



Strathmore
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**ASSESSING THE IMPACT OF VOLATILITY ON EXPECTED STOCK
RETURN**

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**A research project submitted in partial fulfillment of the requirements for the
Degree of
Bachelor of Business Science Actuarial at Strathmore University**

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December, 2015

DECLARATION

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Project contains no material previously published or written by another person except where due reference is made in the Research Project itself.

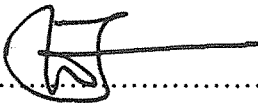
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04/12/2015

This Research Proposal has been submitted for examination with my approval as the Supervisor.

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1 INTRODUCTION

1.1 Background Information

Many researchers have studied movements in aggregate stock market volatility. Some argue that the relation between expected returns and volatility is strong. Pindyck (1984) attributed much of the decline in stock prices during the 1970's to increases in volatility. On the other hand, Porteba and Summers (1986) argued that the time-series properties of volatility make the scenario unlikely. Neither study, however, provides a direct test of the relation between expected return and volatility. The amplitude of the fluctuations in aggregate stock volatility is difficult to explain using simple models of stock valuation. (Schwert G. W., 1989).

The term "frontier markets" is widely attributed to the International Finance Corporation (IFC), which coined it in 1992 to refer to a subset of emerging markets. Standard & Poor's bought the IFC Emerging Markets Database in 2000 and subsequently established a frontier index in 2007. Frontier markets refer to equity markets in small nations that are at an earlier stage of economic and political development than larger and more mature emerging markets. In other words, think of frontier markets as the smaller siblings of emerging.

Frontier equity markets typically have modest market capitalization, limited investability and liquidity, and few market information sources. On the positive side, they generally possess favorable demographics and good long-term growth prospects. As these markets probably constitute the last frontier of investing in an increasingly interlinked global economy, investors should be aware of their risks and rewards, and the options available to invest in them. Frontiers Markets, as defined by the MSCI Frontier Markets index, comprise 26 markets spanning Asia, Eastern Europe, Africa, Latin America and the Middle East

African markets and stock markets are referred to as the final investment frontier (Mattes, 2012). Stock markets in this region are increasingly attracting foreign investors to this growing economy and prospects of development in the near future. Five of Africa's largest and most important markets include South Africa being the largest, Kenya, Nigeria, Mauritius and Nigeria which will be the main focus of the above study.

1.11 Trend in African Equity Markets

Capital markets in Africa have experienced tremendous development since the 1990's. With the exception of South Africa, most African stock markets doubled their market capitalization between 1992 and 2002. Total market capitalization for African markets increased from US\$113,423 million to US\$ 244,672 million between 1992 and 2002 (Adjasi & Yartey, 2007).

Indicators of stock market development show that African markets are small with few listed companies and low market capitalization. Adjasi and Yartey (2007) Egypt, Nigeria, South Africa and Zimbabwe are the exceptions with listed companies of 792, 207, 403 and 79 respectively. The average number of listed companies on sub-Saharan African markets excluding South Africa is 39 compared with 113, with the inclusion of Egypt and South Africa. Market capitalization as a percentage of GDP is as low as 1.4 in Uganda. The Johannesburg Securities Exchange in South Africa has about 90 percent of the combined market capitalization of the entire continent. Excluding South Africa and Zimbabwe the average market capitalization is about 27 percent of GDP. This is in contrast with other emerging markets like Malaysia with a capitalization ratio of about 161 percent.

African stock markets face problems of small size and low liquidity which affects the marketability of the stocks. Other problems facing these markets include investor restrictions, and relative unsophistication that discourage foreign investors. Mattes (2012). In most of these stock markets, trading occurs in only a few stocks which account for a considerable part of the total market capitalization. Beyond these actively traded shares, there are serious informational and disclosure deficiencies for other stocks. Further, supervision by regulatory authorities is often far from adequate. The less developed of the stock markets suffer from a far wider range of such deficits. Due to all the mentioned risks, volatility in this market is high which in most scenarios leads to lower returns.

Despite of the named problems, African stock markets continue to perform remarkably well in terms of return on investment (Adjasi & Yartey, 2007). The Ghana Stock Exchange was adjudged as the world's best-performing market at end of 2004 with a year return of 144 percent in US dollar terms compared with 30 percent return by Morgan Stanley Capital International Global

Index (Databank Group, 2004). Other five African countries —Uganda, Kenya, Egypt, Mauritius and Nigeria apart from Ghana—were amongst the best performers in the year.

Apart from the volatility of returns caused by additional risk, Mattes (2012) found persistence for all African markets in each crisis with most volatility persistence recorded during the credit crisis period which is in line with findings of Schwert G, (1990) or the United States market. This is an indication that fluctuation in volatility in the major African markets is affected by volatility spillover from crises.

1.12 The Role of Equity Markets

The role and importance of stock markets cannot be underestimated. Stock market is an indicator of an economy financial health. It indicates the mood of investor in a country (Nowbusting, 2009). In short stock markets impact the development and growth of any market depending on whether the relationship between stock market development and economic growth is long term or short term. Stock markets have contributed to the financing growth of large corporation in certain African countries (Adjasi & Yartey, 2007). Corporate financing patterns in certain African countries suggest that stock markets are important an important source of finance apart from bank loans, retained earnings and debt.

Apart from being a source of finance, a well-developed stock market should increase saving and effectively allocate capital to productive investments (Nowbusting, 2009) which leads to an increase rate of economic growth. A measure of well-developed stock markets is by size and liquidity. Size as the share of market capitalization over GDP and liquidity calculated as value of shares traded on the stock market exchange divided by the GDP of the country. Stock market development curbs the problem of liquidity risk so that investors may not suffer liquidity shocks.

Another advantage of stock market contributes to the mobilization of domestic savings by enhancing the set of financial instruments available to savers to diversify their portfolios from global risk. By doing so the stock markets provide an opportunity for investors to invest at a relatively low cost (Dailami & Atkin, 1990).

1.2 Problem statement

Major focus in modeling the relationship between stock return and volatility over the past three decades has been laid on the GARCH model which evolved from (Engle, 1982)ARCH model that allow for time varying behavior of volatility. The findings of early studies are mixed for example Pindyck (1984)and Baillie and DeGennaro (1990)found a positive but weak relationship between US stock market return and conditional variance of the returns same results with (Theodossiou & Lee, 1995). In contrast, Nelson (1991)found a negative but insignificant relationship between expected returns and the conditional variance of the US stock Market. It can be noted that empirical findings remain inconclusive, (Li, Yang, Hsiao, & Chang, 2005).

