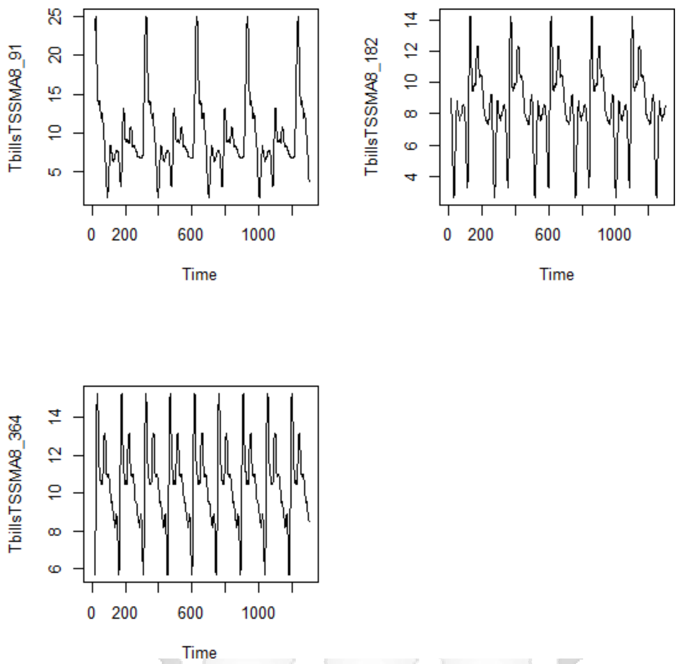

Figure 9: Smoothing of random fluctuations

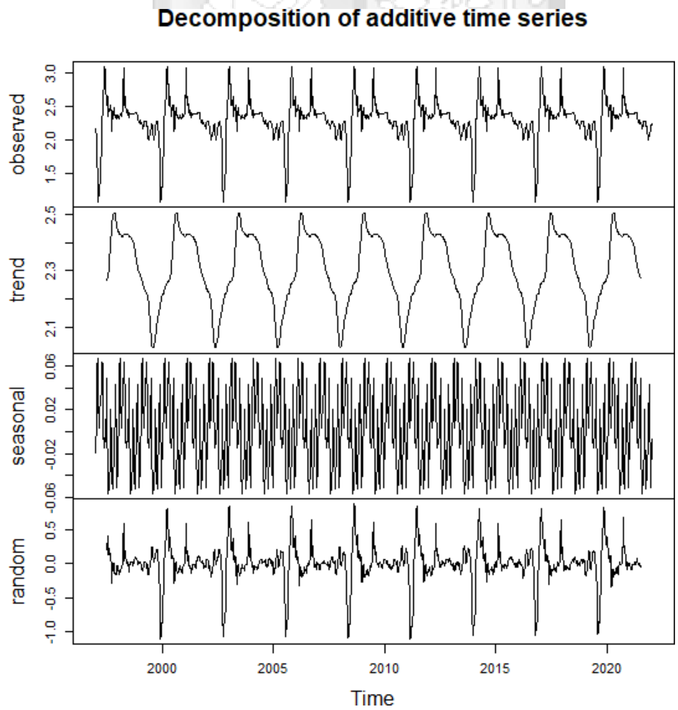

Figure 10: Decomposing of Additive Time Series

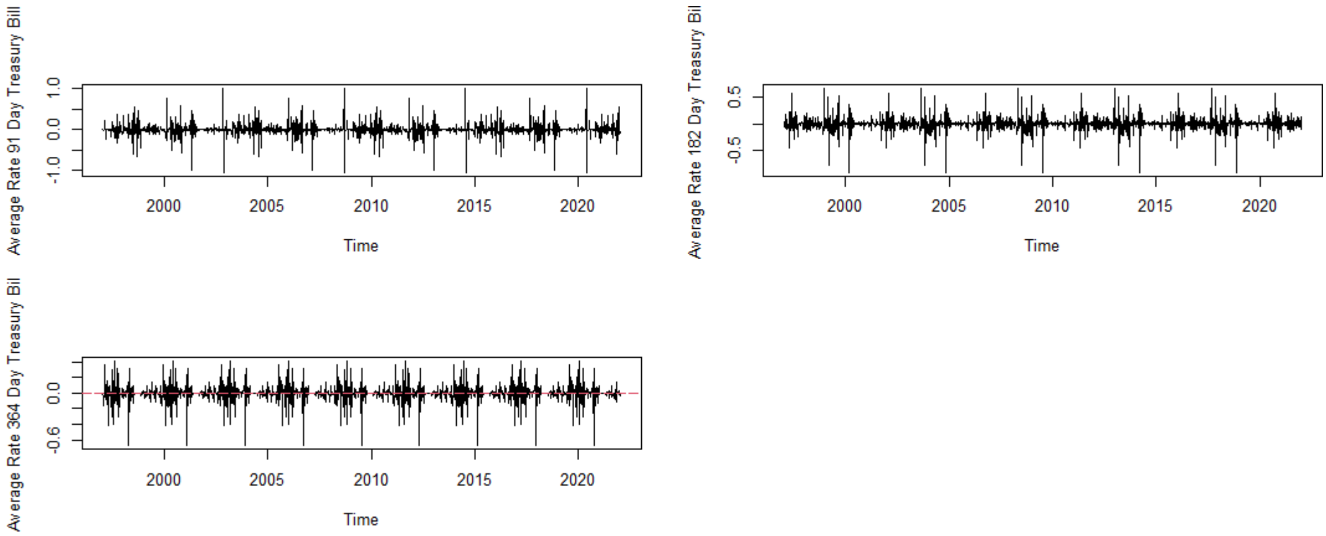

Figure 11: Differenced Yields

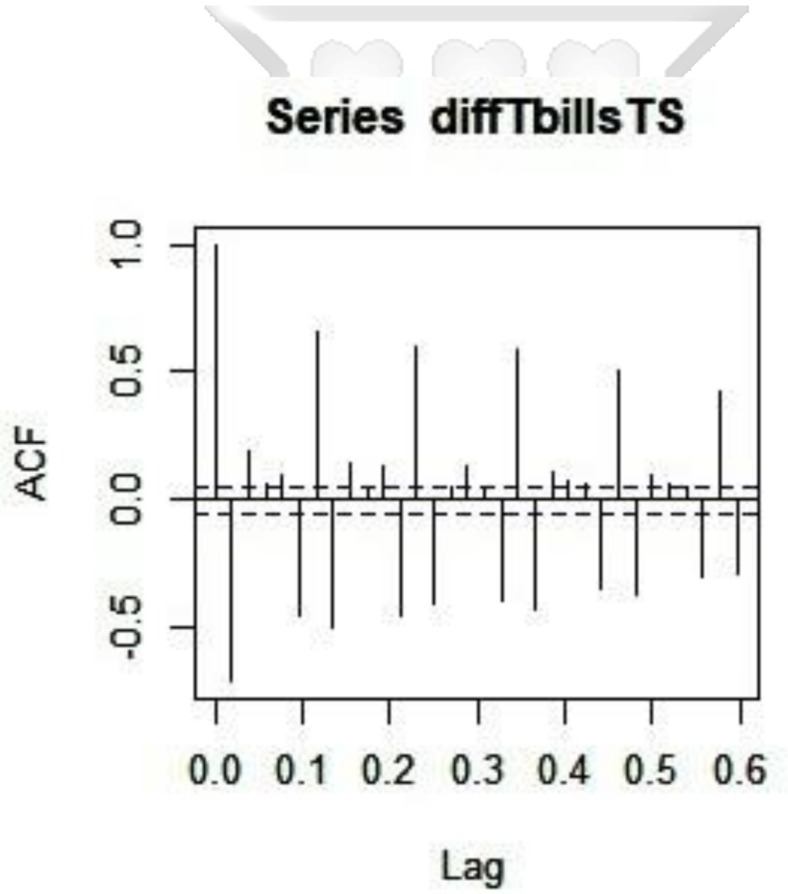

Figure 12: AR Model Fitting

Appendix III

Fitting a Dynamic Nelson Siegel Model

```
library(statespacer)

# Load the
datasetlibrary(
stats)
Tbills <- read.csv("D:/Study/Msc Statistical Science/
   Thesis/DataAnalysis/Average Rates.csv")[ ,-c(1,2)]

data <- Tbills[Tbills$Month >= "2008-01-01" & Tbills$
   Month <= "2021-01-01", ]
years <- data$Month # Used for plots later on
y <- as.matrix(data[-1]) # A matrix of the average rates by tenors

library(psych)
pairs.panels(
y)cor(y)

set.seed(2022)
# Specifying the
list self_spec_list
<- list()

# We want to specify the H matrix
ourselvesself_spec_list$H_spec <-
TRUE

# We have got 6 state parameters: 3 factors and 3
fixed meansself_spec_list$state_num <- 6
```

```
# In total we need 20 parameters:
#  1 for lambda
#  1 for sigma2 (H)
#  6 for the variance - covariance matrix Sigma_eta (Q)
#  9 for the vector autoregressive coefficient
matrix Phi#  3 for the means mu
self_spec_list$param_num <- 20
```

```
# R is a fixed diagonal
matrix self_spec_list$R <-
