

The Wang transform and option pricing

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Introduction

- There are many "transforms" that may be used as pricing tools.
- Wang proposed a general framework – now known as the ‘Wang transform’ – for pricing financial and insurance risk, based on the Choquet integral.
- The Choquet integral is non additive and is therefore useful to price financial or insurance risk.
- The Wang transform can recover
 - the capital asset pricing model
 - the Black-Scholes formula for options and
 - is consistent with Bühlmann’s economic premium principle.
- The Wang transform is only consistent with arbitrage free pricing in the setting of geometric Brownian motion.
 - In this setting it allows for a simple yet intuitive approach to pricing options.
- We use the Wang transform to price some exotic options.

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Distortion function

Definition

Let S be any set and F any subset of S . A set function $\nu : F \rightarrow [0, 1]$ is

- a **capacity** if $\nu(\emptyset) = 0$, $\nu(S) = 1$ and ν satisfies monotonicity with respect to set inclusion, i.e. $A \subseteq B \Rightarrow \nu(A) \leq \nu(B)$.
- **concave** if for all $A, B \in F$

$$\nu(A \cup B) + \nu(A \cap B) \leq \nu(A) + \nu(B).$$

A capacity is often referred to as a **distorted probability measure**, since it is known that there exists a non-decreasing function $g : [0, 1] \rightarrow [0, 1]$ with $g(0) = 0$ and $g(1) = 1$, and a probability measure Q such that $\nu = g \circ Q$.

The function g is called a **distortion function** or a **distortion operator**, as it transforms a probability distribution F_X to a new distribution $g(F_X)$.

Choquet integral

Definition

Let S be any set and F any set of subsets of S . Let $\nu : F \rightarrow [0, 1]$ be a non-increasing set function and $X : S \rightarrow \mathbb{R}$ measurable. The **Choquet integral** of X with respect to ν is given by

$$C_\nu(X) = \int_{-\infty}^0 (\nu(X \geq x) - \nu(S)) dx + \int_0^{\infty} \nu(X \geq x) dx.$$

The Choquet integral has the following properties:

- (i) Monotonicity, i.e. if $X \leq Y$ a.s., then $C_\nu(X) \leq C_\nu(Y)$,
- (ii) Translation invariance, i.e. $C_\nu(X + \theta) = C_\nu(X) + \theta$,
- (iii) Positive homogeneity, i.e. $C_\nu(\lambda X) = \lambda C_\nu(X)$ for all $\lambda > 0$,
- (iv) Sub-additivity, i.e. $C_\nu(X + Y) \leq C_\nu(X) + C_\nu(Y)$ if and only if ν is concave.

The intuition behind using a Choquet integral as a pricing method is shown by the following.

- Let F_X be the cumulative distribution function of a random variable X , i.e.

$$F_X(x) = P(X \leq x).$$

- The decumulative distribution function is given by

$$S_X(x) = P(X > x) = 1 - F_X(x).$$

- Then S_X is a non-increasing function from \mathbb{R}^+ to $[0, 1]$.
- One can show that the expected value of a random variable X can be written as

$$\mathbb{E}[X] = \int_{-\infty}^0 (S_X(x) - 1) dx + \int_0^{\infty} S_X(x) dx.$$

Let g_α be a distortion function. The **risk adjusted premium** $H[\cdot, \alpha]$ of a risk X with respect to g_α is defined by

$$H[X, \alpha] = \int_{-\infty}^0 [g_\alpha(S_X(x)) - 1] dx + \int_0^{\infty} g_\alpha(S_X(x)) dx. \quad (1)$$

$H[X, \alpha]$ can be seen as the certainty equivalent of a risky cash flow – (see Wang ([6]) for details).

In order to derive pricing results, α is derived in such a way that the certainty equivalent is consistent with the market price of the asset.

The following connection between H and \mathbb{E} is useful.

