

Strathmore
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**BITCOIN AS AN ALTERNATIVE ASSET IN EMERGING
MARKETS: PORTFOLIO OPTIMIZATION VIA CONDITIONAL
VALUE-AT-RISK**

MUKURIA MARY WANJIRU

084141

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Strathmore Institute of Mathematical Sciences
Strathmore University

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MARY MUKURIA [Name of Candidate]

 [Signature]

DECEMBER, 2017 [Date]

This Research Project has been submitted for examination with my approval as the Supervisor.

Dr. Caroline Kaniuki [Name of Supervisor]

 [Signature]

December, 2017 [Date]

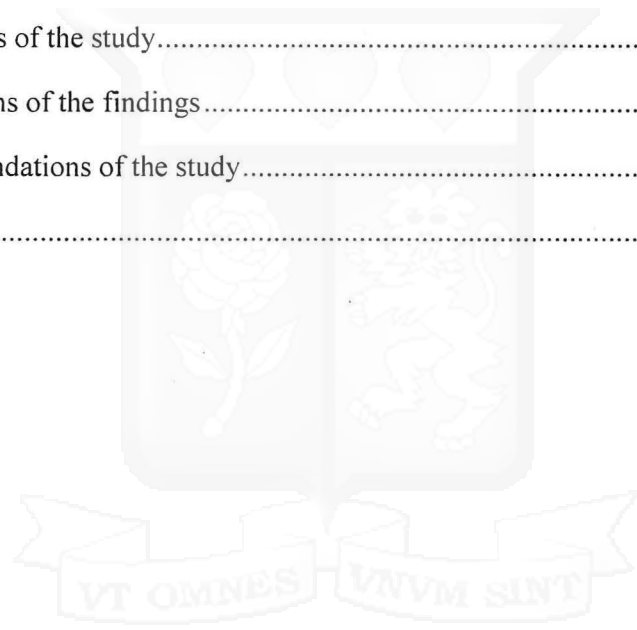
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Abstract

In a bid to diversify their portfolios, investors are venturing into various alternative assets. This study analyses the inclusion of bitcoin, as an alternative asset, in a well-diversified portfolio in South Africa. Recognizing that bitcoin is a relatively new asset, this study has provided detailed information on the features of bitcoin, both as a currency and as an investable asset. The study utilizes a well-diversified portfolio that consists of stocks, bonds, commodities, cash, real estate and international assets. The asset classes in the portfolio are picked from the South-African market as it is a very lucrative emerging market. The study utilizes time series data as the historical data of the asset returns is collected from July 28, 2010 to December 29, 2017. The Mean-CVaR Portfolio Optimization approach is utilized so as to accommodate the highly non-normal return distribution of bitcoin instead of the Mean Variance Optimization approach which assumes that returns are normally distributed. Two different portfolio frameworks are utilized namely; The Minimum CVaR Concentration Portfolio and the Minimum CVaR portfolio under an upper 30% CVaR allocation constraint are used to assess the objectives of the study. Dynamic rebalancing is utilized so as to achieve robust results. The results show that the inclusion of bitcoin increases the risk-return ratios of the different portfolios. The results also show that bitcoin's weight allocation is relatively low, however, even with the low weighting in the portfolios, the risk contribution of bitcoin to the portfolio CVaR is relatively high. The study concludes that bitcoin appears to be an attractive investment that can substantially increase the return of an efficient portfolio as the portfolios with bitcoin outperform their non-bitcoin counterparts. The recommendations of this study are that sophisticated forecasting techniques such as Bayesian methods or Neural Network should be utilized for scenario generation instead of the use of historical data and that the impact of the inclusion of bitcoin to a well-diversified portfolio should be conducted when the asset is at its mature stage.



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List of Abbreviations

| | |
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| BTC | Bitcoin |
| CAPM | Capital Asset Pricing Model |
| CVaR | Conditional Value-at-Risk |
| ER | Expected Regret |
| ES | Expected Shortfall |
| M-CVaR | Mean Conditional Value-at-Risk |
| MSCI | Morgan Stanley Capital International |
| MPT | Modern Portfolio Theory |
| MVO | Mean Variance Optimization |
| P2P | Peer-to-Peer |
| TCE | Tail Conditional Expectation |
| USD | United States Dollar |
| WCE | Worst Conditional Expectation |
| VaR | Value-at-Risk |



CHAPTER ONE

1. INTRODUCTION

1.1 Background of the study

Emerging markets are promising high-growth countries that represent attractive business opportunities for investors (Cavusgil, 1997). Miller (1998) concurs that emerging markets' similarities lie in their potential for future growth and these opportunities for future market expansion distinguish these countries from less developed countries.

One of the world's major and reputable index providers, Morgan Stanley Capital International (MSCI), launched the first comprehensive emerging markets index in 1988. In 1988, the MSCI Emerging Index consisted of ten countries namely; Argentina, Brazil, Chile, Mexico, Malaysia, Philippines, Thailand, Greece, Jordan and Portugal. The MSCI Emerging Markets Index has evolved since its inception in terms of size and the countries included. Since September 2012, it includes 21 countries namely; Brazil, Chile, Colombia, Mexico, Peru, China, India, Indonesia, Korea, Malaysia, Philippines, Taiwan, Thailand, Czech Republic, Egypt, Hungary, Morocco, Poland, Russia, South Africa and Turkey.

Mody (2004) explains that emerging markets are categorized using certain agreed upon characteristics. The World Bank defines developing countries as those with a lower-than-average per capita income. The low income allows for rapid growth, which is an important characteristic. Rapid growth in turn contributes to another characteristic, higher than average returns for investors which makes emerging markets attractive to investors. In developing countries, capital markets are less mature than those in the developed markets. High volatility is also a characteristic of emerging markets that stems from domestic policy instability, external price shocks and natural disasters.

Due to the aforementioned characteristics, the allure of emerging markets is strong as the rapid growth is associated with stronger earnings growth and thus appeals to investors. In 2017, the MSCI Emerging Markets Index advanced 27.1% through the

third quarter indicating that stocks and bonds in developing countries have been surging in 2017 (MSCI, 2017).

In 2013, due to the increased volatility of the South African financial markets, the media reports were encouraging investors to reduce risk through investment diversification and investment options such as art were proposed (Ferdin, Brett, & Jen, 2015). An inclusion of different types of asset classes to a portfolio can have a beneficial impact on portfolio diversification (Belousova & Dortfleiter, 2012).¹ Unlike traditional portfolios, portfolios of this age and time should incorporate a balanced mix of traditional and alternative investments so as to achieve diversification. Due to the volatility in South African financial markets, it is important for an investor to have a good asset allocation plan, as research has found that, asset allocation dictates over 90% of portfolio returns over time.

Bitcoin has become a force of innovation since its introduction in 2008. It has now become a household name in the financial markets and institutions and it has become the best performing asset class. Many investors believe that the cryptocurrencies space is one to watch and they compare it to the technology boom in the late 1900's. The price of bitcoin has increased from zero value at the time of its launch in 2009 to around \$10,000 in November 2017, thus qualifying as a highly lucrative asset. Bitcoin adoption has grown at an astounding rate despite it being a relatively new asset; as at 2016 the total market capitalization of the bitcoin network was over \$8 billion and it is currently actively traded in more than 60 online exchanges in the world.

The adoption of bitcoin in a global market portfolio could be interesting from an investment perspective. Generally, bitcoin is sensitive to market forces of supply and demand, digital-currency specific factors such as attractiveness of bitcoin and global

¹ Investors vet alternative forms of assets as diversifiers, hedge and safe haven assets so as to reduce their risk and increase their returns in investments. Bouri, Molnár, Azzi, Roubaud and Hagfors (2017) define a diversifier as an asset that has weak positive correlation with another asset. Baur and Mc Dermontt (2010) define a hedge asset as an asset that is uncorrelated with another asset or portfolio. It could either be a weak hedge or a strong hedge. Baur and Mc Dermontt (2010) define a safe haven asset as one that is negatively correlated with another asset of portfolio in certain periods only. Hedge and safe haven assets perform well during times of economic trouble. Traditional assets, such as, gold and the US dollar, have been confirmed to be a safe haven for crisis periods (Hillier, Draper, & Faff, 2006; Baur & Lucey, 2010; Ranaldo & Söderlind, 2010; Grisse & Nitschka, 2013; Bouoiyour & Selmi, 2017)

macro-financial development (van Wijk, 2013; Kristoufek, 2013; Bouoiyour & Selmi, 2015; Ciaian, Rajcaniova, & Kancs, 2016).

1.1.1 Bitcoin as an investable asset

Bitcoin is a very unique asset because it is a currency with characteristics of being an investment asset (Yermack, 2013; Baur, Lee, & Hong, 2015). In order to be qualify as a currency, bitcoin must fulfill the three functions of a currency; it should be a medium of exchange that is widely accepted, a store of value that stays stable over time and a unit of account that can compare the cost of goods and services over time and between merchants. The medium of exchange is viewed as the most important function of them all (Mishkin, 2009).

Previous literature has found that bitcoin does not perform substantially well as a medium of exchange and as a unit of account. The consumer transaction of bitcoin is quite low due to the volatility of bitcoin that exposes users to short-term risk, thus bitcoin's underperformance as a medium of exchange (Klabbers, 2017). Secondly, bitcoin's pricing of consumer goods requires very odd numbers with leading zeros and this does not sit well with retail market participants, thus bitcoin's failure as a unit of account (Klabbers, 2017).

Yermack (2013) argued that bitcoin is a speculative investment instead of a true currency. Recent literature shows that bitcoin can be seen more as an investment asset than a currency and that it is useful for risk management and portfolio optimization. Rogojanu & Badea (2014) concluded that bitcoin is similar to gold, but in a virtual environment. Bitcoin is a profitable investment according to numerous literature (Eisl, Gasser, & Weinmayer, 2015 ; Halaburda & Gandal, 2014).

Dyhrberg (2015) compared bitcoin to gold and the USD and concluded that bitcoin has medium of exchange characteristics and that it reacts almost the same as gold, including the hedging capabilities. The study concludes that bitcoin has a place in risk and portfolio management and is ideal for risk averse investors in anticipation of negative shocks. Bitcoin is classified as somewhere in between a currency and a commodity. Over one third of bitcoin users use the currency for investment purposes (Klabbers, 2017).

1.1.2 Bitcoin as a Cryptocurrency

A cryptocurrency is a type of digital currency which relies entirely on cryptography, usually alongside a proof-of-work scheme, in order to create and manage the digital currency (Ahamad, Nair, & Varghese, 2013). Many researchers argue that cryptocurrencies have the potential to both transform and disrupt the existing global financial infrastructure (Raymaekers, 2015). Bitcoin is referred to as a cryptocurrency as it relies on cryptography to validate transactions and govern the production of the currency itself.²

Bitcoin is a decentralized, purely peer-to-peer version of electronic cash, that allows online payments to be sent directly from one party to another without going through a financial institution (Nakamoto, 2008).³ It was founded in 2009 by a pseudonym named Satoshi Nakamoto.⁴ It has no intrinsic value thus, it is not backed by any government or any legal entity and its supply is not determined by a central bank (Elwell, Murphy, & Seitzinger, 2015).⁵ Bitcoin is the first and the largest of the other crypto-currencies (Swan, 2015).⁶

New bitcoins are generated by a competitive process known as mining. Mining can be defined as the process of spending computer power to process transactions and secure the network. Miners are special users on the bitcoin network who gather together blocks of new transactions and compete to verify the validity of transactions (Elwell, Murphy, & Seitzinger, 2015) using specialized hardware. As a reward, miners that successfully verify a block of transactions are rewarded by the network's controlling computer

² It is actually the first implementation of cryptography which was coined by Wei Dai (Bitcoin.org, 2017).

³ The terminology, Bitcoin, can be very confusing because the same word is used to refer to three parts of the concept. Bitcoin can be used to refer to the protocol through which transactions are effected or the actual cryptocurrency (bitcoins). Bitcoin can also be used to refer to the underlying blockchain technology (Bitcoin Blockchain) (Swan, 2015).