diag(1, 6, 6)

# P_inf is a matrix of zeroes, as all state parameters
  arestationary
self_spec_list$P_inf <- matrix(0, 6, 6)

# Needed because we want to use collapse = TRUE
# The fixed means only appear in the state equations,
# not in the observation equations. So the 4th, 5th, and 6
  th stateparameters
#     are     state     _
only    .#self_spec_list$
state_only <- 4:6

self_spec_list$sys_mat_fun <- function(param) {

  # Maturities of the interest
  ratesmaturity <- c(3,6,12)

  # The constant lambda
  lambda <- exp(2 * param
  [1])

  # The variance of the observation
  errorssigma2 <- exp(2 * param[2])
  H <- sigma2 * diag(1, 3, 3)

  # Z matrix corresponding to the
  factorslambda_maturity <- lambda *
```

```
maturity
z <- exp(-lambda_
maturity)Z <- matrix(1,
3, 3)
Z[, 2] <- (1 - z) / lambda_
maturityZ[, 3] <- Z[, 2] - z

# Variance of the state disturbances
```

```
Q <- Cholesky ( param = param [3:8] , decompositions = FALSE
  , format =matrix (1, 3, 3))

# Vector auto regressive coefficient matrix,
 enforcingstationarity
Tmat <- CoeffARMA (A = array (param [9:17], dim = c(3, 3, 1)),
                  variance = Q,
                  ar = 1, ma = 0) $ar [, ,1]

# Initial uncertainty of the
factorsT_kronecker <- kronecker (
Tmat, Tmat)