Proposition

Let Z be a standard normal variable and h a continuous increasing function with range in $[0, \infty)$. If $X = h(Z)$ and $\alpha \in \mathbb{R}$, then

$$H[X, \alpha] = \mathbb{E}[h(Z + \alpha)].$$

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Wang transform

Definition

Let Φ denote the standard normal cumulative distribution function, i.e.

$$\Phi(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$$

and let α be a real-valued parameter. Then $g_\alpha : [0, 1] \rightarrow [0, 1]$, defined by

$$g_\alpha(p) = \Phi[\Phi^{-1}(p) - \alpha],$$

defines a distortion operator, called the **Wang transform**.

Remarks on

$$g_\alpha(p) = \Phi[\Phi^{-1}(p) - \alpha] :$$

- If, for example, X follows a normal distribution, this distortion operator shifts the p^{th} quantile of X ; i.e., the value y such that

$$P(X \leq y) = p,$$

by a positive or negative value α , and then re-evaluates the normal cumulative probability for the shifted quantile.

- If X is not normal, then the Wang transform does not yield closed-form solutions and numerical techniques would need to be applied.
- By shifting the quantiles to the left, i.e. assigning higher probabilities to lower values, the Wang transform allows for pessimistic scenarios and by shifting the quantiles to the right, it allows for more optimistic scenarios.

Useful properties of the Wang transform g_α (see Wang [6] for details):

- g_α is continuous for all α .
- g_α is convex for $\alpha < 0$ and concave for $\alpha > 0$.
- If X has a Normal(μ, σ^2) distribution, then $g_\alpha(F_X)$ has a Normal($\mu + \alpha\sigma, \sigma^2$) distribution.
- If X has a LogNormal(μ, σ^2) distribution, then $g_\alpha(F_X)$ has a LogNormal($\mu + \alpha\sigma, \sigma^2$) distribution.
- The parameter α can be viewed as the market price of risk.

RND vs Distortion

The following proposition shows how a continuous distortion function and the Radon-Nikodým derivative are related.

Proposition

Given any continuous distortion function

$$F_X^*(x) = g(F_X(x)),$$

its Radon-Nikodým derivative is given by the derivative of g with respect to x evaluated at $F_X(x)$.

Proof.

As $F_X^*(x) = g(F_X(x))$, we get

$$f_X^*(x) := \frac{d}{dx} F_X^*(x) = \frac{d}{dx} g(F_X(x)).$$

Using the chain rule,

$$f_X^*(x) = \frac{dg}{dx}(F_X(x)) \cdot \frac{dF_X}{dx}(x).$$

Since, $\frac{dF_X}{dx}(x) = f_X(x)$, we have

$$\frac{f_X^*(x)}{f_X(x)} = \frac{dg}{dx}(F_X(x)),$$

which proves the proposition. □

Proposition

The Radon-Nikodým derivative corresponding to the Wang transform is given by

$$RND_g(x) = e^{\alpha\Phi^{-1}(F_X(x)) - \frac{1}{2}\alpha^2}.$$

Remarks:

- The proof is elementary.
- The result is potentially useful in option pricing!
- One has access to the Radon-Nikodým derivative without going through the standard procedure of changing from real world measure to risk neutral measure, and using Girsanov's theorem and the martingale representation theorem.

Pricing a Security

- S_t is the price of a security at time t .
- W_t is a Brownian motion under P .
- S_t follows GBM:

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

- Let $S_T = h(Z)$, where $h(x) = S_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}x}$.

Then

$$\begin{aligned} H[S_T, -\alpha] &= \mathbb{E}[h(Z - \alpha)] \\ &= \mathbb{E}[S_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}Z - \sigma\sqrt{T}\alpha}] \\ &= S_0 e^{(\mu - \frac{\sigma^2}{2})T - \sigma\sqrt{T}\alpha + \frac{\sigma^2}{2}t}. \end{aligned}$$

By no arbitrage, we want the discounted share price to be a martingale; i.e.,

$$\mathbb{E}[S_T] = e^{rT} S_0.$$

Thus

$$\alpha = \frac{\mu - r}{\sigma} \sqrt{T} \text{ and } S_0 = e^{-rT} \mathbb{E}[S_T] = e^{-rT} H[S_T, -\alpha].$$

Pricing a European Call Option

- The pay-off of a European call option with strike K is $C(T, K) = (S_T - K)^+$.
- Let $C(T, K) = f(Z)$, where $f(x) = (S_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}x} - K)^+$.