⁴ Nakamoto later left the project in late 2010 but many developers took it up and started working on Bitcoin and the community has since grown exponentially. However, his/her anonymity often raised unjustified concerns, especially due the open-source nature.

⁵ "The Bitcoin system's operation is similar to the growth of money under a gold standard" (Elwell, Murphy, & Seitzinger, 2015). On the supply of Bitcoins, instead of a central bank, the supply is programmed to grow at a steady rate and is regulated by the degree of mining activity and then it is capped at a fixed amount.

⁶ Other alternative crypto-currencies include: Ripple, Litecoin, Auroracoin, Peercoin, Dogecoin, Nxt, Mastercoin, Namecoin, Counterparty, Quark, Protoshares, Feathercoin, Primecoin, Infinitecoin amongst many others.

algorithm (Elwell, Murphy, & Seitzinger, 2015).⁷ Elwell et al. (2015) explain that the process of obtaining bitcoins is not as complicated as many would think. First and foremost, one has to download the free and open source software. Once connected to the software, a user can exchange conventional money for a fee on an online exchange.⁸ A second way to obtain bitcoins would be by serving as a miner as mentioned above. Also, a user, like say a merchant, can receive bitcoins from a buyer of his goods or services, in exchange for the sale of goods or services.

Bitcoin is quite interesting and unique because only 21 million bitcoins will ever be created. This should not be viewed as a limitation because transactions can be denominated in smaller sub-units of a bitcoin known as bits and as such, there are 1,000,000 bits in one bitcoin. Bitcoins can be divided into up to eight decimal places (0.00000001) and potentially even smaller units.

Nakamoto (2008) introduced Bitcoin because he realised that online commerce was limited because it entirely relies on financial intermediaries to process the payments as the third parties. Secondly, he highlights the trust-problem. Thirdly, the outrageous transaction fees due to mediation fees are highlighted. These increased transaction costs limit the minimal transaction size thus hindering small casual transactions. The fourth problem stated is the lack of ability to make non-reversible payments for non-reversible services.⁹ Nakamoto (2008) therefore suggests an electronic payment system that is not based on trust, but instead, cryptographic proof that would allow any two parties to comfortably transact with each other without the need of a third party.

⁷ In order to mine and validate a new block of transactions, miners compete to solve a difficult math problem and the miner that manages to solve it first validates the transactions in the block and broadcasts his proof-of-work to the bitcoin network then other miners counter-check the miner's results and in the case that the miner's work is found to be correct, the successful miner is rewarded. The probability of an individual discovering bitcoins through mining is proportional to the amount of computer processing power that needs to be applied meaning this is not an easy task because the more it is difficult to verify transactions, the more the computational cost of the service rises (Elwell, Murphy, & Seitzinger, 2015). The reward is currently 12.5 bitcoins from 25 bitcoins, but is still expected to change over time as it halves every 210,000 blocks.

⁸ The exchange fee decreases as the size of the transactions increases. The price of Bitcoin relative to another currency, say dollars, is determined by demand and supply (Elwell, Murphy, & Seitzinger, 2015).

⁹ Non-reversibility of sales eliminates the possibility of consumer charge-backs which are generally costly to the consumers (Elwell, Murphy, & Seitzinger, 2015). Transactions that are computationally impractical to reverse would protect sellers from fraud and would enable the implementation of escrow mechanisms to protect buyers (Nakamoto, 2008).

Bitcoin has insurmountable advantages which include; lower transaction costs, it enables fast peer-to-peer (P2P) transactions, it offers a less risky payment system, it offers a high degree of privacy which many people value in their commercial and financial transactions, it offers freedom of payment, it enables transparency in transactions and it is free from erosion of power due to inflation.

Bitcoin also has several disadvantages which include; Bitcoin has no government endorsement, unlike legal tenders and thus an unattractive vehicle for holding wealth in the eyes of many people, it has a long-term deflationary bias, it could be attractive to criminals and terrorists because of its privacy and irreversibility of transactions, its price volatility discourages its use as medium of exchange and the development of bitcoin is still ongoing and not in completion.

1.2 Problem Statement

Investors are attracted to growth and there is evidence that emerging markets contain the growth that investors are after. Emerging markets are good long-term investments because unlike developed markets which are experiencing a slowdown in growth, they are experiencing increased growth. Africa, particularly, is gradually becoming a viable place of investment as many financial analysts and investors believe that Africa has better growth potential and offers an opportunity to reap better returns than other emerging markets. Due to the political risk and lack of advanced technological infrastructure in Africa, investors not only deem it riskier than investing in developed markets but they also perceive a higher return potential from Africa than from developed markets (Sjolin, 2014).

In a bid to diversify their portfolios, investors are exploring alternative assets to diversify their traditional assets. Bitcoin is currently the best performing asset class, thus investors are curious about it from an investable asset point of view. Despite an increasing interest in bitcoin, there is minimal academic research on Bitcoin from a financial economics perspective. Limited studies have been conducted that have analysed cryptocurrencies from an investable asset perspective as most literature focuses on the payment system perspective and the technology aspect of bitcoin. Furthermore, prior studies have also been conducted using portfolios created from developed markets

and none of these studies have utilized emerging markets. This research proposes to bridge this gap, by conducting a study of bitcoin's impact to a well-diversified portfolio in South-Africa.

Due to the volatile nature of bitcoin, there has been a lack of consistency in the appropriate optimization method. Previous literature which has covered portfolio optimization with bitcoin differs on the appropriate optimization method. Some of the studies utilized Harry Markowitz' Mean-Variance Optimization (MVO), while some utilized a Modified Mean Variance Optimization or alternatively, the Mean-Conditional Value-at-Risk (M-CVaR). This research discusses and assesses the different portfolio optimization methods in detail.

1.3 Research Objectives

The principal objective of this study is to investigate the impact of an investment in Bitcoin on a well-diversified investment portfolio in an emerging market.

The specific objectives of the study are:

- i. To investigate the impact of the inclusion of bitcoin in an already diversified portfolio on the return and risk measures.
- ii. To analyse the effect of the inclusion of bitcoin on the asset allocation of an already well-diversified portfolio.
- iii. To investigate the effect on the risk contribution of the assets of a well-diversified portfolio once bitcoin is included.

1.4 Research Questions

The guiding questions to this study are:

- i. How are the return and risk measures of an already diversified portfolio affected by the inclusion of bitcoin?
- ii. How is the asset allocation of a well-diversified portfolio affected by the addition of bitcoin?
- iii. How is the risk contribution of assets in a well-diversified portfolio affected by the inclusion of bitcoin?

1.5 Scope of the study

This study aims to investigate the impact of the inclusion of bitcoin in a well-diversified portfolio in an emerging market in Africa. Egypt and South-Africa are the only emerging markets in Africa according to MSCI. South-Africa's financial market is regarded as one of the most sophisticated among emerging markets. Therefore, this study will utilize South Africa, which is a country in Sub-Saharan Africa.

1.6 Significance of the study

The findings of this study will serve various stakeholders in the following ways:

The research will contribute to academic research in the bitcoin space, especially from a financial economics perspective. It adds up to the body of knowledge of bitcoin as an investment asset, and not as a currency. The study also analyses different portfolio optimization methods other than the classical Mean Variance approach. As such, the study will act as a source of future reference for future scholars.

The research will contribute to investors in search of alternative investments to include in their portfolios so as to mitigate risk via risk diversification. This study introduces bitcoin as a possible alternative investment and the results indicate the impact of the inclusion of bitcoin on an investors' well-diversified portfolio. The study is focused particularly on emerging markets, specifically in Sub-Saharan Africa, so as to encourage investors to accept bitcoin as an investable asset, and as one that actually enables diversification in their portfolios. This is also applicable to institutional investors as well.

The study will contribute to the regulators' acceptance of bitcoin in emerging markets, specifically in Sub-Saharan Africa, as the study outlines the advantages of the inclusion of bitcoin in investors' portfolios. The research informs on the level of support that is required to provide investors with the opportunity to invest in bitcoin as it contributes positively to the investors' portfolio performance in general. Generally, this study incentivises regulators to provide better accessibility of bitcoin to emerging market investors.

CHAPTER TWO

2. LITERATURE REVIEW

2.0 Introduction

The aim of this chapter is to review and discuss past literature related to the research area. This chapter mainly reviews previous work on the portfolio selection theory and various portfolio optimization frameworks and provides essential background knowledge on the research subject.

2.1 Theoretical Review

2.1.1 Markowitz Portfolio Theory

Harry Markowitz received a Nobel-prize for his innovative and ground-breaking contributions to financial economics and corporate finance, which developed the underpinnings and formed the foundation of Modern Portfolio Theory. Modern Portfolio Theory (MPT) is an investment framework for the selection and construction of investment portfolios based on the simultaneous maximization of expected portfolio returns and the minimization of investment risk (Fabbozi, Gupta, & Markowitz, 2002). The foundation that Markowitz established was later expanded upon by William Sharpe, his fellow Nobel-prize co-winner, who is known mainly for the 'Capital Asset Pricing Model'.

Markowitz (1952) stated that under certain conditions, an investor's portfolios selection can be reduced to balancing the expected return and the risk of the portfolio. He also stated that portfolio selection should not be based on compiling securities with individually attractive risk-reward opportunities to form a portfolio but on overall risk-reward opportunities portrayed by the portfolio. One of the most important aspect of Markowitz' work was his description of diversification and the impact of portfolio diversification in a portfolio by the number of securities and the relationships of the securities' covariance (Megginson, 1996).

James Tobin, an economist, later derived the 'Efficient Frontier' and the 'Capital Market Line' based on Markowitz' works in 1958. Tobin (1958) suggested that investors, despite their risk tolerance levels, would maintain stock portfolios in the same

proportions, as long as they maintain similar expectations regarding the future and thus their portfolios will only differ in the relative proportions of stocks and bonds. Based on Tobin's and Markowitz' work, William Sharpe, John Lintner and Jan Mossin later developed the Capital Asset Pricing Model (CAPM) which contributed to the theory of capital markets equilibrium and enabled investors to value securities as a function of systematic risk.

Sharpe (1964) advanced the efficient Frontier and the Capital Market Line concepts in his derivation of CAPM. Sharpe's asset pricing theory is regarded to as a 'positive theory' unlike Markowitz' portfolio selection theory, which is a normative theory. The positive theory (CAPM) hypothesizes how investors actually behave while the normative theory (Markowitz' portfolio selection') hypothesizes how investors should behave. Lintner (1965) later derived the CAPM from the perspective of a corporation that had issued shares of stock. Mossin (1966) also derived the CAPM explicitly specifying quadratic utility functions.

2.1.1.1 Risk and Return

The risk of a security can be analysed either in a stand-alone basis or a portfolio basis (Mangram, 2013). Markowitz' portfolio selection theory explains that the most important aspect of the risk of an asset, is not the asset in isolation, but the contribution of each asset to the risk of the aggregate portfolio. MPT assumes that in a portfolio, total risk can be divided into two basic components: systematic risk, also known as market risk or common risk, and unsystematic which is also known as diversifiable risk.

In MPT, risk is synonymous with volatility, which refers to the amount of uncertainty related to changes in the value of a security. Volatility can be measured by a number of portfolio tools including; expected return, variance of an expected return, the standard deviation from an expected return, the covariance of a portfolio of securities and the correlation between investments (Ross, Westerfield, & Jaffe, 2002).

2.1.1.2 Risk-Return Tradeoff

Markowitz stated that the riskier the investment, the greater the required potential return, therefore, an investor will only invest in a risky security only if the expected return is high enough to sufficiently compensate them for assuming the risk (Ross, Westerfield,

& Jaffe, 2002). If investors are willing to bear risk, they expect to receive a risk premium, which is the return in excess of the risk-free rate, that the investment is expected to yield. Mangram (2013) explains that the risk-return tradeoff points only to the possibility of a higher return on investments but does not guarantee a higher return and as such, sometimes risky investments do not pay more than a risk-free investment.