Consistency across markets of previously done work results is a gap identified keeping in mind markets are independent and studies done in international markets does not necessarily prevail in the African Frontier markets. This study aims to adopt a multivariate extension and examine stock return and volatility across frontier markets in Africa.

1.3 Research Objective

To determine the relationship between stock return and volatility in frontier equity markets in sub-Saharan Africa

1.4 Research question

What is the relationship between stock return and volatility in frontier markets in the sub-Saharan region?

1.5 Significance of the study

The findings of this study have some important implications. Based on the relationship determined, investors in the stock market require a suitable time varying risk premium that will explain the stock market behavior and whether it applies across the African region. This information address the concern of investor compensation for each unit of additional taken in the equities market

2. LITERATURE REVIEW

2.1 Introduction

This chapter outlines the theoretical and conceptual underpinnings upon which the premise of the study is built. Section 2.2 discusses and defines the main constructs of the Capital Asset Pricing Model, Consumption CAPM together with Intertemporal Capital Asset Pricing Model and a brief discussion on Arbitrage Pricing model. Section 2.3 presents the GARCH model and a brief overview of expected stock return literature.

2.2 Theoretical background: Risk return relationship basics

Sharpe (1964) and Linter (1965) were the first authors of asset pricing theory who came up the prominent capital asset pricing model used in explaining price of securities. CAPM is an equilibrium model of capital markets that try explain securities values given investor total utility in the case where there are no incentives for further trading. In short, capital market theory attempts to explain how investors place their money on securities traded in the capital markets.

The model specifies expected returns for use in capital budgeting, valuation and regulation. Risk premium on an individual security is a function of systematic risk, measured by covariance with the market. CAPM model can be used to evaluate given estimates of expected returns relative to risk. Also the Model can be used to obtain estimates of expected returns through estimates of risks based on average of past returns. Four decades later since the derivation of CAPM, the CAPM is still widely used in applications, such as estimating the cost of equity capital for firms and evaluating the performance of managed portfolios (Fama & French, 2003).

CAPM as a pricing model itself has not been an empirical success (Fama & French, 2003). Empirical work results show that the relation between average return and market beta is flat; meaning the risk premium per unit risk taken is lower than predicted by the model. In the late 1970's, other works began to uncover other variables omitted in the CAPM model. Variables such as size and various price ratios, Fama and French (1992) add explanation to the explanation of expected average returns provided by market beta. This is due to the fact that the linear relation that arises from the mean variance efficiency of the market portfolio is difficult to justify either the assumption of normality in returns or quadratic preferences to guarantee such efficiency on empirical grounds (Ross, 1976).

These findings begin to question market beta to be efficient enough to describe expected return thus raising serious problems in most applications of the CAPM (Fama & French, 2003). The CAPM model however provides fundamental concepts and a strong portfolio used to build more advanced models. Adding diversified portfolios that capture covariation in returns and variation in average returns left unexplained by the market is the spirit of ICAPM (Fama & French, 2003). The three-factor model is widely used in empirical research that requires a model of expected returns.

An intertemporal model for the capital market is deduced from the portfolio selection behavior by an arbitrary number of investors who act so to maximize the expected utility of lifetime consumption and who can trade consciously in time (Merton, 1973). ICAPM is a consumption based asset pricing model that takes into account major sources of risk in optimizing consumption. ICAPM is an equilibrium model of the capital market which is simple and is consistent with expected utility maximization and the limited liability of assets. It also provides a specification of the relationship among yields that is more consistent with empirical evidence. The model is based on consumer-investor behavior (Merton, 1973). Most investors do not participate in financial markets for one year, but over longer periods of time where investment opportunities might shift as expectations of risk change resulting in situations in which investors may wish to hedge their portfolios.

Unlike the static CAPM model, Merton (1973) showed in ICAPM model in a number of examples that portfolio behavior for an intertemporal investor will be significantly different when he faces a changing investment opportunity set instead of a constant one over multiple periods of time. Furthermore the intertemporal nature of the model allows it to capture effects which would never appear in a static model, and it is precisely this effects which cause significant differences in specification of the equilibrium relationship among asset yields that obtain in the new model and classical model.

Another model that explains the risk return relationship is the Consumption CAPM which is a financial model that extends the concepts of CAPM to include the amount that an individual or firm wishes to consume in future. The consumption CAPM uses consumption beta in its calculation of expected return (Yogo, 2006). In the simplest way, CCAPM differs from CAPM

by only the coefficient beta used in calculation. The consumption beta attempts to measure the covariance between the investor's ability to consume goods and services from investment return from a market index.

However the pricing mechanism using consumption risk alone is not conclusive (Ghosh, Julliard, & Taylor, 2010). It does not explain historically observed levels of return and also since the CCAPM measures risk by nondurable consumption beta; it fails to explain the cross-sectional dispersion of returns between different classes of financial assets (Shapiro & Mankiw, 1986).

Risk-return discussions cannot be complete without the discussion of Arbitrage theory of capital asset pricing theory (APT) which holds that the expected return of an asset can be modeled as a linear function of various macroeconomic factors where sensitivity to changes in each factor is represented by factor specific beta (Ross, 1976). The APT model is an alternative theory of pricing risky assets that retains intuitive results.