Then

$$\begin{aligned}
 H[C(T, K), -\alpha] &= \mathbb{E}[f(Z - \alpha)] \\
 &= \int_{-\infty}^{\infty} \left(S_0 e^{(\mu - \frac{\sigma^2}{2})T - \sigma\sqrt{T}\alpha + \sigma\sqrt{T}z} - K \right)^+ \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
 &= S_0 e^{\mu T - \sigma\sqrt{T}\alpha} \Phi(-z_{\min} + \sigma\sqrt{T}) - K \Phi(-z_{\min}),
 \end{aligned}$$

where

$$z_{\min} = \frac{\ln\left(\frac{K}{S_0}\right) - (\mu - \frac{\sigma^2}{2})T - \sigma\sqrt{T}\alpha}{\sigma\sqrt{T}}.$$

Let $\alpha = \frac{\mu - r}{\sigma} \sqrt{T}$ an discount to get the the Black-Scholes price.

Option Pricing – the general case

The following result is useful in pricing using the Wang transform:

Proposition

Consider an option with underlying f , maturity T , strike K and payoff

$$(f_T - K, 0)^+,$$

where $(x, 0)^+ = \max(x, 0)$. If the underlying f follows geometric Brownian motion, i.e.

$$df_t = \hat{\mu}f_t dt + \hat{\sigma}f_t dW_t,$$

where W is a Brownian motion, $\hat{\mu}$ is the drift and $\hat{\sigma}$ is the volatility, then $e^{-r(T-t)} H[f, -\alpha]$, with

$$\alpha = \frac{\hat{\mu} - r}{\hat{\sigma}} \sqrt{T - t},$$

reduces to the equivalent Black-Scholes price.

Examples

We use the preceding result to price:

Example

(i) **Standard European option**

First, we need to ensure that the proposition holds for vanilla European options. The underlying of a standard normal European option is a stock which follows

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

This is just a straight forward application of preceding Proposition, as we have already noted earlier.

Example

(ii) **Margrabe option**

A Margrabe option gives the holder the right but not the obligation to exchange one asset for another.

Assume we have two assets S_1 and S_2 , which follow

$$dS^{(i)} = \mu_i S^{(i)} dt + \sigma_i S^{(i)} dW_i, \text{ for } i = 1, 2,$$

where the W_i 's are correlated Brownian motions with correlation ρ_{12} . Then the payoff of the Margrabe option at expiry T is given by

$$(S_T^{(1)} - S_T^{(2)}, 0)^+.$$

Using $S^{(2)}$ as the numéraire, we can rewrite the payoff as

$$\left(\frac{S_T^{(1)}}{S_T^{(2)}} - 1, 0 \right)^+.$$

The underlying in this case is $\frac{S^{(1)}}{S^{(2)}}$ and the strike is 1.

We first need to find the process that $\frac{S_t^{(1)}}{S_t^{(2)}}$ follows. Let

$$f(S^{(1)}, S^{(2)}) = \frac{S^{(1)}}{S^{(2)}}.$$

By the multivariate version of Itô's lemma, we get

$$df = \frac{S_t^{(1)}}{S_t^{(2)}} \left([\mu_1 - \mu_2 + \sigma_2^2 - \sigma_1\sigma_2\rho_{12}]dt + \sigma_1 dW_1 - \sigma_2 dW_2 \right).$$

Let $\hat{\sigma} dW^* = \sigma_1 dW_1 - \sigma_2 dW_2$, where

$$\hat{\sigma}^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}$$

and $dW^* \sim \text{Normal}(0, \hat{\sigma}^2 dt)$. Thus,

$$df = \hat{\mu} f dt + \hat{\sigma} f dW^*,$$

where

$$\hat{\mu} = \mu_1 - \mu_2 + \sigma_2^2 - \sigma_1\sigma_2\rho_{12}.$$

Note that since we used $S^{(2)}$ as the numéraire, we have that $r = 0$ and hence, the choice

$$\alpha = \frac{\hat{\mu}}{\hat{\sigma}} \sqrt{T - t},$$

may be used to yield the Black-Scholes price.