2.1.1.3 Diversification

Diversification, a cornerstone of Markowitz' portfolio theory, is defined as a risk reduction concept that involves the allocation of investments among various financial instruments, industries and other investment categories. Simply, diversification can be achieved by investing in different stocks, different asset classes such as bonds and real estate and commodities such as gold and oil (Mangram, 2013). Mangram (2013) explains that the main objective of diversification is to minimize risk and maximize return by investing in different assets that would react differently to the same events. The Diversification effect refers to the relationship between portfolio risk and correlations, in that, when the correlation between assets is imperfect, the result is the diversification effect (Gibson, 1990).

Markowitz (1952) argues that diversification cannot eliminate all risk as it cannot eliminate or reduce systematic risk which comes from external factors such as war, inflation and recessions. However, diversification does reduce the unsystematic risk of securities in a portfolio. Practically, since the returns on different assets are correlated only to some degree, unsystematic risk cannot be completely eliminated regardless of how many types of assets are compiled in a portfolio.

2.1.1.4 Efficient Frontier

The Markowitz Efficient Frontier represents the best combination of securities, that offer maximum expected return at a given level of risk, within a portfolio. Portfolios that lie on the efficient frontier represent the best possible combination of expected return and investment risk. The optimal portfolios that lie along the curve represent the highest expected return for the given amount of risk.

The relationship between the securities in a portfolio affects the risk within the portfolio. The greater the covariance, the smaller the risk within a portfolio. Markowitz' theory

implies that rational investors seek out portfolios that generate the largest possible return with the least amount of risk, which are the portfolios represented on the efficient frontier.

2.1.2 Portfolio Optimization

Portfolio optimization has gradually evolved from Markowitz' perspective of the 'return-variance risk management framework'. Developments in this field are stimulated by adequate modeling of utility functions, risks and constraints as well as the ability to handle large numbers of instruments and scenarios (Krokhmal, Palmquist, & Uryasev, 2001).

In the context of CAPM, Mean-Variance Optimization is used to calculate sharpe ratios and to determine the optimal portfolio given a number of risky assets (Eisl, Gasser, & Weinmayer, 2015). However, the limitation is that the mean-variance approach requires that returns follow a normal distribution. This allows the use of variance as a risk measure, otherwise, the variance is likely to underestimate the potential loss resulting from additional tail-risk and lead to sub-optimal portfolio decisions (Jorion, 2001; McNeil, Frey, & Embrechts, 2005). Due to non-normality, a risk measure that captures downside risk is preferred.

As a result, numerous alternatives to the classical Mean Variance Optimization (MVO) approach have emerged in the literature. Markowitz (1959) examined semi-variance, expected loss, expected absolute deviation, probability of loss and maximum loss as alternative measures of risk. Markowitz H. M. (2010) introduced ; Value-at-Risk (VaR) and Conditional Value-at-Risk Optimization (CvaR) as alternative measures of risk and stated that CvaR is the best alternative if executed properly.

2.1.2.1 Mean-Variance Optimization (MVO)

The mean-variance optimization framework has dominated the asset allocation process for over 50 years (Xiong & Idzorek, 2010). Markowitz (1952) proposed mean-variance both in a positive sense and normative sense, however, no rationale for the normative use of mean-variance was given. However, Markowitz (1959) argued that under certain conditions, a carefully chosen portfolio from a mean-variance efficient frontier will maximize the investors' expected utility.

In the Mean-Variance model, return and risk are estimated from the sample mean and sample variance of returns. It only focuses on the first two moments of the return distribution. The risk of the portfolio is measured by standard deviation. (Prapinmongkolkarn, 2008) explains that due to the use of standard deviation, mean-variance portfolio selection holds two assumptions. It assumes that the portfolio returns follow an elliptically symmetric distribution such as normal distribution or Student t-distribution. It also assumes that the utility of an investor is quadratic. The main objective is to minimize the risk for a given target return.

The Mean-Variance optimization is as follows;

Let \mathbf{w} represent a vector of asset weights, Σ the covariance matrix of the security returns, and \mathbf{R} a vector of expected returns. In order to form an efficient frontier of portfolios with optimal risk-return profiles, for a given risk tolerance q , minimize, using the dual method;

$$\mathbf{w}^T \Sigma \mathbf{w} - q \cdot \mathbf{R}^T \mathbf{w}$$

Subject to constraints:

$$\sum_{i=1}^N w_i = 1$$

$$0 \leq w_i \leq p$$

Where p is some maximum weight that any given asset can take.

Criticisms of MVO

- i. Investor preferences go beyond mean and variance as they are also concerned with significant losses such as the downside risk (Xiong & Idzorek, 2010).
- ii. It is suitable for elliptical distributions but it may lead to incorrect conclusions in the case of non-elliptical distributions (Miskolczi, 2016). If used for non-elliptic distributions, it can severely underestimate extreme events that cause the worst losses (Szegő, 2002).

2.1.2.2 Mean-Conditional Value at Risk (M-CVaR)

In the Mean-CVaR model, the variance is replaced with the CVaR. CVaR is closely related to value-at-risk (VaR) which estimates the maximum potential loss that a portfolio can suffer at a given level of probability over a specified period of time. CVaR is calculated by taking a probability weighted average of the possible losses conditional on the loss being equal or exceeding the specified VaR (Rockafellar & Uryasev, 2000).

CvaR is also known as mean shortfall, tail VaR, or expected tail loss. It is a coherent measure of risk (Pflug, 2000 ; Rockafellar & S., 2000 ; Artzner, Delbaen, Eber, & D., 1999 ; Acerbi & Tasche, 2002).¹⁰ According to previous studies, CVaR portrays more appealing properties than VaR (Pflug, 2000 ; Rockafellar & S., 2000). VaR is defined as the maximum loss in a specified period with a probability level (Szegö, 2002). In 1996 and 2001, the Basel Committee on Banking Supervision chose VaR as the foundation of market risk regulations thus making VaR the standard of risk measure (Danielsson, 2002).

Many researchers have studied the use of VaR in portfolio optimization instead of using standard deviation (Alexander & Baptista, 2004; Benati & Rizzi, 2007; Gaivoronski & Pflug, 2004; Campbell, Huisman, & Koedijk, 2001). However, serious drawbacks of VaR were identified such as it is not an acceptable measure of risk when it is applied to non-elliptical return distributions (Szegö, 2002). VaR cannot measure losses exceeding VaR and an attempt to reduce VaR may actually lead to an increase in losses exceeding VaR (Prapinmongkolkarn, 2008). VaR is also non-smooth and a multi-extreme function with respect to positions, so it is difficult to use VaR in optimization problems.

VaR focuses on only one particular point on the distribution while CVaR is a more complete measure of risk relative to VaR. VaR is criticized for not satisfying one of the most desirable properties of risk measures, the sub-additivity property, ergo incoherent

¹⁰ For a risk measure to qualify as a 'Coherent measures of risk', it has to satisfy four properties: positive homogeneity, subadditivity, monotonicity and translational invariance (Artzner, Delbaen, Eber, & D., 1999). Some of the proposed risk measures include; spectral risk measures, expected regret (ER), tail conditional expectation (TCE), worst conditional expectation (WCE) and Expected Shortfall (ES) or Conditional Value-at-Risk (CVaR).

(Artzner, Delbaen, Eber, & D., 1999).¹¹ VaR also ignores the severity of losses in the far tail of the profit-and-loss distribution. It also has the non-convexity property which makes VaR impossible to use in optimization problems (Szegö, 2002), hence CvaR and Conditional Tail Expectation are used in Finance (Embrechts & Hofert, 2014).

VaR is only acceptable when the joint distribution of return is normal or elliptic. In this scenario, the VaR minimizing portfolio coincides with the Markowitz variance-minimizing portfolio (Prapinmongkolkarn, 2008). Szegö (2002) explains that VaR can only be used when the variance is computationally simpler and not in measuring risk for unusual distributions as was the intent at first. CVaR can remedy the drawbacks of VaR as it can measure tail risk and unlike VaR, CVaR is both sub-additive and convex (Rockafellar & S., 2000). For these reasons, CVaR is more preferred to VaR in portfolio optimization.

M-CvaR takes non-normal return characteristics into consideration and Unlike Mean-Variance optimization, the restriction on assets to have elliptical distributions is dropped. In general, CvaR prefers assets with positive skewness, small kurtosis, and low variance (Xiong & Idzorek, 2010).

Minimizing VaR

VaR can be defined in mathematical terms as shown below. Krokmal, Uryasev and Zrazhevsky (2002) defined $f(\omega, \mathbf{y})$ as the loss associated with the decision vector ω , which are chosen from a certain subset \mathbf{X} of \mathbf{R}^n , and the random vector \mathbf{y} in \mathbf{R}^m . Vector ω can be interpreted as a portfolio, with \mathbf{X} as the set of available portfolios, subject to various constraints and vector \mathbf{y} stands for the uncertainties (random variables) such as market prices.

Due to random variables \mathbf{y} , the loss $f(\omega, \mathbf{y})$ is a random variable with a distribution in \mathbf{R} . The probability distribution of \mathbf{y} , $p(\mathbf{y})$, is assumed to have density. The probability of $f(\omega, \mathbf{y})$ does not exceed a threshold ζ and can be defined as:

The cumulative distribution function $\psi(\omega, \zeta)$ of a loss $z = f(\omega, \mathbf{y})$ is:

¹¹ “Sub-additivity states that the risk the combination of two portfolios is at most the sum of their individual risks, which implies that diversification reduces risk or at least, does not increase risk.” (Ruppert, 2010). VaR discourages diversification and thus is an incoherent measure of risk.

$$\psi(\omega, \zeta) = P\{y | f(\omega, y) \leq \zeta\}$$

Or

$$\psi(\omega, \zeta) = \int_{f(x,y) \leq \zeta} p(y) dy$$

Where; w = decision vector/portfolio weights,

ζ = a specific loss,

y = uncertainties that affect the loss.

The cumulative distribution function $\psi(\omega, \zeta)$ is non-decreasing with respect (w.r.t) to ζ , and is assumed to be continuous w.r.t ζ . Therefore, the Value-at-risk for a given confidence level, $VaR_{\alpha, \{\zeta_{\alpha}(\omega)\}}$, can be defined as :

$$\zeta_{\alpha}(\omega) = \min\{\zeta \in R: \psi(\omega, \zeta) \geq \alpha\}$$

VaR_{α} , is the smallest value, such that probability that loss will be less than or equal to this value is equal to α , whereby this probability level or confidence level (α) equals to 90%, 95% and 99%.

The minimization of VaR can be defined as:

$$\min_{x \in X \subset R^n} \zeta_{\alpha}(\omega)$$

However, VaR is difficult to optimize because it is a non-smooth, non-convex and multiextreme function w.r.t. the decision variables ω (Larsen, Mausser, & Ursayev, 2002).

Minimizing CVaR

CVaR for Continuous Loss distributions

Larsen, Mausser and Ursayev (2002) defined CVaR for continuous distributions as the conditional expected value of losses under condition that the losses exceed VaR_{α} , which is denoted by $\zeta_{\alpha}(\omega)$. The $CVaR_{\alpha}$ function, $\phi_{\alpha}(\omega)$, is defined as:

$$\phi_{\alpha}(\omega) = \frac{1}{1 - \alpha} \int_{f(\omega, y) \geq \zeta_{\alpha}(\omega)} f(\omega, y) p(y) dy$$

Where: $f(\omega, y)$ is the loss function

ω is the decision vector to be chosen from a certain subset \mathbf{X} of \mathbf{R}^n

y is the random vector

$p(y)$ is a density function

$\zeta_\alpha(\omega)$ is VaR estimated at a confidence level α

The integral above can be approximated by transforming it into the function below which is convex and linear w.r.t. α . It can be minimized easily by a linear programming technique and CVaR can be easily estimated by minimizing the function below.

$$\tilde{F}_\alpha(\omega, \zeta) = \zeta + (1 - \alpha)^{-1} \sum_{j=1}^j \pi_j [f(\omega, y)_j - \zeta]^+$$

The Conditional Value-at-Risk ($CVaR_\alpha$) is the expected value of loss, given that the loss is weakly exceeding the Value-at-Risk ($\zeta_\alpha(\omega)$).

