



School of Computing and Engineering Sciences
Bachelor of Science in Telecommunications
End of Semester Examination
CNS 1103 -Differential Calculus

Date: Wednesday 26th July 2023.

Time: 2 Hours

Instruction

1. Answer **QUESTION ONE** and any other **TWO QUESTIONS**

QUESTION ONE [30 Marks]

- a) Find the values of a and b so that the piecewise function below is continuous: [3 Marks]

$$g(x) = \begin{cases} 2x^2 + 3x & \text{if } x \leq -4 \\ ax + b & \text{if } -4 < x \leq 3 \\ -x^3 + 4x^2 - 5 & \text{if } 3 \leq x \end{cases}$$

- b) Given a function $f(x)$, when do we say that the function is continuous:

i. At a point say a ? [2 Marks]

ii. On a closed interval say $[a, b]$ [2 Marks]

- c) Evaluate the following limits:

i. $\lim_{x \rightarrow -1} \frac{3x-4}{8x^2+2x-2}$ [3 Marks]

ii. $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$ [3 Marks]

iii. $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{x^2}$ [4 Marks]

iv. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^3}$ [4 Marks]

- d) Find y' given that:

- i. $y = \sin^3(\cos x)$ [3 Marks]
- ii. $y^3 = \tan(xy) + 3xy$ [3 Marks]
- iii. $x = 3^{x^y}$ [3 Marks]

QUESTION TWO [20 Marks]

a) From first principles differentiate the following:

- i. $y = -x^3 - 2x^2 + x$ [3 marks]
- ii. $y = \frac{2}{\sqrt{x}}$ [3 Marks]
- iii. $y = \cos(x)$ [5 Marks]
- iv. $y = x^{-2}$ [3 Marks]

b) Find y' given that

- i. $y = (\sqrt{x} - \frac{1}{\sqrt{x}})^4$ [3 Marks]
- ii. $x = 7^{\sin(2y)}$ [3 Marks]

QUESTION THREE [20 Marks]

a) Evaluate the following limits:

- i. $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$ [3 Marks]
- ii. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}}$ [1 Mark]

b) Find y' given that:

- i. $y = \frac{(3x+1)^4}{(5x-2)^3}$ (Simplify your answer) [4 Marks]
- ii. $x^{yx} = y^{xy}$ [4 Marks]
- iii. $y = \frac{\ln(\sin(x))}{e^{\cos(x)}}$ [4 Marks]
- iv. $x^{\sin^y(x)} = y^{\sin^x(y)}$ [4 Marks]

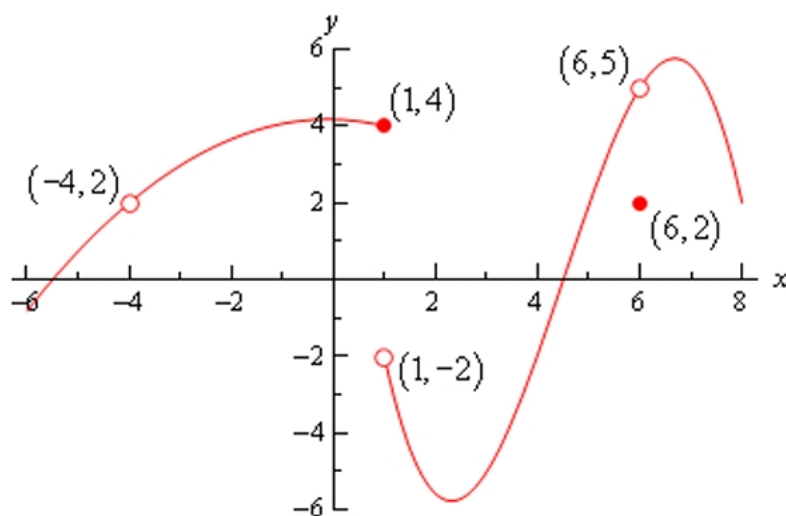
QUESTION FOUR [20 Marks]

- a) For $\lim_{x \rightarrow 1} T(x)$ to exist, what values must q take given that:
[3 Marks]

$$T(x) = \begin{cases} qx^2 - 5 & \text{if } x < 1 \\ 10 & \text{if } x = 1 \\ \frac{1}{x} - 2q & \text{if } x > 1 \end{cases}$$

- b) Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$ and $\lim_{x \rightarrow a} f(x)$. Further, determine if the limit either exists or not at the points. [9 Marks]

- i. $a = -4$
- ii. $a = 1$
- iii. $a = 6$

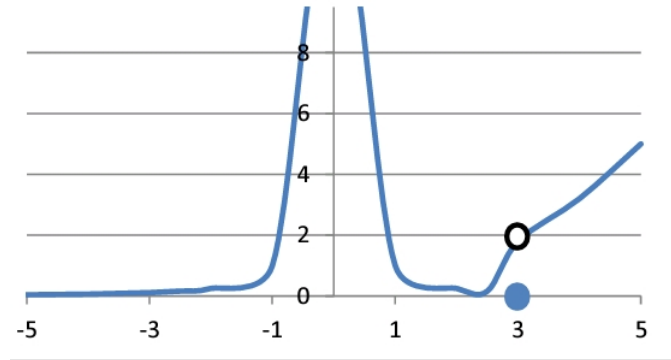


- c) Determine if each function given below is continuous. If the function is not continuous, find the point of discontinuity and classify the discontinuity exhibited.

i. $f(x) = \frac{1}{(x-1)^2}$ [2 Mark]

ii. $h(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$ [2 Marks]

- d) Determine where the function whose graph is shown below is discontinuous and identify the type of discontinuity exhibited. [4 Marks]



QUESTION FIVE [20 Marks]

- a) Find y' given the following:

- i. $y = \sin^6(\sin(5x))$ [4 Marks]
- ii. $2y = \frac{4^x}{x^4} + \tan(xy)$ [4 Marks]
- iii. $xy = \frac{\cos^4(3x)}{e^{6x}}$ [4 Marks]

- b) Show that:

- i. $\frac{d}{d\theta}(\sec(\theta) = \tan(\theta)\sec(\theta)$ hence evaluate $\frac{d}{d\theta}(\sec^3(3\theta))$ [4 Marks]
- ii. $\frac{d}{dt}(\cot(t)) = -\operatorname{cosec}^2(t)$ hence evaluate $\frac{d}{dt}(\cot(\ln(t)))$ [4 Marks]