Simulations

- We would like to see what prices the Wang transform could yield.
- Consider a Margrabe option to exchange $S^{(1)}$ for $S^{(2)}$ in half a years time.
- We compare the Wang price to the Black Scholes price for varying α in the case of the above-described Margrabe option.
- This is shown in Figure 1.
- As can be seen in Figure 1, it is possible to obtain a wide variety of prices using the Wang transform, while the Black Scholes price remains constant.

- The following parameters were used in the simulations:
 - $r = 8\%$,
 - $S_0^{(1)} = 20$, $\mu_1 = 16\%$ and $\sigma_1 = 20\%$,
 - $S_0^{(2)} = 25$, $\mu_2 = 10\%$ and $\sigma_2 = 15\%$, and
 - $\rho = 0.2$.

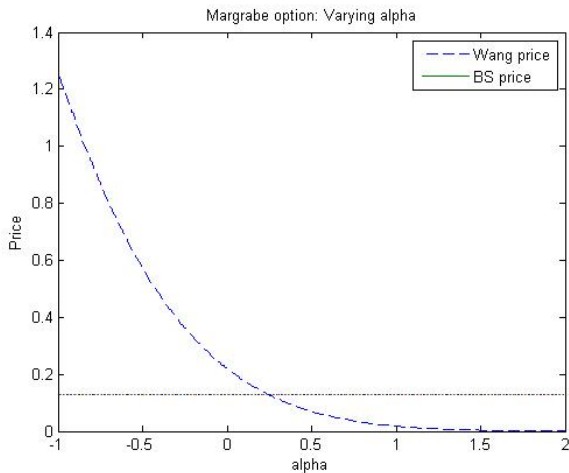


Figure: Wang price vs alpha

Example

(iii) **Geometric basket option**

A basket option is an option whose underlying is a sum or average of different assets.

- To price an arithmetic basket option, which uses an arithmetic average as the underlying, Monte Carlo techniques are often used.
- We consider a geometric basket option, which uses a geometric average as the underlying.

The payoff of this option takes the form

$$(\sqrt{S^{(1)}S^{(2)}} - K, 0)^+.$$

The processes $S^{(1)}$ and $S^{(2)}$ follow geometric Brownian motion, i.e.

$$dS^{(i)} = \mu_i S^{(i)} dt + \sigma_i S^{(i)} dW_i, \text{ for } i = 1, 2,$$

where the W_i 's are correlated Brownian motions with correlation ρ_{12} .
The underlying in this case is $\sqrt{S^{(1)}S^{(2)}}$ and the strike is K .

We first need to find the process that $\sqrt{S^{(1)}S^{(2)}}$ follows. Let

$$f(S^{(1)}, S^{(2)}) = \sqrt{S^{(1)}S^{(2)}}.$$

By the multivariate version of Itô's lemma, we get

$$df_t = f_t \left(\left[\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 + \frac{1}{8}\sigma_1^2 + \frac{1}{8}\sigma_2^2 + \frac{1}{8}\sigma_1\sigma_2\rho_{12} \right] dt + \frac{1}{2} [\sigma_1 dW_1 + \sigma_2 dW_2] \right).$$

Let $\hat{\sigma} dW^* = \frac{1}{2}(\sigma_1 dW_1 + \sigma_2 dW_2)$, where

$$\hat{\sigma}^2 = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12})$$

and $dW^* \sim \text{Normal}(0, \hat{\sigma}^2 dt)$. Thus,

$$df_t = \hat{\mu} f_t dt + \hat{\sigma} f_t dW^*,$$

where

$$\hat{\mu} = \frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 + \frac{1}{8}\sigma_1^2 + \frac{1}{8}\sigma_2^2 + \frac{1}{8}\sigma_1\sigma_2\rho_{12}.$$

Setting

$$\alpha = \frac{\hat{\mu}}{\hat{\sigma}} \sqrt{T-t},$$

reduces $e^{-r(T-t)} H[f, -\alpha]$ to the Black-Scholes price.