The argument can be expressed as follow by a simple factor model

Equation 1

$$x_i = E_i + \beta_i \delta + \epsilon_i$$

Where δ is a mean zero common factor and ϵ_i is mean zero that permits the law of large number to hold. Under the APT model, an asset is mispriced if its current price diverges from the price predicted by the model. A correctly priced asset here may be in fact a synthetic asset- a portfolio consisting of correctly priced n assets i.e. uses no wealth and is well-diversified. Thus the influence of the well-diversified portfolio is that it has no systematic risk. (Noise is negligible). The arbitrageur creates the portfolio by identifying x correctly priced assets and then weighting the assets such that the portfolio beta factor is the same for the mispriced asset (Ross, 1976)

2.3 Empirical Literature

Since Markowitz (1952) settled on the idea that investors would demand higher returns on a market portfolio than a risk-free investment, the relationship between risk and return has been subjected to extensive theoretical and empirical enquiry (Mandika, 2010). Different authors explain the different ways in which key variables relate to the expected return on a stock. Fama and French (1992) tried to find out whether two variables, Size and book to market equity can capture the cross-sectional variation in the average stock return using size, book to market equity and earnings-price ratios. For their study, they had strong arguments for cross sectional of expected stock returns,

Firstly, The Sharpe-Linter-Black (SLB) asset pricing model has a central prediction that market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market implies that (a) expected returns on securities are positive linear function of their market β s and (b) market β s suffice to describe the cross-section of expected returns. However there are contradictions of the SLB model. The most prominent is the size effect of Banz (1981). He finds out that market equity,(ME), adds to the explanation of the cross-sectional of average returns. Average returns on small (low ME) stocks are too high given their β estimates, and the average returns on stocks are too low.

Another contribution of the SLB model is the positive relation between leverage and the average Bhandari (1988). Leverage is associated with risk and expected return, but in the SLB model, leverage risk is captured by market β . Bhandari finds , however, that leverage helps explain the cross-section of average stock returns in the tests that include size (ME) as well as β .

Another line of thought finds that average returns on U.S. stocks are positively related to the ratio of a firm's book value of common equity, BE, to its market value, ME. Stattman (1980), Roseign Reid and Lanstein (1985). Chan, Hamao, and Lakonishok (1991) find that book-to-market equity, BE/ME, also has a strong role in explaining the cross-section of average returns on Japanese stocks.

It is however reasonable to expect that some of the line of thoughts are redundant in describing average returns. The long term goal is to provide an economic foundation for the empirical relations between average stock return and size, and average return and book-to-market-equity (Fama & French, 1995). Unlike the simple relation between β and average return, the univariate

relations between average return and size, leverage, E/P, and book-to-market equity are strong. In multivariate tests, the negative relation between size and average return is robust to the inclusion of other variables.

The positive relation between book-to-market equity and average return also persists in competition with other variables. Moreover, although the size effect has attracted more attention, book-to-market equity has a consistently stronger role in average returns. The bottom-line results are: (a) β does not seem to help explain the cross-section of average stock returns and the combination of size and book-to-market equity seems to absorb the roles of leverage and E/P in average stock returns, at least during our 1963-1990 sample periods.

If assets are priced rationally, our results suggest that stock risks are multidimensional. One dimension of risk is proxied by size, ME. Another dimension of risk is proxied by BE/ME, the ratio of the book value of common equity to its market value. It is possible that the risk captured by BE/ME is the relative distress factor of Chan and Chen (1991). They postulate that the earning prospects of firms are associated with a risk factor in returns. Firms that the market judges to have poor prospects, signaled here by low stock prices and high ratios of book-to-market equity, have higher expected stock returns (they are penalized with higher costs of capital) than firms with strong prospect. It is also possible, however, that BE/ME just captures the unraveling (regression toward the mean) of irrational market whims about the prospects of firms.

The close links between the leverage and book-to-market results suggest that there are two equivalent ways to interpret the book-to-market effect in average returns. A high ratio of book equity to market equity (a low stock price relative to book value) says that the market judges the prospects of a firm to be poor relative to firms with low BE/ME. Thus BE/ME may capture the relative-distress effect postulated by Chan and Chen (1991). A high book-to-market ratio also says that a firm's market leverage is high relative to its book leverage; the firm has a large amount of market-imposed leverage because the market judges that its prospects are poor and discounts its stock price relative to book value. In short, our tests suggest that the relative distress effect, captured by BE/ME, can also be interpreted as an involuntary leverage effect, which is captured by the difference between A/ME and/BE.

The results to here are easily summarized:

- (1) When we allow for variation in β that is unrelated to size, there is no reliable relation between β and average return.)
- (2) The opposite roles of market leverage and book leverage in average returns are captured well by book-to-market equity.
- (3) The relation between E/P and average return seems to be absorbed by the combination of size and book-to-market equity.

In a nutshell, market β seems to have no role in explaining the average returns on NYSE, AMEX, and NASDAQ stocks for 1963-1990, while size and book-to-market equity capture the cross-sectional variation in average stock returns that is related to leverage and E/P.

(Baille & DeGennaro, 1990) Explored empirical evidence to evaluate the relationship between stock returns and its volatility. The author applied GARCH in mean models to examine this relationship. Through the use of daily and monthly portfolio returns, it was concluded that the relationship is weak and suggest that investors consider some other risk measure to be superior to variance.

Various authors like Sharpe (1964) and Black and Scholes in asset pricing models have related the return on assets to its variance or its covariance. Other empirical studies attributed much of the decline in stock prices in 1970's with increase in volatility, while other studies Kraus and Litzenberger (1977) have questioned the use of variance as a measure of risk.

The paper used a relatively new approach to assess the empirical evidence for a relationship between returns on a portfolio of equities and the standard deviation. The assumption made is that the market returns generating process is a regression model with moving average disturbances and innovations that have either conditional normal distribution with time dependent variances. The equation of the conditional variance is assumed to follow a generalized ARCH (autoregressive conditional heteroskedasticity) model with included explanatory variables that directly estimates the effect of these variables on volatility.

The GARCH in mean model with a conditional student t density is found to provide a good description of daily and monthly returns data. Controlling for excess kurtosis by use of the student t density is found to be important, and the estimated models show very little evidence for a statistically significant relationship between a stock portfolio's return and its own volatility. Our results suggest that traditional two-parameter models relating portfolio means to variances are inappropriate and indicate the need for research into other measures of risk.

Evidence was found that the expected market premium (the expected return on a stock minus Treasury bill yield) is positively related to the predictable volatility of stock returns. There is also evidence that unexpected stock market returns are negatively related to the unexpected change in the variance of stock returns. This negative relation provides indirect evidence of a positive relation between expected risk premium and volatility.

Equation 2

$$E(R_{mt} - R_{ft} - \sigma_{mt}) = \alpha + \beta \sigma^p_{mt}, \quad p = 1, 2,$$

R_{mt} is the return on a stock market portfolio, R_{ft} is the risk-free interest rate, σ_{mt} is an ex ante measure of the portfolio's standard deviation. If $\beta=0$ in (1), the expected risk premium is unrelated to the ex-ante volatility. If $\alpha=0$ and $\beta>0$, the expected return is proportional to the standard deviation ($p=1$) or variance ($p=2$) of stock market returns.

