SCHOOL OF COMPUTING AND ENGINEERING SCIENCES

BACHELOR OF SCIENCE IN INFORMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBER SECURITY

END OF SEMESTER EXAMINATION

ICS 1205 / CNS 1205: LINEAR ALGEBRA

DATE: 11th March 2024 Time: 2 Hours

Instructions

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

QUESTION ONE [30 MARKS]

(a) Given that $\lambda = -1$ is an eigenvalue of the matrix $A = \begin{pmatrix} x & 2 \\ 2 & x \end{pmatrix}$, determine the possible values of x.

[3 Marks]

(b) A structure is subjected to three forces F_1, F_2 and F_3 in newtons, satisfying the simultaneous equations:

$$F_1 - F_2 + 2F_2 = 3$$
$$-2F_1 + F_2 + F_3 = -2$$
$$F_1 + F_2 - F_3 = 2$$

Use Gauss Jordan method to solve the equations.

[4 Marks]

(c) Determine for what values of λ and μ the following equations

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$

have:

i. no solution
ii. a unique solution
iii. infinite number of solutions **[6 Marks]**

(d) Is the system of vectors $X_1 = [2 \quad 2 \quad 1]^T$, $X_2 = [1 \quad 3 \quad 1]^T$, $X_3 = [1 \quad 2 \quad 2]^T$ linearly dependent?

[5 Marks]

(e) Given the matrices

$$A = \begin{pmatrix} 4 & -3 & 7 \\ -2 & -5 & 1 \\ -1 & 6 & 8 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 1 & 4 & -3 \\ 2 & 1 & 5 \\ -3 & -4 & -2 \end{pmatrix}$$

Determine:

i. AB **[2 Marks]**
ii. $N = A + B$ **[1 Mark]**
iii. N^T **[1 Mark]**

(f) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 4 & -5 \\ 1 & -2 \end{pmatrix}$.

[5 Marks]

(g) Given that $\begin{pmatrix} a+bj & b \\ aj & b-aj \end{pmatrix} = \begin{pmatrix} 6j & 2d \\ 2cj & ej^2 \end{pmatrix}$. Solve for a, b, c, d and e.

[3 Marks]

<u>QUESTION TWO [20 MARKS]</u>

(a) Given the matrix

$$A = \begin{pmatrix} 6 & 6 & -3 \\ -3 & 6 & 6 \\ 6 & -3 & 6 \end{pmatrix}$$

verify that $AA^T = \lambda I$ where I is the identity matrix and λ is a constant.
Hence solve the equations:

$$6x_1 + 6x_2 - 3x_3 = 0$$
$$-3x_1 + 6x_2 + 6x_3 = 9$$
$$6x_1 - 3x_2 + 6x_3 = 18$$

[8 Marks]

(b) In a manufacturing process, the cost of 5 tonnes of steel, 4 tonnes of copper and 1 tonne of aluminium is $Ksh.\ 34$ million; the cost of 10 tonnes of steel, 9 tonnes of copper and 4 tonnes of aluminium is $Ksh.\ 88$ million; while the cost of 10 tonnes of steel, 13 tonnes of copper and 15 tonnes of aluminium is $Ksh.\ 192$ million. Use Cramer's rule to determine the cost of one tonne of each type of metal. **[12 Marks]**

QUESTION THREE [20 MARKS]

(a) Find for what values of K the set of equations

$2x-3y+6z-5t=3$, $y-4x+t=1$, $4x-5y+8z-9t=K$ has

i. no solution
ii. infinite number of solutions

[6 Marks]

(b) Test the consistency of the following system of equations and hence find the solution.

$$4x_1 - x_2 = 12$$
$$-x_1 + 5x_2 - 2x_3 = 0$$
$$-2x_2 + 4x_3 = -8$$

[8 Marks]

(c) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ show that $A^2 - 5A - 2I = 0$, where I is the unit matrix of order 2, and hence find A^{-1}. **[6 Marks]**

QUESTION FOUR [20 MARKS]

(a) The eigenvalues of a 2×2 matrix A are $\lambda_1 = 3$ and $\lambda_2 = 2$ with corresponding eigenvectors $x_1 = [-2 \quad 1]^T$ and $x_2 = [-1 \quad 1]^T$. Determine the

i. The modal matrix M and spectral matrix Λ of A.
ii. the matrix A.
iii. A^2 **[8 Marks]**

(b) A flour-milling firm produces two types of flour; maize flour and wheat flour, which have a scale of $Ksh.\ 50$ and $Ksh.\ 80$ per kilogram respectively. The resources used to produce each type of flour and the maximum time available for each task per week is as shown in the table below.

	Hours required for each type		
	Transport	Milling	Storage
Maize	3	1	2
Wheat	2	1	1
Maximum hours per week	600	240	300

i. Formulate a linear programming (LP) model for the above problem. **[3 Marks]**

ii. Using the simplex method, determine the optimum monthly production plan which maximizes sales for the milling firm. **[9 Marks]**

QUESTION FIVE [20 MARKS]

(a) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

and test for the orthogonality of the vectors. **[10 Marks]**

(b) Verify Cayley Hamilton Theorem for the matrix

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

Also express $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ as quadratic polynomial in A. **[10 Marks]**

END