Figure 2 shows the flexibility of the Wang transform to price a geometric basket option with strike $K = 22$ and maturing in half a years time. The same parameters were used as in the preceding example.

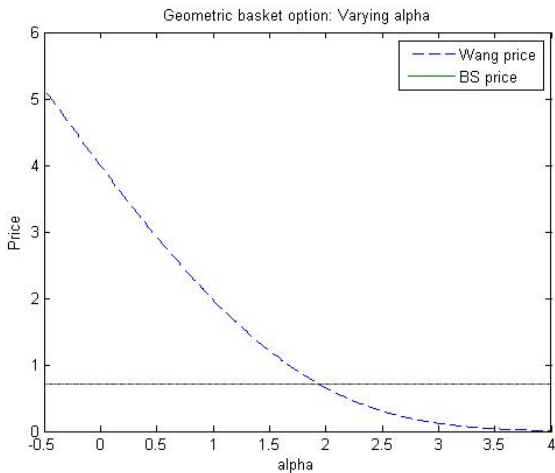


Figure: Wang price vs alpha

The Wang transform and arbitrage-free pricing

- Pelsser [5] showed that the Wang transform is consistent with arbitrage-free pricing in the setting of geometric Brownian motion with constant coefficients.
- What happens if you let the traded asset be described by a general stochastic process?
- Suppose we have a traded asset X_t the price of which follows the stochastic differential equation

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t,$$

where W_t is a Brownian Motion.

Pelsser also showed:

Proposition

The Wang transform is consistent with arbitrage-free pricing if and only if the following conditions on $\mu(t, X_t)$ and $\sigma(t, X_t)$ are satisfied

$$\frac{\partial}{\partial x} \left(\mu(t, X_t) \frac{\frac{\partial F}{\partial x}}{\phi(\Phi^{-1}(F))} \right) = 0$$

and

$$\frac{\partial}{\partial x} \left(\sigma(t, X_t) \frac{\frac{\partial F}{\partial x}}{\phi(\Phi^{-1}(F))} \right) = 0.$$

Note that a necessary condition for this to hold is that the ratio $\frac{\mu(t, X_t)}{\sigma(t, X_t)}$ is a function of time only.

Esscher transform

The Esscher transform was developed to approximate the aggregate claim amount distribution around a point of interest x_0 , by applying an analytic approximation, the Edgeworth series, to the transformed distribution with the parameter θ chosen such that the new mean is equal to x_0 .

Definition

Given a cumulative distribution function (cdf) $F(x)$ and a parameter θ , the **Esscher transform** is defined by

$$dF^\theta(x) = \frac{e^{\theta x} dF(x)}{\int e^{\theta y} dF(y)}.$$

If $F(x)$ admits a density $f(x)$, then $F^\theta(x)$ has the density

$$f^\theta(x) = \frac{e^{\theta x} f(x)}{\int e^{\theta y} f(y) dy}$$

Properties

- F_X and its Esscher transform F_X^θ have the same null sets and are thus *equivalent distributions*.
- For a normal cdf with mean μ and variance σ^2 , its Esscher transform is a normal cdf with mean $\mu + \theta\sigma^2$ and variance σ^2 .

Esscher price

Definition

The real-valued function, given by

$$\pi_X^{Ess}(\theta) = \int_{-\infty}^{\infty} x dF_X^\theta = \frac{\mathbb{E}[Xe^{\theta X}]}{\mathbb{E}[e^{\theta X}]},$$

is known as the **Esscher premium** or the **Esscher price** with parameter θ .

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Esscher-Girsanov transform

Instead of considering the random variable X , Goovaerts and Laeven consider the extended real-valued function $\Phi^{-1}(F_X(x))$.

If F_X is continuous, then $\Phi^{-1}(F_X(x))$ is normally distributed with mean 0 and variance 1.