According to Merton (1980), the relation between the market risk premium and volatility with a model similar to (1). Merton uses contemporaneous, rather than ex ante, measures of volatility, so his measures include both ex ante volatility and unexpected change in volatility. The argument is that a positive relation between unexpected return and ex ante volatility will induce a negative relation between the excess holding period return and unexpected change in volatility. Therefore combining the two components of volatility obscures the ex ante relation.

When market volatility is stochastic, intertemporal models predict that asset risk premium is not only determined by covariation of returns with the market return, but also covariation with the state variables that govern market volatility and also determine expected returns of individual assets or portfolio of assets. Merton (1973)

To study this prediction, Adrian and Rosenberg (2006) modeled the log-volatility of the market portfolio as the sum of a short- and a long-run volatility component. This approach

parsimoniously captures shocks to systematic risk at different horizons. Findings show that prices of risk are negative and significant for both volatility components implies that investors pay for insurance against increases in volatility, even if those increases have little persistence. The short-run component captures market skewness risk, which we interpret as a measure of the tightness of financial constraints. The long-run component relates closely to business cycle risk.

Being a significant cross sectional asset pricing factor, market volatility should be modeled with minimal errors. Ang et al (2006) showed their two factor model with the market return and market volatility but does not reduce pricing errors compared to the capital asset pricing model (CAPM), though not by as much as the Fama-French model. In contrast, the benchmark asset pricing model with the market return and two volatility components as cross sectional pricing factors achieves lower pricing error than the Fama and French (1993) model for size and book-to-market sorted portfolios. The findings were that the short-run and long-run volatility components are negative, highly significant prices of risk is robust across sets of portfolios, sub-periods and volatility model specifications.

To interpret the economics of short- and long-run volatility as pricing factors, (Adrian & Rosenberg, 2006) related these two factors to a measure of the tightness of financial constraints and to the business cycle. The skewness of market returns as an indicator of the tightness of financial constraints was used since return skewness arises endogenously in pricing theories with financial constraints (Hong and Stein (2003), Yuan (2005)). Intuitively, shocks to market skewness are particularly costly when financial constraints of investors are binding. Industrial production growth is our proxy for the business cycle; we use this measure because market volatility moves with the business cycle (Schwert G. W., 1989).

During empirical analysis, it was found out that the risk premium of the short-run component correlates highly with the risk premium of market skewness, while the risk premium of the long-run component correlates highly with the risk premium of industrial production growth. Furthermore, market skewness is a significant pricing factor in the cross section of size and

book-to-market sorted portfolios; however, including the short-run volatility component as additional factor makes skewness insignificant. The significance of industrial production innovations are eliminated by the inclusion of the long-run component as a pricing factor.

The pricing results deepened the understanding of the relationship between risk and return. An extensive literature shows that the time series relation between market risk and return is ambiguous. French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Wu (2001)). The cross sectional pricing approach allows one to distinguish between the asset pricing effects of shocks to volatility and the static risk-return tradeoff. Volatility risk premium compensate investors for the risk that volatility might increase in the future. The finding that prices of risk are negative and significant for both volatility components implies that investors are willing to pay for insurance against increases in volatility risk, even if those increases have little persistence. (Adrian & Rosenberg, 2006)

In modeling market risk, it was important incorporate these features and specify the dynamics of the market return in excess of the risk-free rate R^M and its conditional volatility $\sqrt{v_t}$ as:

Equation 3(a)

$$\text{Market return: } R_{t+1}^M = \mu_t M + \sqrt{v_t} \varepsilon_{t+1}$$

Equation 3(b)

$$\text{Market volatility: } \ln \sqrt{v_t} = s_t + l_t$$

Equation 3(c)

$$\text{Short-run component: } s_{t+1} = \theta_4 s_t + \theta_5 \varepsilon_{t+1} + 1 + \theta (\varepsilon_{t+1} - \sqrt{2}/\pi)$$

Equation 3(d)

$$\text{Long-run-component: } l_{t+1} = \theta_7 + \theta_8 l_t + \theta_9 \varepsilon_{t+1} + \theta_{10} (\varepsilon_{t+1} - \sqrt{2}/\pi)$$

In equation (3a), ε_t is a normal i.i.d. error term with zero expectation and unit variance, and μ_t^M is the one-period expected excess return. The log-volatility in equation (3b) is the sum of two components s_t and l_t . Each component is an AR(1) processes with its own rate of mean reversion. Without loss of generality, let l_t be the slowly mean-reverting, long-run component

and s_t be the quickly mean-reverting, short-run component ($\theta_4 < \theta_8$). The normalization of the unconditional mean of s_t let it to be zero.

The terms in equations (3c) and (3d) are the shocks to the volatility components. Their expected values are equal to zero, given the normality of ϵ . For these error terms, equal sized positive or negative innovations result in the same volatility change, although the magnitude can be different for the short- and long-run components (θ_6 and θ_{10}). We also allow for an asymmetric effect of returns on volatility by including the market innovation in equations (3c) and (3d) with corresponding coefficients θ_5 and θ_9 .

Empirical results demonstrate that shocks to systematic volatility are more important determinants of equity returns than has been previously shown. It was found out that prices of risk are negative and significant for both volatility components indicating that investors require compensation in order to hold assets that depreciate when volatility rises, even if the volatility shocks have little persistence. This analysis links the short-run volatility component to market skewness risk, which interprets it as a measure of the tightness of financial constraints. The long-run volatility component relates closely to business cycle risk. The three factor pricing model with the two volatility components and the market return as pricing factors compares favorably to benchmark models in explaining the cross section of equity return.

Due to earlier authors' failure to uncover a positive risk-return relation in the stock market across time, investigation of the aggregate volatility arises to clarify whether omitted variables could have resulted to this problem (Guo & Savickas, 2006). This is due to three main reasons. First, it is priced because many investors hold poorly diversified portfolios. Idiosyncratic risk has little or no correlation to market risk and can substantially be mitigated or eliminated using adequate diversification. Second, it is a proxy for the divergence of opinion, which could lead to stock being overvalued initially and subsequently suffer capital losses if short sales constraints are binding (Miller, 1977). Third, it tracks conditional variance of the risk factors of multifactor of intertemporal capital asset pricing model (ICAPM) model omitted by CAPM Lehmann (1990).