Rockafellar and Uryasev (2000) came up with a much simpler function that did not involve VaR in it.

$$F_\alpha(\omega, \zeta) = \zeta + (1 - \alpha)^{-1} \int_{y \in \mathbf{R}^n} [f(\omega, y) - \zeta]^+ p(y) dy$$

Where: $[f(\omega, y) - \zeta]^+ = \max[f(\omega, y) - \zeta, 0]$

It can be proved that (Rockafellar & Uryasev, 2000):

- i. The function $F_\alpha(\omega, \zeta)$ is convex w.r.t. ζ .
- ii. VaR is a minimum point of this function w.r.t. ζ .

$$A_\alpha(\omega) = \underset{\zeta \in \mathbf{R}}{\operatorname{argmin}} F_\alpha(\omega, \zeta)$$

and $\zeta_\alpha(\omega) = \text{left endpoint of } A_\alpha(\omega)$

$$\zeta_\alpha(\omega) \in \underset{\zeta \in \mathbf{R}}{\operatorname{argmin}} F_\alpha(\omega, \zeta)$$

- iii. Minimizing $F_\alpha(\omega, \zeta)$ w.r.t. ζ gives CVaR.

$$\phi_\alpha(\omega) = F_\alpha(\omega, \zeta_\alpha(\omega)) = \min_{\zeta \in R} F_\alpha(\omega, \zeta)$$

Minimizing the $CVaR_\alpha$ of the loss associated with ω over all $\omega \in X$ is equivalent to minimizing $F_\alpha(\omega, \zeta)$ over all $(x, \zeta) \in X \times R$ in the sense that;

$$\min_{x \in X} \phi_\alpha(\omega) = \min_{(x, \zeta) \in X \times R} F_\alpha(\omega, \zeta)$$

$F_\alpha(\omega, \zeta)$ can be easily minimized because of its convexity with respect to (ω, ζ) while $\phi_\alpha(\omega)$ may be hard to minimize because of its definition in terms of $VaR_{\alpha, \{\zeta_\alpha(\omega)\}}$, which is non-convex.

(Krokhmal, Uryasev, & Zrazhevsky, 2002) prove that solving the following three problems will achieve the same efficient frontier:

$$\begin{aligned} & \min_x \phi(\omega) - \mu R(\omega), \quad \mu \geq 0 \\ & \min_x \phi(\omega), \quad R(\omega) \geq p, \quad \omega \in X \\ & \min_x -R(\omega), \quad \phi(\omega) \leq \varpi, \quad \omega \in X \end{aligned}$$

Where: $\phi(\omega)$ is the CVaR risk function

$R(\omega)$ is the reward function

μ is the optimal reward multiplier

p is the required return

ϖ is the acceptable risk

By varying the parameters μ , p and ϖ , the three problems above will produce the same efficient frontier. The implications of the first two problems is that a portfolio that has minimum variance and that achieves the required return can be constructed. The last problem implies that a portfolio that has a minimum expected loss or maximum expected return and has acceptable risk can be constructed.

In simple terms, optimal portfolios are formed by minimizing $CVaR_\alpha$:

$$\min_{\omega} CVaR_\alpha(\omega)$$

Subject to:

$$\omega^T \hat{\mu} = \bar{r}$$

$$\omega^T \mathbf{1} = 1$$

Where $\hat{\mu}$ =vector of expected returns,

\bar{r} = expected total return of the portfolio

2.2 Empirical Review

Eisl, Gasser and Weinmayer (2015) explored the impact an investment in bitcoin can have on an already well-diversified investment portfolio. A Conditional Value-at-Risk Framework was adopted instead of the classic mean-variance approach due to the non-normal nature of bitcoin returns. In order to evaluate the diversification effect of including bitcoin into portfolios based on the mean-CVaR approach, the study adopted the view of a US investor and well diversified portfolios that included various broad indices for equity, fixed income, money market, commodity real estate and alternative investment opportunities were constructed from July 18, 2010 to April 30, 2015. A back-testing technique was also applied to calculate monthly out-of-sample portfolio returns and risk return ratios based on the CVaR. The results indicate that bitcoin should be included in optimal portfolios. It is noted that an investment in bitcoin increases the CVaR of a portfolio but this additional risk is overcompensated by high returns leading to better risk-return ratios. The results of the study were largely robust with regard to the optimization framework applied.

Gangwal (2016) conducted a research on the effects of adding bitcoin to a portfolio of an international investor. The portfolio consisted of stocks, bonds, Baltic index, MXEF, gold, real estate and crude oil and daily data available from July 2010 to August 2016 was utilized. Optimal portfolios are constructed to reflect the lucrative opportunities associated with investment in bitcoin which are unexplored. The study then replicated these portfolios of international investors holding a wide array of asset classes and added bitcoin via back-testing on the daily returns data. The study's conclusion was that adding bitcoin to the portfolio yielded a higher Sharpe ratio meaning that bitcoin's returns offset

its high volatility. The study states that the better risk adjusted return should be persuasive enough for people to start considering bitcoins as a new asset class.

Carpenter (2016) conducted a study on portfolio diversification with bitcoin. The study found that bitcoin exhibits low correlations in comparison to traditional assets and tends to have exceedingly high volatility and returns. In order to imitate the constraints of an average US investor, the paper assumed long only positions and no leverage and in order to determine the efficacy of a portfolio containing bitcoin and to avoid lookahead bias, a backtesting framework was implemented. The study used a modified Mean Variance framework to show that bitcoin is a viable diversification tool. The paper also stated that bitcoin's investment appeal could have been skewed by the return activity that occurred during a speculative bubble in 2013.

Klabbers (2017) investigated whether bitcoin as an investment asset offers diversification benefits and specifically whether bitcoin has hedge or safe haven properties. The international portfolio used includes several stock indices, several government bonds, a global commodity index and a global real estate index from 22 July, 2015. The study uses the Mean Variance framework in combination with the Monte Carlo Simulation to address the estimation of risk of bitcoin. The findings of the paper are consistent and show that bitcoin is an effective diversifier with a weight allocation of on average between 0% and 5%. In the study, bitcoin did not show any hedge or safe haven properties for a global market portfolio.

In a bid to analyse bitcoin from an investor's perspective, Schut (2017) constructed an efficient frontier with a wide variety of asset classes so as to determine what Bitcoin risk-return characteristics add to a diversified portfolio. A wide variety of U.S asset classes were used from January 2014 to July 2017. The results indicate that there is no place for bitcoin within the global minimum-variance portfolio, but there are certain benefits of adding bitcoin to an investment portfolio. The study found that bitcoin had a low correlation with the other assets thus providing high diversification opportunities but bitcoin still showed a level of volatility too high to be included in the lowest-risk portfolio. Approximately 2% of the weights in the optimal portfolio was attributed to bitcoin as the high volatility only allows for a small allocation to the asset class.

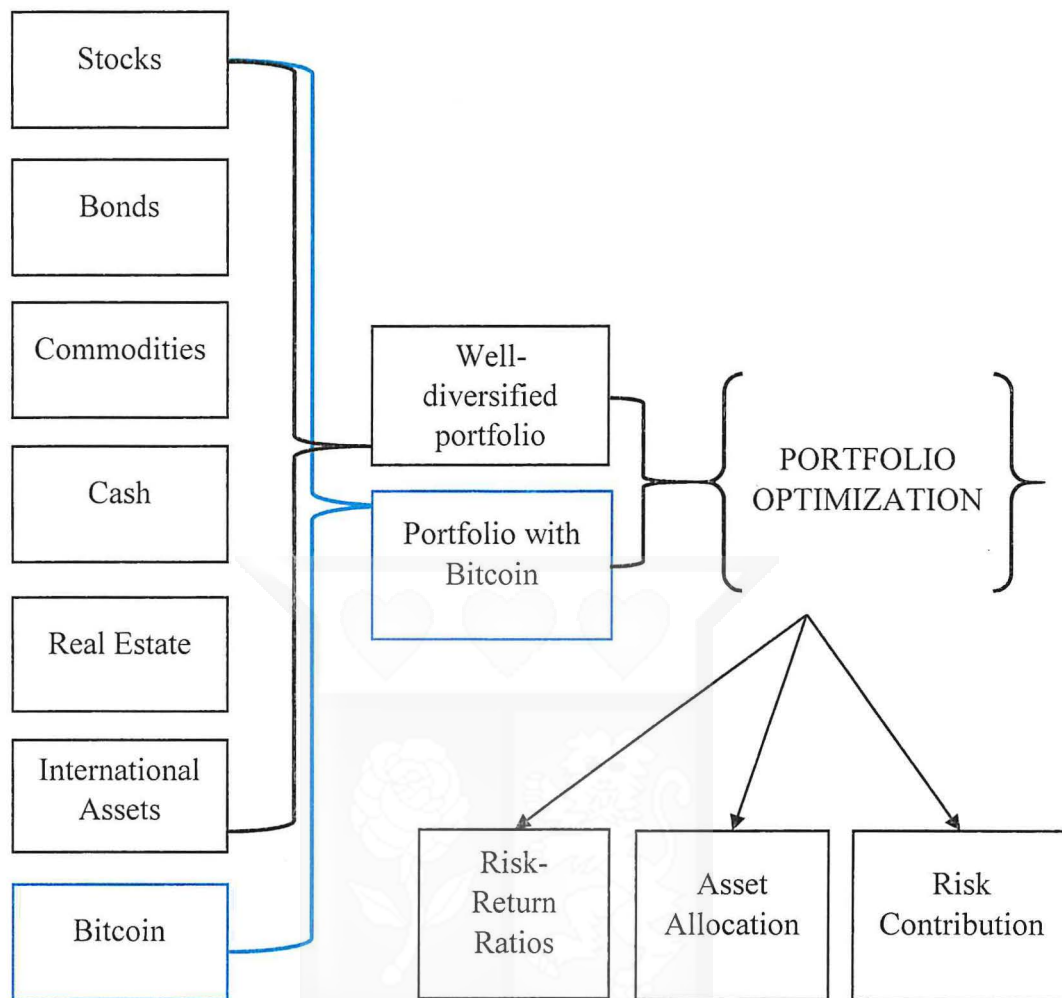
Contrary to Klabbers (2017) and Carpenter (2016), this research will not utilize MVO due to the erroneous assumption that stock returns are normally distributed and individuals have a quadratic utility function. Bitcoin's excess volatility implies a non-normal distribution thus the use of MVO should be discouraged. This study will utilize the Mean-Conditional VaR (M-CVaR) Optimization method.

2.3 Research Gap

The study aims to fill the knowledge gap on Mean-Conditional Value at Risk Optimization. The theory of the optimization method is explained in detail in the literature review section. Unlike Mean-Variance Optimization, which has been used in many publications to perform portfolio optimization, the Mean-Conditional VaR Optimization method allows for unique assets, which are not normally distributed, such as bitcoin to be assessed. Conditional VaR is used as the risk measure in place of variance which is used in Mean-Variance Optimisation. The use of M-CVaR optimization overcomes limitations of Mean-Variance Optimisation. M-CVaR optimization caters to the fact that investor preferences go beyond mean and variance as they are also concerned with significant losses such as the downside risk. M-CVaR optimization is also suitable for non-elliptical distributions unlike Mean-Variance optimization which leads to incorrect conclusions when used in non-elliptical distributions because it severely underestimates extreme events that cause the worst losses.

This study also acknowledges that similar studies that assess bitcoin as an investable asset have focused on portfolios with assets from developed markets. This study therefore aims to fill the knowledge gap by exploring investments in emerging markets. Investors perceive that emerging markets offer a higher return potential than developed markets. The portfolio utilized from this study is constructed assuming the position of a South African investor. There has also been limited academic research on bitcoin as an investable asset as majority of the publications concentrate on bitcoin from a technological or legal perspective. Research of bitcoin as a crypto-currency has also been done however, the research on bitcoin as an asset is under-explored. This study

aims to fill this knowledge gap by exploring bitcoin in a business science perspective



and providing the most essential details on the asset.