Definition

Given a continuous cdf $F(x)$ and parameters $h, v \in \mathbb{R}$, the **Esscher-Girsanov transform** is defined by

$$dF_X^{(h,v)}(x) = \frac{e^{hv\Phi^{-1}(F_X(x))}}{\mathbb{E}[e^{hv\Phi^{-1}(F_X(X))}]} dF_X(x) = e^{hv\Phi^{-1}(F_X(x)) - \frac{1}{2}h^2v^2} dF_X(x). \quad (2)$$

Esscher-Girsanov price

- For a normal cdf with mean μ and variance σ^2 , its Esscher-Girsanov transformation is normal with mean $\mu + hv\sigma$ and variance σ^2 .
- If we let $v = \sigma$, then the Esscher-Girsanov transform reduces to the Esscher transform.

Definition

The **Esscher-Girsanov price** of the random variable X , with parameters $h \leq 0$ and $v > 0$, is given by

$$\pi_X^v(h) = \mathbb{E}[Xe^{hv\Phi^{-1}(F_X(x)) - \frac{1}{2}h^2v^2}].$$

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Proposition

Let $\alpha = hv$ and let X be a random variable with pdf $f_X(x)$. The pdf generated by the Wang transform with parameter α is equal to the pdf generated by the Esscher-Girsanov transform with parameters h and v .

Proof: By the previous proposition, the pdf generated by the Wang transform is given by

$$f_X^{WT}(x) = e^{\alpha\Phi^{-1}(u) - \frac{1}{2}\alpha^2} f_X(x) = e^{hv\Phi^{-1}(u) - \frac{1}{2}h^2v^2} f_X(x),$$

which is the Esscher-Girsanov transformed pdf of X .

Proposition

Let $\alpha = hv$ and let X be a random variable with pdf f_X and continuous cdf F_X . The Esscher-Girsanov price for a risk X with parameters h and v is equal to Wang's risk adjusted premium for X with parameter α .

Proof: Wang's risk adjusted premium for X is given by

$$\pi_X^{WT}(x) = - \int_{-\infty}^0 \Phi(\Phi^{-1}(F_X(x)) - \alpha) dx + \int_0^{\infty} [1 - \Phi(\Phi^{-1}(F_X(x)) - \alpha)] dx.$$

Let $c(x) = \Phi^{-1}(F_X(x)) - \alpha$, then

$$\pi_X^{WT}(x) = - \int_{-\infty}^0 \int_{-\infty}^{c(x)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt dx + \int_0^{\infty} \int_{c(x)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt dx.$$

Next, we need to change the order of integration. Letting $k(t) = F_X^{-1}(\Phi(t + \alpha))$ and $\gamma = \Phi^{-1}(F_X(0)) - \alpha$, we get

$$\begin{aligned}\pi_X^{WT}(x) &= - \int_{-\infty}^{\gamma} \int_{k(t)}^0 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dx dt + \int_{\gamma}^{\infty} \int_0^{k(t)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dx dt \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} k(t) dt \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} F_X^{-1}(\Phi(t + \alpha)) dt.\end{aligned}$$

Substituting $y = t + \alpha$, we get

$$\pi_X^{WT}(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-\alpha)^2} F_X^{-1}(\Phi(y)) dy. \quad (3)$$

On the other hand, the Esscher-Girsanov price for X is given by

$$\begin{aligned}\pi_X^{EG}(x) &= \mathbb{E}[Xe^{hv\Phi^{-1}(F_X(x)) - \frac{1}{2}h^2v^2}] \\ &= \int_{-\infty}^{\infty} xe^{hv\Phi^{-1}(F_X(x)) - \frac{1}{2}h^2v^2} f_X(x) dx.\end{aligned}\quad (4)$$







Let $y = \Phi^{-1}(F_X(x))$, then $dy = \sqrt{2\pi}e^{\frac{1}{2}y^2}dF_X$. Substituting this into (4), we get

$$\pi_X^{EG}(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-hv)^2} F_X^{-1}(\Phi(y)) dy.\quad (5)$$

As (3) is the same as (5), we have that

$$\pi_X^{WT}(x) = \pi_X^{EG}(x).$$

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Thank You

Thank you