Early literature was used to construct the proxies for idiosyncratic volatility and stock market volatility. Similar to Campbell, Lettau, Malkiel, and Xu (2001) and Goyal and Santa-Clara (2003), we define realized value-weighted idiosyncratic volatility (IV) as

Equation 4

$$IV_t = \sum_{i=1}^{N_t} \omega_{it} \left[\sum_{d=1}^{D_{it}} \eta_{id}^2 + 2 \sum_{d=1}^{D_{it}} \eta_{id} \eta_{id} - 1 \right]$$

Where N_t is the number of stocks in quarter t , D_{it} is the number of trading days in a quarter t , ω_{it-1} is the market capitalization of stock I at the end of quarter $t-1$, and η_{id} is the idiosyncratic shock to the excess of the return on stock I in the day d of quarter t . Following Merton (1973) and Andersen, Bollerslev, Diebold and Labys (2003), stock market volatility is defined as

Equation 5

$$MV_t = \sum_{d=1}^{D_t} (E_{md} - emd)^2$$

where E_{md} is the excess stock return in day d of quarter t and emd is its quarterly average. However, the volatility measure in (2) is potentially downward-biased because of two reasons. First the quarterly average of daily returns is a noisy measure of the expected daily return. Second, it does not account for the positive correlation in the daily return.

In this article, there is evidence that value-weighted idiosyncratic stock volatility is a strong predictor of excess stock market returns when combined with stock market volatility. Contrary to the nondiversification hypothesis, a high level of IV is usually associated with low expected future stock returns. Moreover, its forecasting abilities are very similar to those of the consumption-wealth ratio and some standard measures of aggregate stock market liquidity. Overall, the results indicate that IV is a pervasive macro variable that captures systematic movements in stock returns; in particular, it might be a proxy for volatility of a risk factor of a multifactor or ICAPM model omitted from the CAPM.

The results also shed light on the out-of-sample stock return predictability, for which recent authors e.g. Goyal and Welch (2003) found little support using conventional forecasting variables. The difference is explained by the fact that our forecasting variables drive out most variables used by the earlier authors, including dividend yield, term premium, and default premium choices. The analysis in this article can be extended in several directions. First, the results appear to be consistent with two alternative explanations that IV is a proxy for liquidity risk and that IV is a proxy for the dispersion of opinion. However, they do not distinguish between these two hypotheses, because liquidity and the dispersion of opinion are two closely

related empirical concepts. Use the underlying economic theories can help to develop more powerful tests to distinguish the two hypotheses

3. METHODOLOGY

3.1 Introduction

The methodology will adopt the ARCH models by (Engle, 1982) and (Bollerslev, 1986). The time-varying pattern of stock market volatility has been widely recognized and modeled as a conditional variance in the parametric GARCH framework. (Li, Yang, Hsiao, & Chang, 2005). This chapter consists of the research design, the sampling design, data, model specification and conclusion sections.

3.2 Research Design

This study is explanatory in nature as it seeks to assess the extent to which variation in expected return attributed to volatility factors. The explanatory design was also selected since based on previous studies, volatility in the stock market contribute to variation of stock return. Based on research findings, a working hypothesis can be formulated on the dynamic relationship between volatility and expected returns in African frontier markets. The study is also quantitative in nature.

3.3 Sampling Design

This study analyses the relationship between volatility and expected return using monthly data for the period (January 2008 to December 2014). The Frontier Markets include will Kenya, Mauritius, Morocco, Nigeria and Tunisia. The MSCI Frontier Markets Africa Index was launched on December 18, 2007. Data prior to the launch data is back-tested (i.e. calculations of how the index might have performed over that time period had the index existed). There are frequently material differences between back-tested performance and actual results. Past performance- whether actual or back tested- is no indication or guarantee of future performance.

3.4 Data

The research will extract end-of-month and end-of-week index values. Which we use to compute weekly returns respectively. We compute returns as;

Equation 6

$$\ln(pt) - \ln(pt - 1)$$

Where P_t is the index value at time t

We use monthly returns because monthly returns are less noisy and appear to have better ability to reveal volatility clustering; similarly, monthly frequency is the most popular in the stock volatility literature (Poon and Taylor, 1992; Choudhry, 1996; Kim et al., 2004; Tah, 2013). Nonetheless, we also analyze weekly returns to check the robustness of our findings from monthly returns. Since the study will focus in Africa Frontier Markets, We use the MSCI Frontier Markets Africa Index since it captures large and mid-cap representation across 5 frontier markets (FM) Africa countries. With 35 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country.

For the individual indices for each of the frontier markets, the sources for data include the NSE all share index for Kenya, NSE all share index for Nigeria, SEMDEX index for Mauritius.

3.5 Model specification

The study adopts the GARCH-type models (GARCH and EGARCH) which are the most prevalent among empirical studies on stock market volatility. The GARCH models are relevant since they appropriately capture the features of volatility that has been discussed in literature. Another feature that gives motivation of the GARCH model is known as ‘volatility clustering’ which describes the tendency of large changes in asset prices to follow large changes and small changes to follow small changes. Thus for this phenomenon to be parametrised, the GARCH models are applied

3.5.1 Stationarity tests

Empirical work based on time series data assumes that the underlying time series is stationary. The study will therefore run two tests of stationarity for confirmation of stationarity. Two tests, Augmented Dickey-Fuller and Phillips-Pearson tests should be able to show whether the series are stationary in levels and the results shall be shown in a table during analysis.

