



School of Computing and Engineering Sciences (SCES)
Bachelor of Informatics and Computer Science (BICS)
Bachelor of Science in Computer Networks and Cyber Security (BCNS)
End of Semester Examination
ICS 1205 - Linear Algebra
CNS 1205 - Linear Algebra

Date: Thursday 19th March 2026

Time: 08 : 30 to 10 : 30

Instructions

1. This exam consists of five printed pages.
2. Answer QUESTION ONE (COMPULSORY) and any other two questions.

QUESTION ONE (COMPULSORY) - 30 Marks

- (a) For the matrix A , determine the values of θ and β that make it a skew-symmetric matrix while for the matrix B and in your wisdom, state whether it is symmetric or skew-symmetric giving a reason for or against. [4 Marks]

i) $A = \begin{pmatrix} 0 & -3 & 2\theta + 2 \\ 3 & 0 & 3 \\ -2 & -2\beta - 3 & 0 \end{pmatrix}$ ii) $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- (b) Are the following statements true or false?

- i) The REF of a matrix is unique. [1 Mark]
- ii) If a matrix has a pivot in every column, the corresponding linear system ($AX = B$) is always consistent. [1 Mark]

- iii) A basic variable in a linear system is a variable that corresponds to a non pivot column of the coefficient matrix. [1 Mark]
- (c) Show that $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ represents the equation of a line passing through the points (x_1, y_1) and (x_2, y_2) . [4 Marks]
- (d) If $\begin{pmatrix} -4.5 \\ -4 \\ 1 \end{pmatrix}$ is an eigenvector of $\begin{pmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{pmatrix}$, find the corresponding eigenvalue. [2 Marks]
- (e) Some matrix A has $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ as two of its eigenvectors corresponding to the eigenvalue 7 and with a trace of two. Find the determinant of matrix A . [3 Marks]
- (f) Given $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, use the Cayley-Hamilton theorem to find an equivalent but simplified expression for the polynomial; [4 Marks]
- $$A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I_3.$$
- (g) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Show that:
- A is diagonalizable if $(a - d)^2 + 4bc > 0$. [3 Marks]
 - A is not diagonalizable if $(a - d)^2 + 4bc < 0$ over $\mathbb{R}$. [1 Mark]
- (h) The 2×2 matrix B is given as $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$.
- Find scalars h and k such that $(B - hI_2)B = -kI_2$. [2 Marks]
 - Use (i) above to find B^{-1} . No credit will be given for finding B^{-1} using the direct method. [2 Marks]
- (i) Show that $H = \begin{pmatrix} 1 & -2 & k \\ k & 2 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ is non singular $\forall k \in \mathbb{R}$. [2 Marks]

QUESTION TWO - 20 Marks

- (a) For all $n \in \mathbb{Z}^+$, find an expression for A^n given that

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

[10 Marks]

- (b) Two matrices P and Q are given as

$$\text{i) } P = \begin{pmatrix} 1 & 2 & 1 \\ 2 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \quad \text{ii) } Q = \begin{pmatrix} 15 & -4 & -1 \\ h & 4 & 2 \\ 17 & -4 & -3 \end{pmatrix}$$

It is further given that the *relation* $PQ = \lambda I_3$ holds true.

Determine the values of λ, h and k then together with the *relation*, deduce P^{-1} . [6 Marks]

- (c) Consider the matrix $R = \begin{pmatrix} 7 & 6 \\ 6 & 2 \end{pmatrix}$. Use the Cayley-Hamilton theorem to show that $R^4 = \lambda R + \mu I$, giving the values of λ and μ . [4 Marks]

QUESTION THREE - 20 Marks

- (a) Solve the following system:

[8 Marks]

$$\begin{aligned} 2a + 5b - 8c + 2d + 2e &= 0 \\ 6a + 2b - 10c + 6d + 8e &= 6 \\ 3a + 6b + 2c + 3d + 5e &= 6 \\ 3a + b - 5c + 4e + 3d &= 3 \\ 6a + 7b - 3c + 6d + 9e &= 9 \end{aligned}$$

- (b) The system of simultaneous equations

$$\begin{pmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

where $t \in \mathbb{Z}$ and a, b and c are real constants does not have a unique solution but is consistent.

- i) Determine the value of t .

[5 Marks]

- ii) Show clearly that $a + b = c$.

[7 Marks]

QUESTION FOUR - 20 Marks

- (a) Some 3×3 matrix is given below

$$M^T = \begin{pmatrix} -4 & -1 & -7 \\ -4 & 0 & -6 \\ 4 & 1 & 7 \end{pmatrix}$$

Given that I is the 3×3 identity matrix, determine the possible values of the constant λ , so that $M + \lambda I$ is singular. [8 Marks]

- (b) Find an LU decomposition of the matrix [9 Marks]

$$\begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{pmatrix}$$

hence solve the linear system $AX = B$ given that $B = \begin{pmatrix} 8 \\ 7 \\ 14 \\ -7 \end{pmatrix}$.

- (c) Are the following statements true or false? [3 Marks]
- i) Not every matrix has an LU decomposition; it requires the matrix to be transformable to row echelon form without row swaps (or pivoting).
 - ii) LU factorizations are generally not unique.
 - iii) An upper triangular matrix has zeros below the main diagonal, not above.

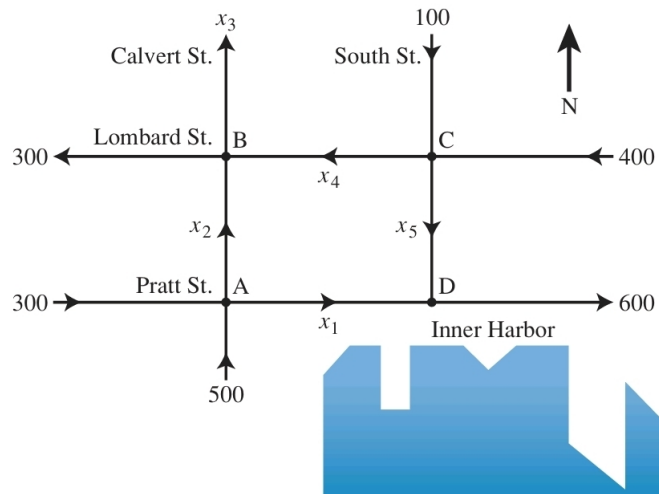
QUESTION FIVE - 20 Marks

- (a) Find the inverse of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{pmatrix}$ hence or otherwise solve the system

$$\begin{aligned} a + 2b + c &= 1 \\ 2a + 3b + c &= 4 \\ 3a + 4b + 2c &= 4 \end{aligned}$$

[10 Marks]

- (b) The network in figure below shows the traffic flow (in vehicles per hour) over several one-way streets in downtown Baltimore during a typical early afternoon.



Write equations that describe the traffic flow, and then find the general solution of the system. Determine the maximum permissible traffic flow along the stretch from C to D . [10 Marks]

Best of luck!