Tinv <- solve (diag (1, dim (T_kronecker)[1], dim (T_
 kronecker) [2]) -T_kronecker)
vecQ <- matrix (
Q) vecPstar <- Tinv
%*% vecQ
P_star <- matrix (vecPstar, dim (Tmat)[1], dim (Tmat) [2])

# Adding parts corresponding to the fixed means to the
 systemmatrices
Z <- cbind (Z, matrix (0 , 3 , 3)) # Not used in the
 observationequation
Q <- BlockMatrix (Q, matrix (0, 3, 3)) # Fixed, so no
 variance inits errors
a1 <- matrix (param [18:20], 6, 1) # Fixed means go into the
  initialguess
Tmat <- cbind (Tmat, diag (1, 3, 3) - Tmat)
Tmat <- rbind (Tmat, cbind (matrix (0, 3, 3), diag (1, 3, 3)))
P_star <- BlockMatrix (P_star, matrix (0, 3, 3))
```

```
  # Return the system matrices
  return(list(H = H, Z = Z, Tmat = Tmat, Q = Q, a1 = a1, P_
    star = P_star))
}

self_spec_list$transform_fun <- function(
  param) {lambda <- exp(2 * param[1])
```

```
  sigma2 <- exp (2 * param [2])
  means  <- param [ 18: 20]
  return (c(lambda, sigma2, means))
}

#Play around with this values for the 20 parameters
initial <- c(1, 2, 1, 1, 1, 0, 0, 0, 4, 0, 0, 0, 3, 0, 0, 0, 2, 0,
   0, 0)

fit <- statespacer(y = y,
                   self_spec_list = self_spec_list,
                   collapse = FALSE,
                   initial = initial,
                   method = "BFGS",
                   verbose = TRUE)
```