3.5.2 GARCH representation

The dynamic form of the GARCH that this study adopts is given by:

Equation 7

$$RT = \mu + \delta \ln(\sigma_t | t - 1) + ut$$

R_t is the rate of return at time t , with a constant mean of μ and conditional variance ($\sigma_t^2|t-1$) with a coefficient δ . The white noise process, $\varepsilon_t = \varepsilon_t|t-1$ is the random component of return such that $\varepsilon_t|t-1$ which is positive. The GARCH model allows the conditional variance to be dependent upon previous own lags, so that the conditional variance equation in the simplest case is now

Equation 8

$$\sigma_t^2|t-1 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2|t-2$$

This is a GARCH (1, 1) model, σ_t^2 is known as the conditional variance since it is a one-period estimate for variance calculated based on past information thought relevant. Using the GARCH model, it is possible to interpret the current fitted variance, h_t , as a weighted function of a long term average value dependent on the information about volatility during the previous period (αu_{t-1}^2) and the fitted variance from the model during the previous period ($\beta \sigma_{t-1}^2$). Note that the GARCH model can be expressed in a form that shows that it is effectively ARMA model for conditional variance. To see this, consider that the squared return at time t relative to the conditional variance is given by

Equation 9

$$\varepsilon_t = u_t^2 - \sigma_t^2$$

Or

Equation 10

$$\sigma_t^2 = u_t^2 - \varepsilon_t$$

Using the latter expression to substitute in for the conditional variance we get;

Equation 11

$$u_t^2 - \varepsilon_t = \alpha + \alpha u_{t-1}^2 + \beta (u_{t-1}^2 - \varepsilon_{t-1})$$

In general, GARCH (1, 1) model will effectively capture the volatility clustering in the data, and rarely is any higher order model estimated in academic finance literature.

The conditional variance is changing, but the unconditional variance of u_t is constant and given by

Equation 12

$$\text{Var}(u_t) = \frac{\alpha_0}{1 - (\alpha_1 + \beta)}$$

As long as $\alpha_1 + \beta < 1$. For $\alpha_1 + \beta \geq 1$, the unconditional variance of u_t is not defined, and this would be termed 'non-stationarity in variance'. $\alpha_1 + \beta = 1$ would be known as a 'unit root in variance', also termed 'Integrated GARCH' or IGARCH. Non-stationarity in variance does not have a strong theoretical motivation for its existence, as would be the case for non-stationarity in the mean (e.g. of a price series). Furthermore, a GARCH model whose coefficients imply non-stationarity in variance would have some highly undesirable properties. One illustration of these relates to the forecasts of variance made from such models. For stationary GARCH models, conditional variance forecasts converge upon the long-term average value of the variance as the prediction horizon increases. For IGARCH processes, this convergence will not happen, while for $\alpha_1 + \beta > 1$, the conditional variance forecast will tend to infinity as the forecast horizon increases!

3.5.3 Estimating a GARCH model

(1) Specify the appropriate equations for the mean and the variance – e.g. an

AR(1)-GARCH(1, 1) model

Equation 13

$$y_t = \mu + \phi y_{t-1} + u_t, u_t \sim N(0, \sigma_t^2)$$

Equation 14

$$\sigma_t^2 = \alpha + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2$$

(2) Specify the log-likelihood function (*LLF*) to maximize under a normality assumption for the disturbances

Equation 15

$$L = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^T (y_t - \mu - \phi y_{t-1})^2 / \sigma_t^2$$

(3) The computer will maximize the function and generate parameter values that maximize the *LLF* and will construct their standard errors.

One of the primary restrictions of GARCH models is that they enforce a symmetric response of volatility to positive and negative shocks. This arises since the conditional variance in the above equations is a function of the magnitudes of the lagged residuals and not their signs since the lagged error term are squared and the sign is lost. In the case of equity returns, such asymmetries are typically attributed to leverage effects, whereby a fall in the value of the firm's stock causes the firm's debt to equity ratio to rise. This leads to shareholders, who bear the residual risk of the firm to perceive their future cash flow stream as being relatively more risky. To curb this problem, exponential GARCH (EGARCH) was proposed.

3.5.4 The EGARCH model

The exponential GARCH model was proposed by (Nelson D. B., 1991). There are various ways to express the conditional variance equation, but one possible specification is given by

Equation 16

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \theta_t - 1\sqrt{\sigma_{t-1}^2} + \alpha |\theta_t - 1\sqrt{\sigma_{t-1}^2}|$$

The model has several advantages over the pure GARCH specification. First, since the $\log(\sigma_t^2)$ is modeled, then even if the parameters are negative, σ_t^2 will be positive. There is thus no need to artificially impose non-negativity constraints on the model parameters. Second, asymmetries are allowed for under the EGARCH formulation, since if the relationship between volatility and returns is negative, γ , will be negative. In this study, both GARCH and EGARCH models shall be applied to allow for comparison with previous literature in the above models.

3.6 Conclusion

The aim of the analysis using the GARCH type models is to investigate the dynamic of stock returns and volatility in the Frontier Equity Markets in Africa. The model will examine whether the shocks to equity returns are driven more by past conditional variance or by new disturbances. The model will also determine whether the results are consistent with the inference that investors require larger risk premium on equities if they anticipate greater price volatility in future.

4. DATA ANALYSIS

4.1 Introduction

This chapter presents the analysis of the data findings on the impact of volatility on expected return on three stock markets in sub-Saharan Africa. This study was conducted on selected frontier markets that were identified by the MSCI global index in 2008.

The study used a return based approach on the identified indices instead of the price based approach because of two reasons. Firstly a return is complete and scale-free summary of the investment opportunity. Second, returns are easier to handle than prices because returns have more attractive statistical properties. The selected indices are Nairobi Securities Exchange All Share index for Kenya, Nigeria Stock Exchange All Share index for Nigeria and lastly Mauritius Stock exchange.

4.1 Pre-estimation tests and findings

The frequency of the data for analysis is weekly that gives a more robust picture of the volatility patterns. The weekly returns are calculated in equation 7. Descriptive analysis of the weekly return series are given in table 4.1. RT1 represents the Kenyan market weekly returns, RT2 represents the Nigerian scenario and RT3 represents Mauritius. As observed, the mean return is positive across the three markets. The maximum and minimum returns are not equal in magnitude.

The returns don't exhibit a normal distribution pattern; rather they are positively skewed with a kurtosis above 5. Following a Jarque-Berra statistic, the corresponding p-value confirms the rejection of the null hypothesis of a normal distribution pattern of the weekly returns.