2.4 Conceptual framework

The conceptual framework depicts the diagrammatic representation of this study. This study assesses the impact of the inclusion of bitcoin in a well-diversified portfolio. The main objectives are to investigate how the inclusion of bitcoin to a well-diversified portfolio affects the risk-return ratios, the asset allocation and the risk contribution. In order to make a proper conclusion on the impact of bitcoin inclusion, the well-diversified portfolio is the control group while the portfolio with bitcoin is the experimental group.

CHAPTER THREE

3. RESEARCH METHODOLOGY

3.0 Introduction

This chapter outlines the proposed research methodology of the research study. It involves a blueprint for the collection, measurement and analysis of data. This section of the study indicates the procedures and techniques that will be used in the collection, processing and analysis of data.

3.1 Research Design

The aim of this study is to investigate the impact of the inclusion of bitcoin on a well-diversified investment portfolio. To achieve the proposed research objective, this study followed an experimental design. The experimental group was the portfolio with bitcoin while the control group was the well-diversified portfolio. Bitcoin is administered to the well-diversified portfolio so as to investigate the impact of its inclusion. The two portfolios namely; the portfolio with bitcoin and the portfolio without bitcoin, are measured on the same variables namely; risk-return ratios, the asset allocation and risk contribution.

This study utilized time series data as the data was collected across time and the data is spaced at equal intervals of time. The study is predominantly quantitative in nature as it involves utilizing data that is measurable.

3.2 Population and Sampling

The target population of this study comprised all emerging markets in the world. Based on the data available from MSCI Emerging Market Index, the number of emerging countries in the world is 24. These countries include, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Russia, Qatar, South Africa, Taiwan, Thailand, Turkey and United Arab Emirates. The South African market, which is in Sub-Saharan Africa, is used in this study.

3.3 Data Collection

Data

To achieve the objective of this study, secondary data of various asset classes in the South African Market was utilized. In order to identify the appropriate asset classes that would be used to create a well-diversified portfolio, a sample of five diversified portfolios managed by large and prominent institutions was observed. Growth funds and income funds were excluded since majority of the assets are bonds with varying maturity

| <i>Fund Managers</i> | <i>Fund Name</i> | <i>Equity</i> | <i>Bonds</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Commodities</i> | <i>Int. Assets</i> |
|------------------------|------------------|---------------|--------------|-------------|--------------------|--------------------|--------------------|
| Allan Gray Unit Trust | Balanced Fund | 46.70% | 9.50% | 9.70% | 0.90% | 4.30% | 27.00% |
| Coronation Management | Balanced Plus | 42.80% | 13.60% | 3.40% | 13.20% | 0.20% | 24.20% |
| Investec Fund Managers | Managed Funds | 28.80% | 6.60% | 36.90% | 0% | 1.90% | 25.80% |
| Old Mutual Unit Trust | Balanced Fund | 38.60% | 7.40% | 15.50% | 7.30% | 2% | 29.20% |
| STANLIB | Balanced Fund | 30.08% | 11.08% | 27.82% | 0% | 2.28% | 14.66% |

periods.

Table 1: Sample of Asset Allocation by Top Institutional portfolios in South Africa

Fund Managers' Asset Allocation in South Africa

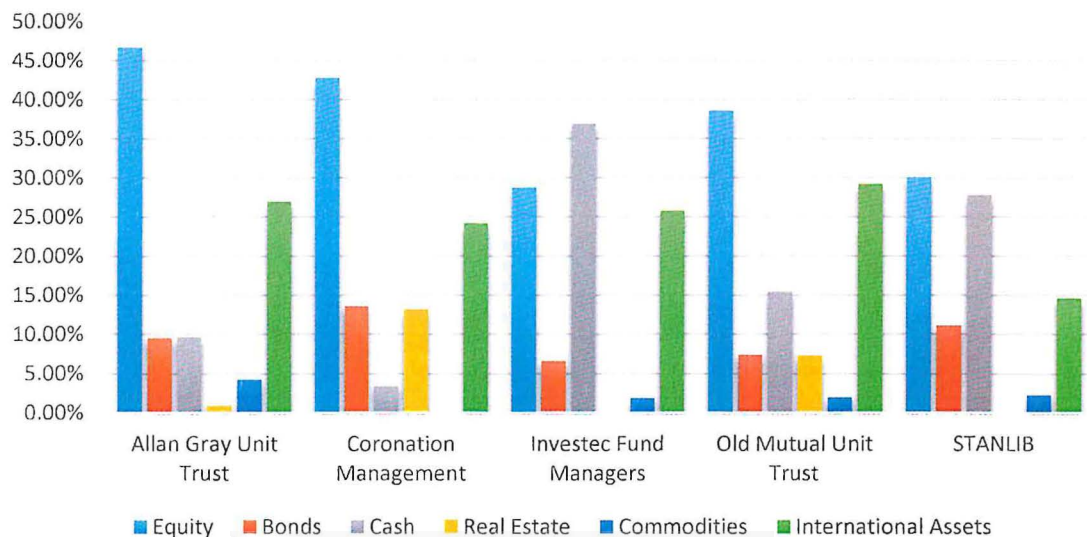


Figure 1: Fund Managers' Asset Allocation in South Africa

As seen above, most South African institutions focus their asset allocation on the following asset classes; equity, international assets, cash, bonds, real estate and commodities, in which gold has the highest percentage. Therefore, following evidence that a typical South African Investment portfolio would consist of the above listed asset classes, this study utilized a portfolio with the above mentioned assets.

The following indices were utilized to construct the well-diversified portfolio; The FTSE/JSE All Share Index was used to represent equities, the SA 10-YR Index to represent bonds and the South Africa Housing Index to represent real estate. Commodities were represented by gold and cash was represented by the Euro-money South Africa. The S&P 500 Index was used as a representation of international assets. All the aforementioned data was sourced from Investing.com, a reliable source.

For Bitcoin data, the study uses the CoinDesk Bitcoin USD Price Index, which is a simple average of global Bitcoin/USD exchange prices. Since bitcoin's historical price data becomes available on July 23, 2010 on CoinDesk, the study will examine monthly returns from July 23, 2010 to December 29, 2017. All the data was carefully collected through examination of the sources of data to ensure that the information gathered from the afore-mentioned sources was credible and reliable.

3.4 Data Analysis

In order to meet the objective of this study, which is to analyse the impact of an inclusion of bitcoin into an already well-diversified portfolio, the following procedure was followed so as to form a deduction.

3.4.1 Estimation Procedure

Asset returns were calculated. This study utilized logarithmic returns, which is the preferred method for return calculations in finance.

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where: R_t is the return of the asset class at time t

P_t is the closing price of the asset at time t

P_{t-1} is the closing price of the asset class at time $t-1$

Once the asset returns were achieved, the descriptive analysis of these returns is conducted. The descriptive statistics were captured from all the assets used in the portfolio optimization frameworks and they included; mean, median, standard deviation, variance, skewness, kurtosis, minimum and maximum. This step was very essential as it provided a first-glance nature of the data and it provided valuable information about the data.

The stylized facts of the time series data were also analysed so as to know the nature of the returns. Normality tests using the Shapiro-Wilk Normality Test was conducted for the above stated purpose.

3.4.2 Portfolio Optimization with CVaR

Conditional Value-at-Risk (CVaR) can either be objective or constraint of the optimization problem (Rockafellar & Uryasev, 2000). As the objective of the optimization problem, the goal is to minimize the risk of the portfolio, which is measured by CVaR given a required expected return. As the constraint, the goal is to maximize the expected return of the portfolio given an acceptable level of risk as measured by CVaR.

Boudt, Ardia, Mullen, Carl, and Peterson (2010) proposed several portfolio allocation strategies that use an appropriate transformation of the portfolio Conditional Value at Risk (CVaR) budget as a-n objective or constraint in the portfolio optimization problem. Risk budgets are used as tools to estimate and manage risk allocation of portfolios. This study aims to create risk allocation optimized portfolios using the constraints proposed by Boudt et al. (2010). To achieve the objectives of this study, the software application R was utilized.

3.4.2.1 Setting the Objective Functions

Weight Constraints

This study utilized basic weight constraint types namely; the leverage and box constraints. Leverage constraints specify that the sum of the weights have to be between a certain minimum and a certain maximum. Box constraints on the other hand specify that the individual asset weights have to be between a certain minimum and a certain maximum. Given the weight constraints, the value of the function to be minimized is called (Boudt, Ardia, Mullen, Carl, & Peterson, 2010).

Risk Allocation Constraints

This study also specified several risk allocation constraints that restrain the percentage contribution of risk that a single asset class can contribute. The constraints are utilized to constrain the risk contribution to a specified percentage of total risk. The specified percentage could be 30% or 40% for example (Boudt, Ardia, Mullen, Carl, & Peterson, 2010). This study utilized a 30% constraint on one of the portfolio frameworks.

3.4.3 Portfolio Optimization

Dynamic rebalancing of the risk budget optimised portfolio was utilized in this study. The aim of rebalancing was to realign the weightings of the different assets in the portfolio. The rebalancing was done through moving window estimation, using a 12-months moving average.

3.4.3.1 Portfolio Optimization Frameworks

In order to evaluate the effect of including bitcoin into a well-diversified portfolio, various portfolio frameworks are utilized. These frameworks are utilized to construct different portfolios.

Framework 1: Minimum CVaR Concentration portfolio

Under the minimum CVaR portfolio framework, optimization was applied to determine the optimal portfolio allocation that minimized the 95% portfolio CVaR. Using optimization, the best asset allocation of the portfolio was achieved.

Framework 2: Minimum CVaR portfolio under an upper 30% CVaR allocation constraint

Under the minimum CVaR portfolio under an upper 30% CVaR allocation constraint framework, optimization was applied to determine the optimal portfolio allocation that minimizes the 95% portfolio CVaR subject to the 30% CVaR allocation constraint.

The constraint value was settled on after observing that some of the assets in the Minimum CVaR Concentration portfolio, were responsible for contributing more than 40% to the portfolio CVaR. The aim of the 30% constraint is to hinder any asset from contributing more than 30% to total portfolio risk.

3.4.4 Performance Evaluation

The return and risk measures of the different portfolios was computed and analysed. The return and risk measures of the different portfolios was used to evaluate the impact of the inclusion bitcoin into a well-diversified portfolio. This evaluation was used to answer whether the inclusion of bitcoin would increase or reduce the risk and return ratios of the different portfolio frameworks.

The different asset allocation of portfolios was also assessed utilising both portfolios with bitcoin and those without bitcoin. The main focus was the weight allocation of all the asset classes, especially bitcoin, given its unique nature. This evaluation was used to answer how the inclusion of bitcoin to a well-diversified portfolio would affect the asset allocation of a well-diversified portfolio.

The risk contribution of the assets in portfolios was also assessed by analysing the different portfolio frameworks. The main focus was to evaluate how much risk each asset class contributed to the portfolio, especially bitcoin. This evaluation was used to

answer how the inclusion of bitcoin to a well-diversified portfolio would alter the risk contribution of the asset classes.



CHAPTER FOUR

4. DATA RESULTS AND ANALYSIS

4.0 Introduction

This chapter presents and discusses the findings of the collected data and aims to answer the research questions outlined in the study.

First, descriptive statistics of the asset returns are presented. The optimal portfolios of portfolio optimization using dynamic rebalancing are presented. The impact of the inclusion of bitcoin is assessed using a comparison of the different portfolio frameworks, with and without bitcoin. The assessment of the inclusion of bitcoin on a well-diversified portfolio aims to answer the research objectives of this study. Results of how the return and risk measures of an already diversified portfolio are affected by the inclusion of bitcoin, how the asset allocation of a well-diversified portfolio is affected by the addition of bitcoin and how the risk contribution of assets in a well-diversified portfolio is affected by the inclusion of bitcoin are presented.