R Codes

Parameter values

```
# The fitted parameters
parameters <- cbind(
  c("lambda", "sigma2", "mu1", "mu2", "mu3"),
  fit $ system _ matrices $ self _spec ,
  fit$standard_errors$self_spec
)
colnames(parameters) <- c("Parameter", "Value", "Standard Error")
parameters

# Vector autoregressive coefficient matrix = T
fit$system_matrices$T$self_spec[1:3, 1:3]

# Variance of the state disturbances = Q
fit$system_matrices$Q$self_spec[1:3, 1:3]

# The fixed diagonal matrix of 1s = R
fit$system_matrices$R$full

#Z matrix corresponds to the factors, which correspond to the beta
    coefficients
fit$system_matrices$Z$self_spec[,2]

# Initial uncertainty of the factors
fit$system_matrices$P_star$self_spec[1:3, 1:3]

#Fixed means
fit$system_matrices$a1$self_spec[1:3,]
```

R Codes

Forecasting using a Dynamic Nelson-Siegel Model with a TV-VAR component

```
lambda  <-  parameters[1,2]  #H

#Z matrix corresponds to the factors, which correspond to the beta
   coefficients
fit$system_matrices$Z$self_spec[,2]



sigma2  <-  parameters[2,2]

#Fixed means
mu_x <-  fit$system_matrices$a1$self_spec[1:3,]

r  =  as.numeric(length(data[,1]))

# Filtered estimate of a state vector at time T- end of sample
X <- matrix(fit$predicted$yfit[r,],  nrow  = 3, ncol  = 1)
X

# DNS factor  loading  matrix
NS.B<-function(lambda,  tau){
  col1  <-  rep.int(1, length(tau))
  col2  <-  (1-exp(-lambda*tau))/(lambda*tau)
  col3   <-  col2  -  exp (- lambda *  tau )
  return(cbind(col1,col2,col3))
}

#Estimated  parameters
#Z matrix corresponds to the factors, which correspond to the beta
   coefficients
A  <-  diag(fit$system_matrices$Z$self_spec[,2])
A
```

```
sigma2  <-  parameters[2,2]

#Fixed means
MU<- matrix(fit$system_matrices$a1$self_spec[1:3,], ncol
    = 1, nrow =3)
MU

lambda <- round((as.numeric(parameters[1
,2])),2)lambda

Phi0  <-  (diag(3) -
A)%*%MUPhi1  <-  A

# factor loading matrix
v.mat  <- as.numeric(c(0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 4, 5, 7,
   10, 20))
nmat  <-  length(v.mat)
B     <-  NS.B(lambda, v.mat)


# Filtered estimate of a state vector at time T- end of
sampleprevX  <- as.vector(fit$predicted$yfit[r,])
prevX

# forecasting
horizonnhor  <-  12
# months

# placeholder for predicted states and yield curve
forecastsXf <- matrix(NA, nhor, 3)
yf  <- matrix(NA, nhor, nmat)
```

```
# dynamic
forecastingfor(h
in 1:nhor){
  # prediction
  Xhat <- Phi0 + Phi1 %*% prevX
```

```
  # forecast yield curve
  y_fit  <- B %*% Xhat

  # use forecasted state at h as previous state at h+1
  prevX <- Xhat

  # save
  Xf[h,] <- Xhat
  yf[h,] <- y_fit
}

yf # Yields
Xf #Factors
factor_forecast <- data.frame(Xf)
month <-colnames(factor_forecast)<-c("Month 1","Month 2","Month 3")
matplot(factor_forecast, type = "l", xlim = c(1,6), xlab = "time(
   months)")

yield_forecast <- data.frame(yf)
colnames(yield_forecast)<-c("Month 1","Month 2","Month 3","Month 4","
   Month 5","Month 6","Month 7","Month 8","Month 9","Month 10","
   Month 11","Month 12","Month 13")
matplot(yield_forecast, type = "l", xlim = c(1,6), xlab = "time(
   months)")
```