Table 4.1: Descriptive Statistics

	RT1	RT2	RT3
Mean	0.001245	-0.00150	0.000143
Median	0.002020	4.16E-05	-0.00018
Maximum	0.153113	0.156150	0.079098
Minimum	-0.10449	-0.14238	-0.15723
Std. Dev.	0.024329	0.035951	0.020848
Skewness	0.021759	-0.17071	-1.05736
Kurtosis	9.156077	6.276070	14.21175
Jarque-Bera	615.8617	176.2995	2115.350
Probability	0.000000	0.000000	0.000000
Sum	0.485569	-0.58294	0.055622
Sum Sq. Dev.	0.230241	0.502774	0.169074
Observations	390	390	390

From figure 4.1, 4.2 and 4.3 below, the returns exhibits no strong signs of correlation. One of the assumptions of choosing the model was a null hypothesis that there is no serial correlation of the residual or error term.

Figure.4.1: Weekly NSEASI returns from January 2008 to June 2015

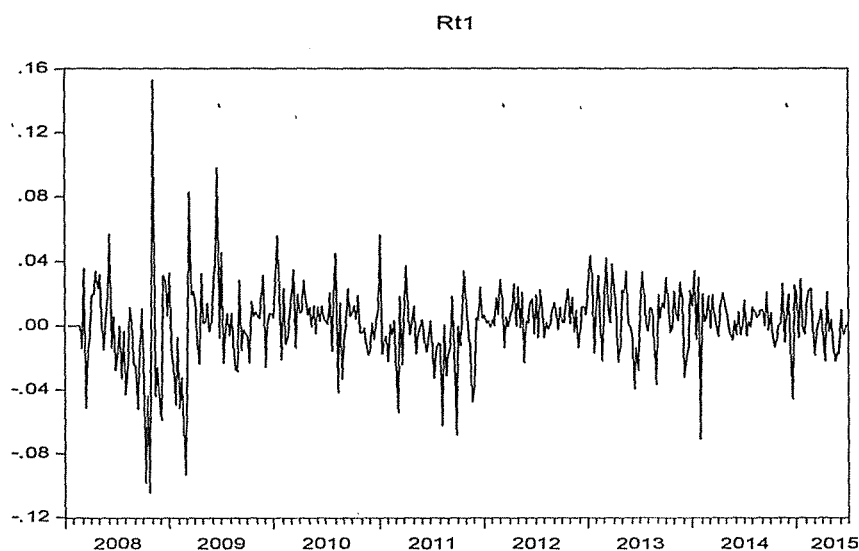


Figure 4.2: Weekly NGSEINDX returns from January 2008 to June 2015

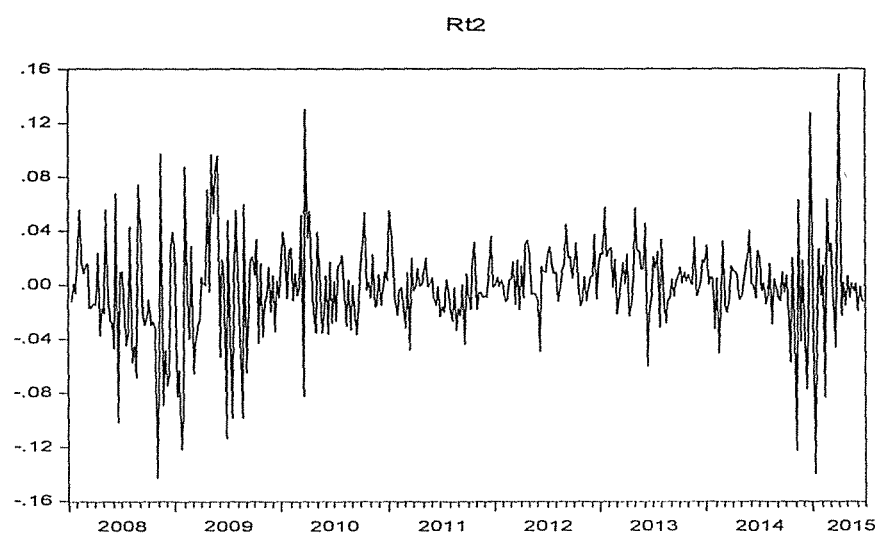
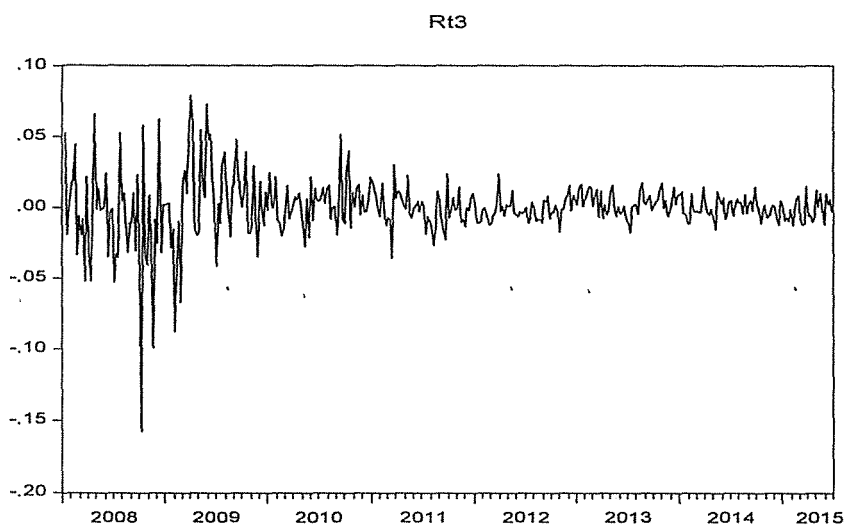


Figure 4.3: Weekly SEMDEX returns from January 2008 to June 2015



Empirical work based on time series data assume that the underlying time series is stationary meaning the mean and autocovariances of the series do not depend on time. The Augmented Dickey-Fuller unit root test for stationarity is preferred for this study. For robustness check, the Phillips-Peron test is also carried out. Table 4.2 shows the results of the ADF tests for Kenya, Nigeria and Mauritius respectively: The null hypothesis of the presence of a unit root was rejected at 1% level. The Phillips-Peron test presented similar results.