4.1 Descriptive statistics

Table 2, 3 and 4 provide descriptive statistics on all assets used in the portfolio optimization frameworks. Table 2 includes descriptive statistics of all the asset classes apart from bitcoin.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- Dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> |
|--------------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|
| Mean | 0.0088 | 0.0012 | -0.0033 | -0.0003 | 0.0046 | 0.0115 |
| Median | 0.0106 | 0.0000 | -0.0035 | 0.0074 | 0.0042 | 0.0115 |
| Standard Deviation | 0.0330 | 0.0482 | 0.0469 | 0.0646 | 0.0040 | 0.0290 |
| Variance | 0.0011 | 0.0023 | 0.0022 | 0.0042 | 1.6E-05 | 0.0008 |
| Kurtosis | -0.5448 | 2.0848 | -0.0989 | 0.3969 | 0.4590 | 1.0399 |
| Skewness | -0.0187 | 0.6788 | -0.0132 | 0.0044 | -0.6399 | -0.0370 |
| Minimum | -0.0660 | -0.1270 | -0.1292 | -0.1588 | -0.0071 | -0.0647 |
| Maximum | 0.0894 | 0.1779 | 0.1042 | 0.1586 | 0.0116 | 0.1023 |

Table 2: Descriptive Statistics

From the descriptive statistics one can observe several characteristics of the asset classes. In particular, the mean and median values of the asset returns approximate zero. This observation implies that the return series are symmetrical. The skew values range between -0.6399 (real estate) and 0.6788 (bonds) which also show that the data is symmetrical. The kurtosis values range between -0.5448 (stocks) and 2.0848 (bonds).

Table 3 provides descriptive statistics on bitcoin which are relatively exceptional. Bitcoin varies from different asset classes in that it has a significantly higher standard deviation (0.3141) and variance (0.0986). These exceptionally high values reflect the risk in bitcoin investment. It has the highest kurtosis (8.5639) and is positively skewed (2.1514).

| | <i>BTC</i> |
|--------------------|------------|
| Mean | 0.1159 |
| Median | 0.0814 |
| Standard Deviation | 0.3141 |
| Variance | 0.0986 |
| Kurtosis | 8.5639 |
| Skewness | 2.1514 |
| Minimum | -0.4298 |
| Maximum | 1.7456 |

Table 3: Bitcoin Descriptive Statistics

The high kurtosis values expresses the presence of significant extreme risks. The high skew value of bitcoin can be compared to the high skew values of sophisticated strategies such as volatility investments which are used to hedge financial portfolios against crises (Briere, Burgues, & Signori, 2010). Bitcoin also has the highest range (-0.4298 to 1.7456) which exhibits how volatile it is compared to other asset classes. The results therefore show that bitcoin could act as a partial hedge against crises, however more research should be conducted on this.

A GRAPH OF THE PORTFOLIO RETURNS

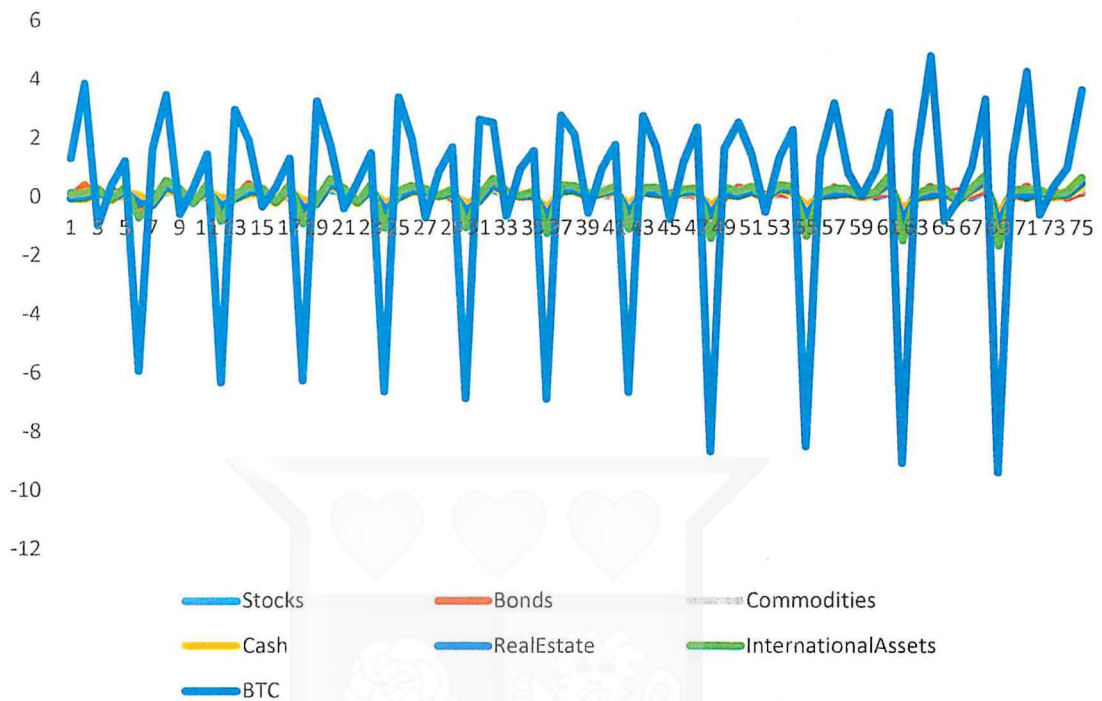


Figure 2: Portfolio Returns inclusive of Bitcoin

Table 4 presents the bitcoin returns correlations with the other assets returns. The table indicates that bitcoin has low correlations with the other asset classes. Bitcoin is negatively correlated with stocks, commodities and cash. The highest correlation is between bitcoin and international assets (0.2319). Generally, bitcoin correlation coefficients are exceptionally small. This implies that bitcoin has potential to act as a diversifier in this portfolio.

Figure 2 visually presents the return series of the different asset classes, which are stocks, bonds, commodities, cash, real estate, international assets and bitcoin. The figure shows the monthly return series of bitcoin compared to those of the other asset classes. Bitcoin's volatile nature is evident from the figure above.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- Dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> | <i>BTC</i> |
|------------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|------------|
| Stocks | 1.0000 | -0.2470 | 0.3201 | -0.0671 | 0.0260 | 0.4281 | -0.1152 |
| Bonds | -0.2470 | 1.0000 | -0.2774 | 0.1122 | 0.1442 | -0.1313 | 0.0384 |
| Commo- Dities | 0.3201 | -0.2774 | 1.0000 | 0.0055 | -0.0216 | 0.0851 | -0.0328 |
| Cash | -0.0671 | 0.1122 | 0.0055 | 1.0000 | -0.0339 | 0.0538 | -0.0148 |
| Real Estate | 0.0260 | 0.1442 | -0.0216 | -0.0339 | 1.0000 | -0.0707 | 0.0899 |
| Int. Assets | 0.4281 | -0.1313 | 0.0851 | 0.0538 | -0.0707 | 1.0000 | 0.2319 |
| BTC | -0.1152 | 0.0384 | -0.0328 | -0.0148 | 0.0899 | 0.2319 | 1.0000 |

Table 4: Correlation Matrix of the Asset Classes

Figure 3 visually presents the correlation of the different asset classes.

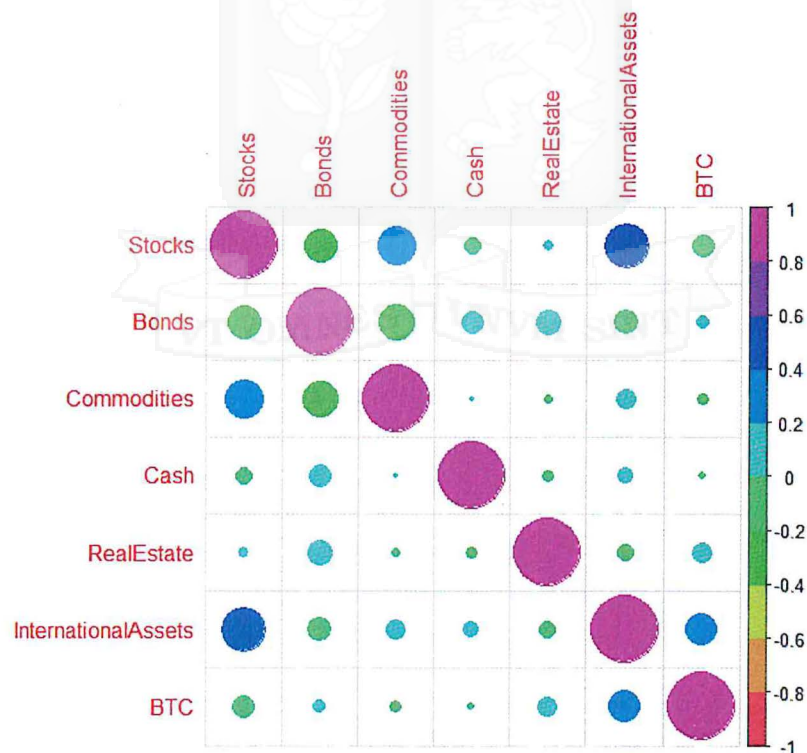


Figure 3: Correlation plot of the Return Series

Table 5 shows normality tests of the asset classes. This study utilizes the Shapiro-Wilk Normality test. The null hypothesis is that the data is normally distributed. The null hypothesis is rejected if the alpha or p-value is less than 0.05, which is the significance level of this study.

| | <i>W</i> | <i>p-value</i> |
|-------------|----------|----------------|
| Stocks | 0.9918 | 0.917 |
| Bonds | 0.9533 | 0.008 |
| Commodities | 0.9925 | 0.942 |
| Cash | 0.9771 | 0.190 |
| Real Estate | 0.9559 | 0.011 |
| Int. Assets | 0.9770 | 0.189 |
| BTC | 0.8394 | 1.49E-07 |

Table 5: Shapiro-Wilk Normality Test

The p-values of stocks, commodities, cash and international assets are greater than the significance level therefore the null hypothesis fails to be rejected. However, the p-values of bonds, real estate, and bitcoin are less than the significance level therefore the null hypothesis is rejected. This reaffirms the lack of use of mean-variance optimization as it assumes that all the data is normally distributed. The different portfolio frameworks are used to present the different risk and return measures, asset allocation of the asset classes and the risk contribution of each of the assets. There are two Minimum CVaR Concentration portfolios; one with bitcoin and one without and similarly two Minimum CVaR Portfolios under an upper 30% CVaR allocation Constraint; one with bitcoin and one without.

4.2 Portfolio Optimization

4.2.1 Risk and Return Measures

A comparison of the risk and return measures is conducted so as to analyse how the inclusion of bitcoin impacts the different portfolios.

4.2.1.1 Minimum CVaR Concentration portfolio

Table 6 presents that the return of the portfolio with bitcoin, which has a higher return than the portfolio without bitcoin. It also shows that the risk of the portfolio with bitcoin is higher as well. Therefore, the inclusion of bitcoin in a portfolio increases the return of a portfolio as well as the risk.

| | <i>Without BTC</i> | <i>With BTC</i> |
|---|------------------------|---------------------|
| Annualized Portfolio Rebalancing Return | 0.0501 | 0.1527 |
| Annualized Portfolio Standard Deviation | 0.0540 | 0.1091 |

Table 6: Comparison of the Min CVaR Concentration Portfolios Return and Risk Measures

Figure 4 represents the visual representation of the return and risk measures of the Minimum CVaR Concentration Portfolio.