Appendix IV: Ethical Clearance Confirmation

6th June 2022

Ms Bosire, Luycer
lucyer.bosire@strathm
ore.edu

Dear Ms Bosire,

RE: Forecasting the Term Structure of Interest Rates Using Bayesian ModelsPost 2007-2008 Financial Crisis.

This is to inform you that SU-IERC has reviewed and **approved** your above **SU Masters'** research proposal. Your application reference number is **SU-IERC1370/22.** The approval period is **6th June 2022 to 5th June 2023.**

This approval is subject to compliance with the following requirements:

i. Only approved documents including (informed consents, study instruments, MTA) will be used
ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by SU-IERC.
iii. Death and life-threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to SU-IERC within 48 hours of notification
iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to SU-IERC within 48 hours
v. Clearance for export of biological specimens must be obtained from relevant institutions.
vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.
vii. Submission of an executive summary report within 90 days upon completion of the study to SU-IERC.

Prior to commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology, and Innovation (NACOSTI) https://research- portal.nacosti.go.ke/ and obtain other clearances needed.

Yours sincerely,

for:**Dr Ben Ngoye,**
Secretary; SU-IERC

Cc: Prof Fred Were,
Chairperson; SU-IERC

Ole Sangale Rd, Madaraka Estate. PO Box 59857-00200,
Nairobi, Kenya. Tel +254 (0)703 034000
Email admissions@strathmore.edu
www.strathmore.edu

Appendix V: Similarity Report

Curiginal

Document Information

Analyzed document	Bosire__Luycer_Nyanchama__124384__Thesis.pdf (D138890561)
Submitted	2022-06-01T18:46:00.0000000
Submitted by	
Submitter email	Luycer.Bosire@strathmore.edu
Similarity	9%
Analysis address	library.strath@analysis.urkund.com

Sources included in the report

SA	**Thesis_draft_URKUND.pdf** Document Thesis_draft_URKUND.pdf (D40785672)	1
SA	**FinalThesis.pdf** Document FinalThesis.pdf (D75797712)	1
W	URL: https://cran.r-project.org/web/packages/statespacer/vignettes/selfspec.html Fetched: 2021-03-28T13:23:46.5300000	2
W	URL: https://www.biostat.uzh.ch/fileadmin/biostat/user/solutions-nup.pdf Fetched: 2019 10-10T15:11:12.5570000	1
SA	**nedved_firstpart.pdf** Document nedved_firstpart.pdf (D24679725)	2
SA	**soas_report - final.pdf** Document soas_report - final.pdf (D73091821)	3
SA	**International Bond Return Predictability.pdf** Document International Bond Return Predictability.pdf (D109923646)	1
SA	**2343619_ECON5102p.docx** Document 2343619_ECON5102p.docx (D40897883)	1
SA	**Report_MF_Final_Work_v2.pdf** Document Report_MF_Final_Work_v2.pdf (D57051432)	1
W	URL: https://rdrr.io/cran/statespacer/src/inst/doc/selfspec.R Fetched: 2022 05-31T17:36:52.0200000	3