		Kenya		Nigeria		Mauritius	
		t-		t-		t-	
		Statistic	Prob.*	Statistic	Prob.*	Statistic	Prob.*
ADF	Test			-		-	
statistic		-16.1118	0.00	18.6701	0.00	17.2345	0.00
Test	critical			-		-	
values:	1%	-3.44695		3.44695		3.44695	
	5%	-2.86875		2.86875		2.86875	
	10%	-2.57068		2.57068		2.57068	

4.3 GARCH estimation

The mean and conditional variance equations under the GARCH-M model are those given in Equation 7 and Equation 8 respectively. Table 4.3 provides the estimates for the model's parameters. A positive volatility coefficient, δ , indicates negative and insignificant relationship (at a 5% level) between conditional variance and return for all the three markets.

$$RT = \mu + \delta \ln(\sigma^2_{t|t-1}) + u_t$$

$$\sigma_{t|t-1} = \omega + \alpha u^2_{t-1} + \beta \sigma_{t-1|t-2}$$

Table 4.3: GARCH (1, 1)–M Results

Parameter	Kenya		Nigeria		Mauritius	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
μ	-0.024853	0.061	-0.032944	0.027	0.010932	0.201
δ	-0.003618	0.026	-0.004708	0.016	0.001132	0.212
ω	0.000099	0.000	0.000054	0.001	0.000001	0.230
α	0.696565	0.000	0.261181	0.000	0.053119	0.000
β	0.308545	0.000	0.720835	0.000	0.937257	0.000
$\alpha + \beta$	1.01		0.98		0.99	
Log-L	956.96		817.35		1106.16	

Table 4.3 also shows the ARCH and GARCH coefficients of the conditional variance equation α and β respectively are both highly statistically significant. The observation for Kenya and Mauritius is that α is relatively larger which means that the conditional variance is driven by new disturbances. As for Nigeria, this is the opposite where β is larger indicating that the conditional variance is driven by past conditional variance. The sum of these two coefficients is close to 1 which implies that the disturbances to the conditional variance are highly persistent in all the three markets observed. The p-values of the results for Mauritius are above 5% implying that the

coefficient of the independent variables (mean and conditional variance) do not entirely affect the dependent variable (return on the index)

The mean and conditional variances for the three markets with substituted coefficients are in the equations below for Kenya, Nigeria and Mauritius in that order.

Equation 16

$$RT = -0.024853 - 0.003618 \ln(\sigma^2_{t|t-1})$$

Equation 17

$$\sigma^2_{t|t-1} = 0.000099 + 0.696565 u^2_{t-1} + 0.308545 \sigma^2_{t-1|t-2}$$

Equation 18

$$RT = -0.032944 - 0.004708 \ln(\sigma^2_{t|t-1})$$

Equation 19

$$\sigma^2_{t|t-1} = 0.000054 + 0.261181 u^2_{t-1} + 0.720835 \sigma^2_{t-1|t-2}$$

Equation 20

$$RT = 0.010932 - 0.001132 \ln(\sigma^2_{t|t-1})$$

Equation 21

$$\sigma^2_{t|t-1} = 0.000001 + 0.053119 u^2_{t-1} + 0.937257 \sigma^2_{t-1|t-2}$$

4.4 EGARCH estimation

Equation 7 and Equation 16 outline the mean and conditional variance equations of the EGARCH framework. The results of the EGARCH estimation are displayed in Table 4.4.

Parameter	Kenya		Nigeria		Mauritius	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
μ	-0.01610	0.172	-0.05342	0.000	-0.00466	0.402
δ	-0.00244	0.087	-0.00731	0.000	-0.00045	0.437
ω	-1.57462	0.000	-0.67862	0.000	-0.00557	0.546
γ	-0.06645	0.196	-0.10632	0.001	-0.05098	0.000
α	0.63985	0.000	0.41655	0.000	-0.02678	0.024
β	0.85873	0.000	0.94824	0.000	0.99737	0.000
Log-L	959.78		820.88		1113.56	

The mean equation indicates a negative risk-return relationship which is significant at the 5% level. The value of the coefficient δ is -0.00244 for Kenya, -0.00731 for Nigeria and lastly -0.00045 for Mauritius. There is a marginal improvement of in the log likelihood function in the EGARCH over the GARCH-M specification indicating that asymmetry does not affect the relationship between risk and return (956.96 to 959.76 for Kenya, 817.37 to 820.88 for Nigeria and 1103.16 to 1113.56 for Mauritius). . The p-values of the results for Mauritius are above 5% implying that the coefficient of the independent variables (mean and conditional variance) do not entirely explain the dependent variable (return on the index).

5. CONCLUSION

5.1 Volatility Pricing

The study realized that volatility is not priced in the weekly returns of Nairobi Securities Exchange All share index for Kenya and Nigeria Stock Exchange All share index for Nigeria. A negative relationship is found between the weekly returns and the conditional volatility which is in line with the empirical results of Nelson (1991). This finding is in contrast of the results found by Pindyck (1984) and Baillie and DeGennaro (1990) who found a positive relationship. In both the GARCH and EGARCH framework, the study found that the relationship is insignificant at 5% level.

A possible explanation for this finding would be the effect of the financial crisis of 2008. Due to the collapse of many financial markets, the financial crisis led to a crisis of confidence in the financial market sector. The financial crisis of 2007-08 has taught us that confidence in financial markets, once shattered can't be quickly restored. In an interconnected world where globalization is taking full effect, a seeming liquidity crisis can very quickly turn into a solvency crisis for financial institutions. But the silver lining is that after every crisis in the past, markets have come out strong to forge new beginnings.

5.2 Policy implication

The results of the study may prove useful for developers and analysts of financial assets. Practitioners who attempt to value other asset classes whose returns share characteristics with those of stocks may also benefit. Specifically portfolio managers may find the study relevant when carrying out a forward-looking valuation of a well-diversified portfolio. They can then exploit this knowledge to obtain the best rates of return from their portfolio. This addresses the risk premium the investors are willing to accept for an additional risk taken up in any of the three markets. The study may also have implications on stock return predictability. However analysts in asset management of investment banking should not place high emphasis on relationship between risk and return. The results show that this relationship does not hold over time.

5.3 Areas for further study

Forecasting may also provide an exciting avenue in which to advance this work. This study proves that conditional variance is stationary. This implies that conditional variance forecasts will converge on their unconditional value as the horizon increases. In order that the prediction error is kept as small as possible, researchers should exercise great caution when extrapolating far from the sample time period.

Another area of further studies could be an analysis of return behavior against liquidity, or book to market values or externalities e.g. presidential general elections

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