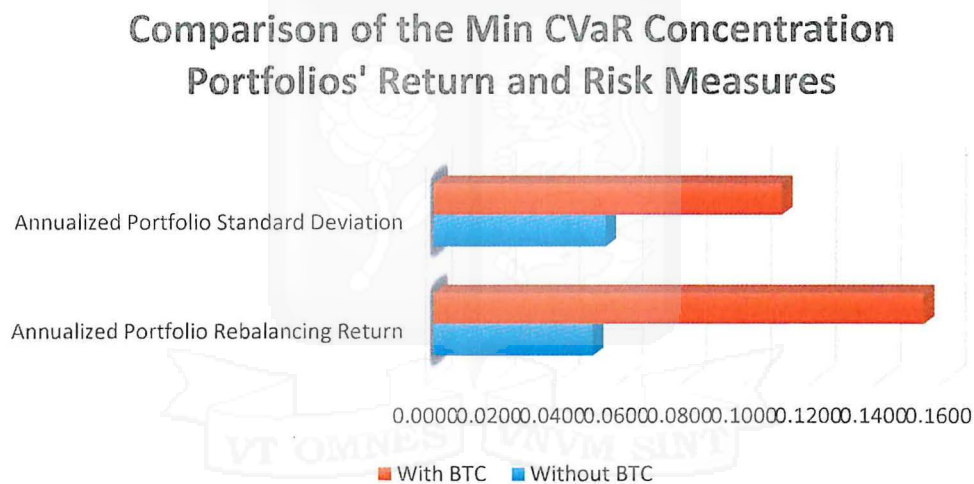


Figure 4: Comparison of the Min CVaR Concentration Portfolios Return and Risk Measures

Table 7 shows the downside risk measures of the two portfolios. It is evident that the downside risk measures of the portfolios with bitcoin, with the exception of modified VaR, are higher than those of the portfolio without bitcoin. This expresses that the inclusion of bitcoin exposes the portfolio to more risk making the portfolio risky.

| <i>Downside Risk Measures:</i> | <i>Without BTC</i> | <i>With BTC</i> |
|--------------------------------|------------------------|-----------------|
| Semi Deviation | 0.0111 | 0.0175 |
| Gain Deviation | 0.0104 | 0.0295 |
| Loss Deviation | 0.0101 | 0.0129 |
| Downside Deviation (MAR=10%) | 0.0134 | 0.0151 |
| Downside Deviation (Rf=0%) | 0.0091 | 0.011 |
| Downside Deviation (0%) | 0.0091 | 0.011 |
| Maximum Drawdown | 0.0488 | 0.0537 |
| Historical VaR (95%) | -0.0228 | -0.0302 |
| Historical ES (95%) | -0.0282 | -0.0348 |
| Modified VaR (95%) | -0.0212 | -0.0205 |
| Modified ES (95%) | -0.0292 | -0.0556 |

Table 7: Comparison of the Min CVaR Concentration Portfolios Downside Risk Measures

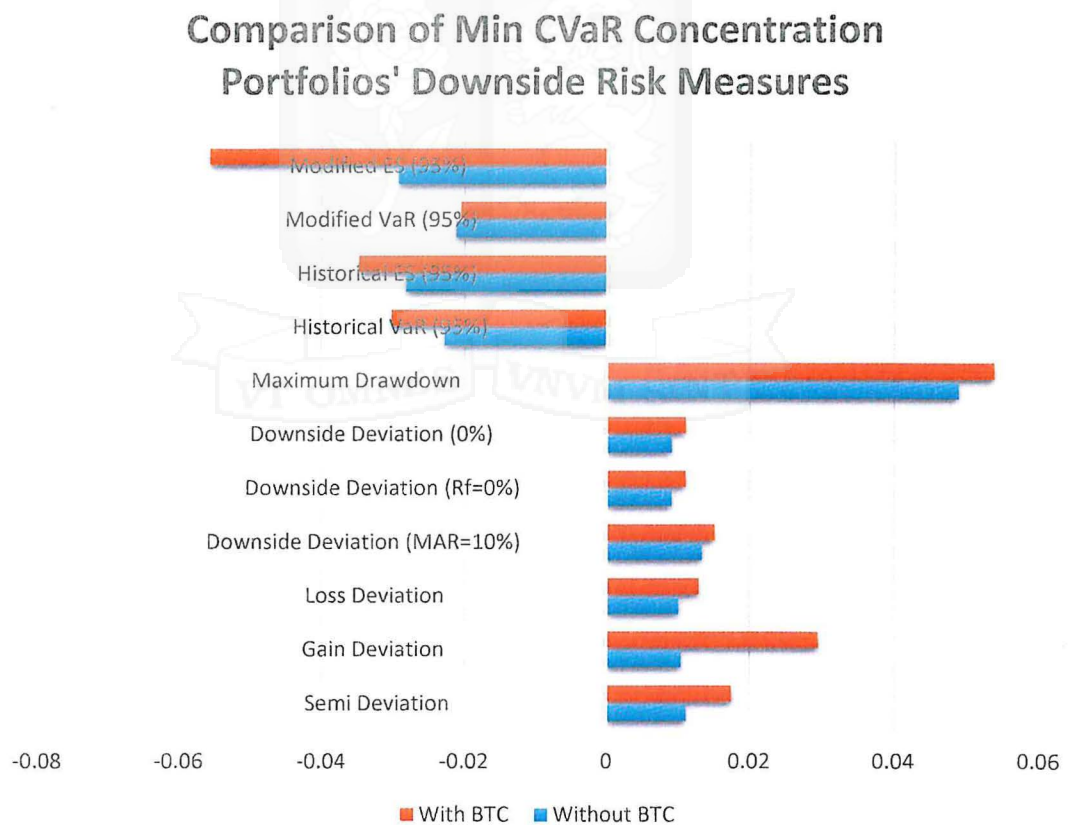


Figure 5: Comparison of the Min CVaR Concentration Portfolios Downside Risk Measures

Figure 5 provides a visual presentation of the comparison of the downside risk measures of the portfolio with and without bitcoin.

4.2.2.2 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint

Table 8 represents the return and risk measures of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with and without bitcoin. The return and risk measures of the portfolio with bitcoin are slightly higher than the portfolio without bitcoin.

| | <i>Without BTC</i> | <i>With BTC</i> |
|--|------------------------|-----------------|
| Annualized Portfolio Rebalancing Return: | 0.0633 | 0.0789 |
| Annualized Portfolio Standard Deviation: | 0.0453 | 0.0567 |

Table 8: Comparison of the Minimum CVaR Portfolios under an upper 30% CVaR allocation Constraint Risk and Return Measures

Figure 6 is a simple visual presentation of the risk and return measures of the Minimum CVaR Portfolios under an upper 30% CVaR allocation Constraint.

Comparison of the Risk and Return Measures of the Min CVaR Portfolios Under an Upper 30% CVaR allocation Constraint



Figure 6: Min CVaR Portfolio under an upper 30% CVaR allocation Constraint Risk and Return Measures

Table 9 represents a comparison of the downside risk measures of the portfolio with bitcoin and the portfolio without bitcoin. The downside risk measures of the portfolio with bitcoin, with the exception of modified expected shortfall, are greater than those of

the portfolio without bitcoin. This expresses more risk to the portfolio that contains bitcoin.

| <i>Downside Risk Measures:</i> | <i>Without BTC</i> | <i>With BTC</i> |
|--------------------------------|------------------------|-----------------|
| Semi Deviation | 0.0097 | 0.012 |
| Gain Deviation | 0.0086 | 0.0104 |
| Loss Deviation | 0.0114 | 0.0086 |
| Downside Deviation (MAR=10%) | 0.0114 | 0.0129 |
| Downside Deviation (Rf=0%) | 0.0076 | 0.0088 |
| Downside Deviation (0%) | 0.0076 | 0.0088 |
| Maximum Drawdown | 0.0462 | 0.0508 |
| Historical VaR (95%) | -0.014 | -0.0231 |
| Historical ES (95%) | -0.0259 | -0.027 |
| Modified VaR (95%) | -0.0181 | -0.021 |
| Modified ES (95%) | -0.0319 | -0.0284 |

Table 9: Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint Downside Risk Measures

A comparison between the Minimum CVaR Concentration Portfolio and the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint shows that after the inclusion of bitcoin, the latter framework is able to constrain the risk of the portfolio. The former has a standard deviation of (0.1091) while the latter has a standard deviation of (0.0567) implying less risk.

Figure 7 visually represents a comparison of the downside risk measures of the portfolio with bitcoin and the portfolio without bitcoin.

Comparison of the Min CVaR Portfolios under an upper 30% CVaR allocation Constraint Downside Risk Measures

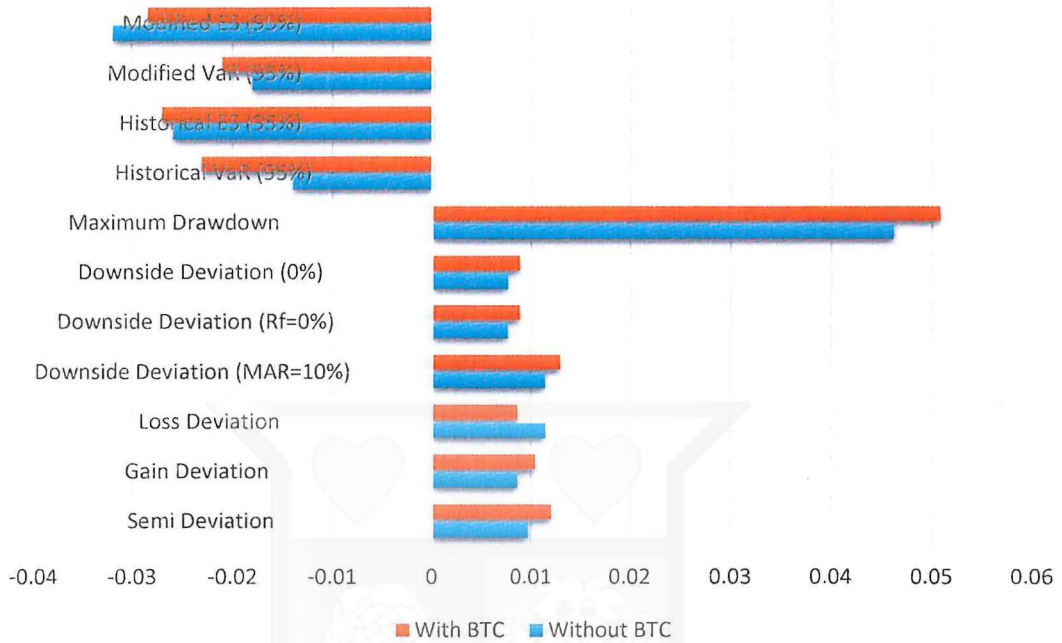


Figure 7: Comparison of the Min CVaR Portfolios under an upper 30% CVaR allocation Constraint Downside Risk Measures

4.2.2 Asset Allocation

A comparison of the asset allocation is conducted so as to analyse how the inclusion of bitcoin impacts the different portfolios.

4.2.2.1 Minimum CVaR Concentration portfolio

Tables 10 and 11 display the weights of the Minimum CVaR concentration portfolio with and without bitcoin over the different rebalancing dates.

4.2.2.1.1 Minimum CVaR Concentration portfolio without Bitcoin

In this portfolio, it is evident that international assets consistently have the highest weight allocation all through the rebalancing dates, with a range of 22% to 42%. Stocks come second with consistently higher weight allocation than the other assets, with a range of 14% to 24%.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|
| 2013-12-01 | 0.2398 | 0.1184 | 0.0620 | 0.0800 | 0.2296 | 0.2611 |
| 2014-12-01 | 0.1386 | 0.2282 | 0.0720 | 0.0941 | 0.0495 | 0.4179 |
| 2015-12-01 | 0.1820 | 0.1884 | 0.1300 | 0.0660 | 0.1060 | 0.3180 |
| 2016-12-01 | 0.2167 | 0.2057 | 0.1097 | 0.0316 | 0.2124 | 0.2242 |
| 2017-12-01 | 0.2040 | 0.1140 | 0.1560 | 0.1360 | 0.0000 | 0.3800 |

Table 10: Weights of the Minimum CVaR Concentration Portfolio without Bitcoin

Figure 8 presents the visual presentation of the above asset allocation.

