STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES

BACHELOR OF BUSINESS SCIENCE FINANCIAL ENGINEERING

SPECIAL EXAMINATION

BSM 4124: COMPUTATIONAL METHODS IN FINANCE

DATE: 25th March 2024 TIME: 2 HOURS

INSTRUCTIONS

I. This examination consists of FIVE questions. Answer **Question 1** (**COMPULSORY**), choose **2 optional** questions out of **Question 2 to 5.**

II. Ensure you have clearly **indicated the question number** on EVERY QUESTION.

QUESTION ONE [30 marks]

i. Define the concept of arbitrage (2 marks)

ii. Define the concept of risk-neutrality as it applies to financial modelling (3 marks)

iii. Derive the binomial option pricing formula via the *risk-neutral valuation* technique, assuming that at time t, the price of an asset is $S(t) = S$, and r represents the interest rate for both lending and borrowing. (5 marks)

iv. Detail the four-step process followed by finite difference techniques in order to arrive at approximate solutions for option pricing (4 marks)

v. How would you describe the truncation/discretization errors and stability of the three popular finite difference methods, i.e., the explicit, implicit, and Crank-Nicolson methods? (6 marks, 2 marks per method)

vi. Discuss the steps underlying the Monte Carlo valuation of derivatives (3 marks)

vii. Describe the antithetic variate method of variance reduction for *path-dependent options* whose
payoff at time T is given by

$$\psi(S_s, s \leq T)$$

And $(S_t, t \geq 0)$ is the lognormal diffusion

$$S_t = x \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right)$$

(3 marks)

viii. Describe the importance sampling variance reduction procedure, including its key characteristics that differentiate it with other variance reduction techniques (4 marks)

<u>QUESTION TWO [15 marks]</u>

Finite method discretization techniques are frequently applied to improve the numerical of continuous time models that would have otherwise proven intractable. To this effect, describe the following finite difference discretization techniques as applied in the discretization of stochastic differential equations, with an application to discretizing the *Black-Scholes-Merton PDE* for purposes of option pricing

- Explicit method
- Implicit method
- Crank-Nicolson method

Where the Black-Scholes-Merton PDE is given as

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

With time t, asset value S, and r represents the risk-free rate. [15 marks, 5 marks per technique]

QUESTION THREE [15 marks]

i. Derive the binomial option pricing (BOPM) formula via the *replicating strategy* assuming that at time t, the price of an asset is $S(t) = S$, α represents the units of stock, B the cash amount in a riskless bond, and r represents the interest rate for both lending and borrowing. (5 marks)

ii. Show that by substituting the values of α and B from the replication BOPM, where α represents the units of stock and B the cash amount in a riskless bond, into $V(S,t) = \alpha S + B$, we get the same value for $V(S,t)$ as the *risk-neutral BOPM for a European call option*. (10 marks)

QUESTION FOUR [15 marks]

i. Why is Monte Carlo Variance reduction important in financial applications? (2 marks)

ii. Differentiate (in summary) any three popular variance reduction techniques in Monte Carlo simulation (3 marks)

iii. Describe (in detail) the Control Variate variance reduction technique and apply this to an *Asian put option* using the Kemna-Vorst Estimation method, where the price of an Asian put option with fixed strike is given by;

$$M = E(e^{rT}(K - \frac{1}{T}\int_0^T S_s\, ds)_+)$$

And $(S_t, t \geq 0)$ is the Black-Scholes process;

$$S_t = x\exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$$

(10 marks)

QUESTION FIVE [15 marks]

i. Derive the Euler discretization method for a generalised/arithmetic Brownian motion process (5 marks).

ii. Describe the Euler discretization method as applied to approximate the price of a *European option on a common stock* (10 marks)

[TOTAL: 60 MARKS]

END OF EXAM