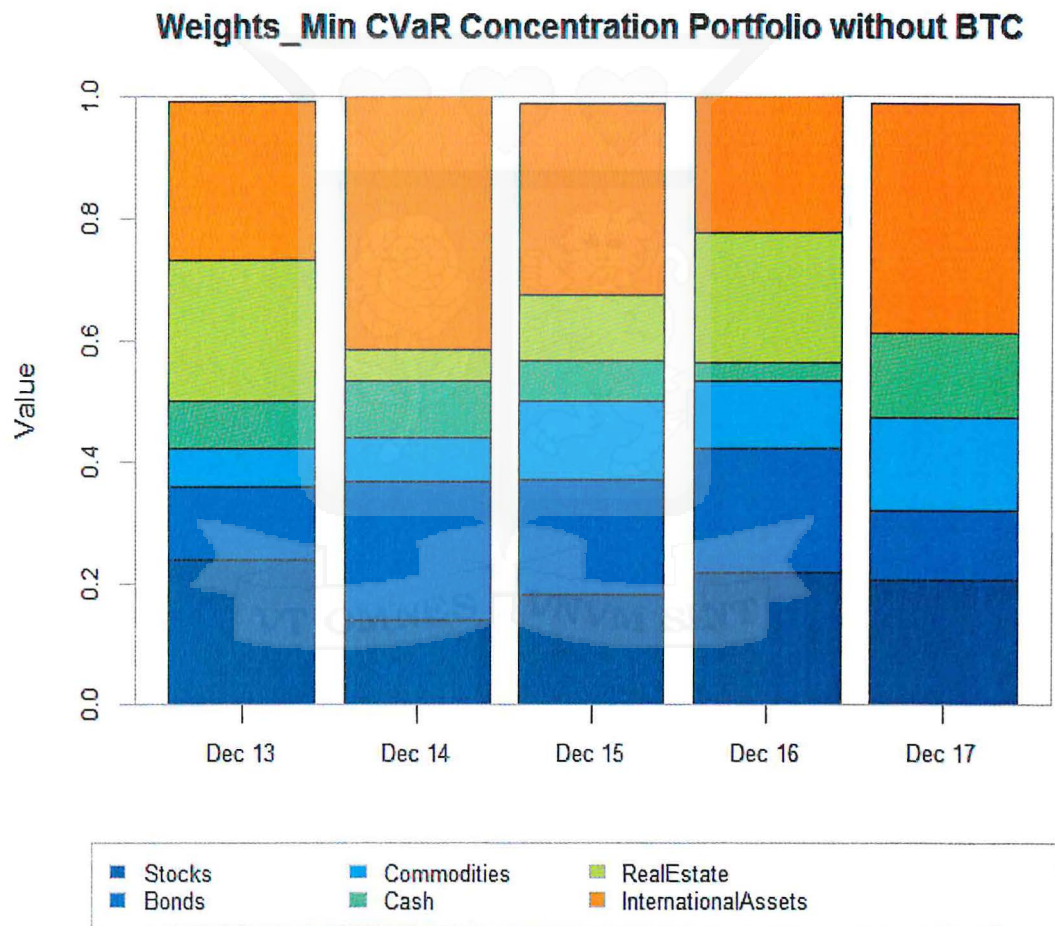


Figure 8: Weights of the Minimum CVaR Concentration Portfolio without Bitcoin

4.2.2.1.2 Minimum CVaR Concentration portfolio with Bitcoin

In this portfolio, the weight allocation of the different asset classes varies and there is no particular trend. Bitcoin's weight allocation is relatively low across the board and ranges from 3% to 5.7%.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo -dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> | <i>BTC</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|------------|
| 2013-12-01 | 0.2040 | 0.0000 | 0.1100 | 0.1620 | 0.2040 | 0.2960 | 0.0300 |
| 2014-12-01 | 0.0969 | 0.2119 | 0.0940 | 0.1200 | 0.0180 | 0.4349 | 0.0300 |
| 2015-12-01 | 0.1961 | 0.2038 | 0.1628 | 0.1359 | 0.0580 | 0.1945 | 0.0422 |
| 2016-12-01 | 0.3229 | 0.2441 | 0.0873 | 0.1025 | 0.0360 | 0.1553 | 0.0577 |
| 2017-12-01 | 0.2653 | 0.2380 | 0.1649 | 0.0960 | 0.0360 | 0.1600 | 0.0409 |

Table 11: Weights of the Minimum CVaR Concentration Portfolio with Bitcoin

Figure 9 presents the visual presentation of the above asset allocation.

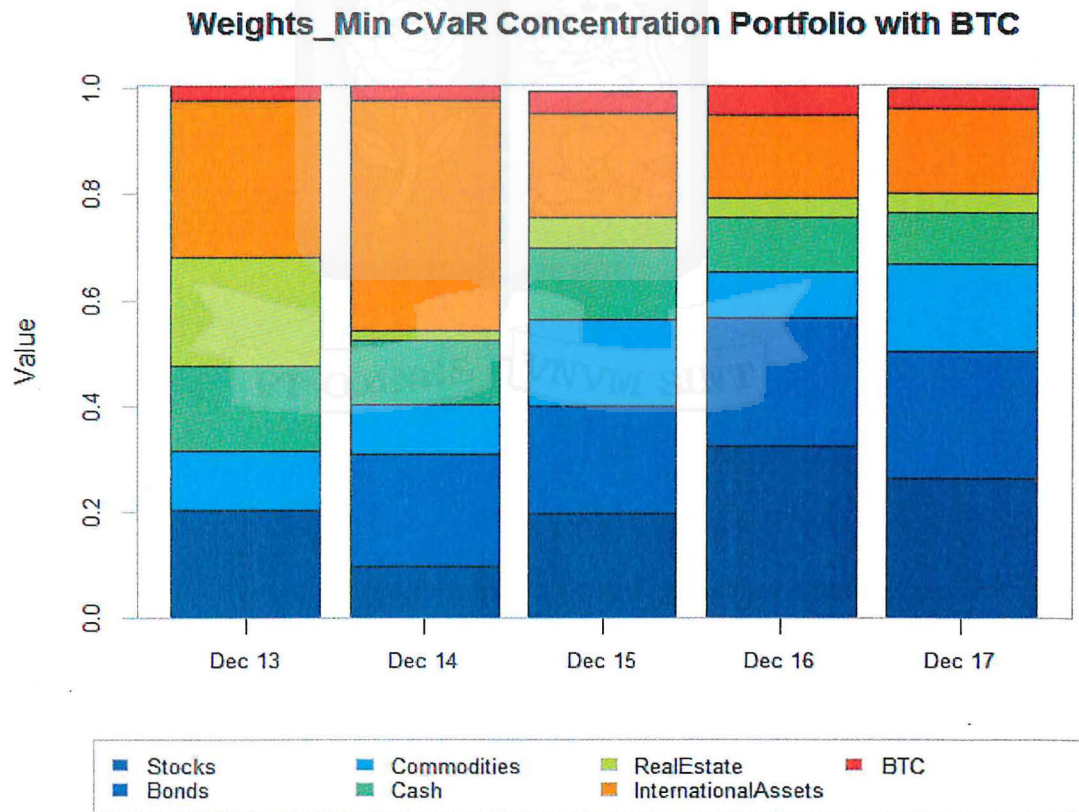


Figure 9: Weights of the Minimum CVaR Concentration Portfolio with Bitcoin

4.2.2.2 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint

Tables 12 and 13 display the weights of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with and without bitcoin over the different rebalancing dates.

4.2.2.2.1 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

In this portfolio, the weight allocation of the different asset classes varies and there is no particular trend. Real estate weightings however exhibit high inconsistency with the highest being 73% in the second rebalancing date and the lowest being 6% in the third rebalancing date.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|
| 2013-12-01 | 0.3915 | 0.2081 | 0.1240 | 0.0708 | 0.1325 | 0.0737 |
| 2014-12-01 | 0.0820 | 0.0540 | 0.0200 | 0.0220 | 0.7300 | 0.0980 |
| 2015-12-01 | 0.3040 | 0.2660 | 0.0040 | 0.0940 | 0.0620 | 0.2740 |
| 2016-12-01 | 0.1320 | 0.0867 | 0.0938 | 0.0527 | 0.4424 | 0.1899 |
| 2017-12-01 | 0.2957 | 0.0337 | 0.1889 | 0.1422 | 0.0418 | 0.2990 |

Table 12: Weights of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

Figure 10 presents the visual presentation of the above asset allocation.

Weights_Min CVaR 30% Constraint without BTC

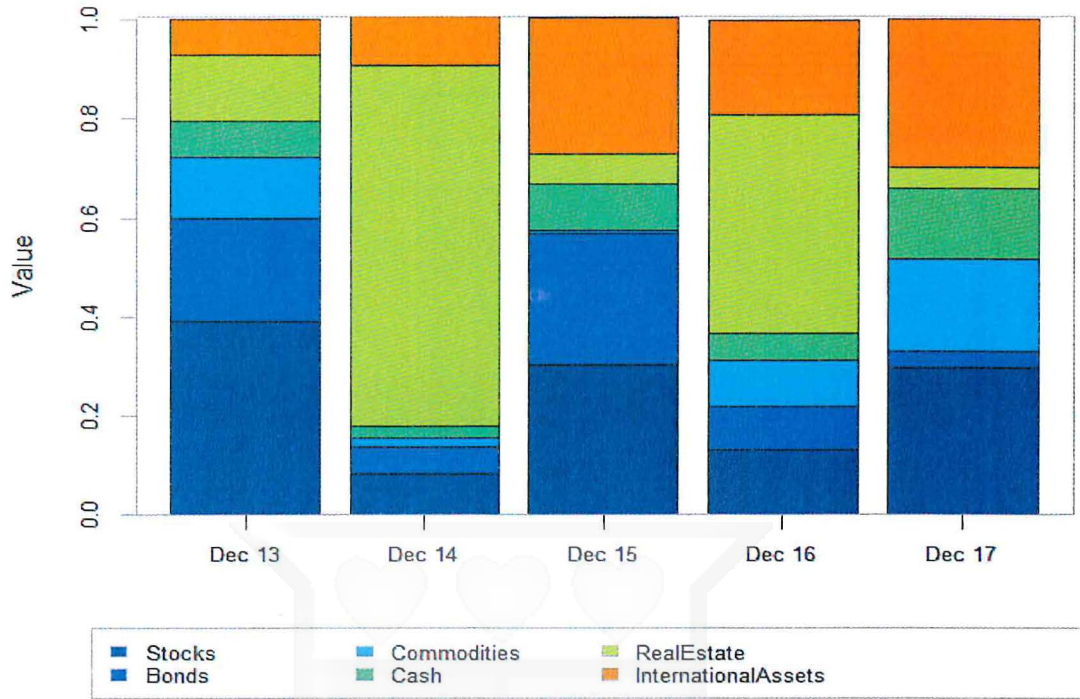


Figure 10: Weights of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

4.2.2.2.2 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

In this portfolio, the weight allocation of the different asset classes varies and there is no particular trend. Bitcoin’s weight allocation is relatively low across the board and ranges from 0.6% to 3.6%.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> | <i>BTC</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|------------|
| 2013-12-01 | 0.2952 | 0.0539 | 0.0833 | 0.1440 | 0.1340 | 0.2478 | 0.0343 |
| 2014-12-01 | 0.3900 | 0.2932 | 0.0080 | 0.0480 | 0.0538 | 0.2020 | 0.0100 |
| 2015-12-01 | 0.1440 | 0.0840 | 0.0560 | 0.1000 | 0.4000 | 0.1820 | 0.0300 |
| 2016-12-01 | 0.0580 | 0.1958 | 0.2108 | 0.1280 | 0.0280 | 0.3680 | 0.0060 |
| 2017-12-01 | 0.2081 | 0.1748 | 0.1200 | 0.0580 | 0.2120 | 0.1874 | 0.0360 |

Table 13: Weights of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

Figure 11 presents the visual presentation of the above asset allocation.

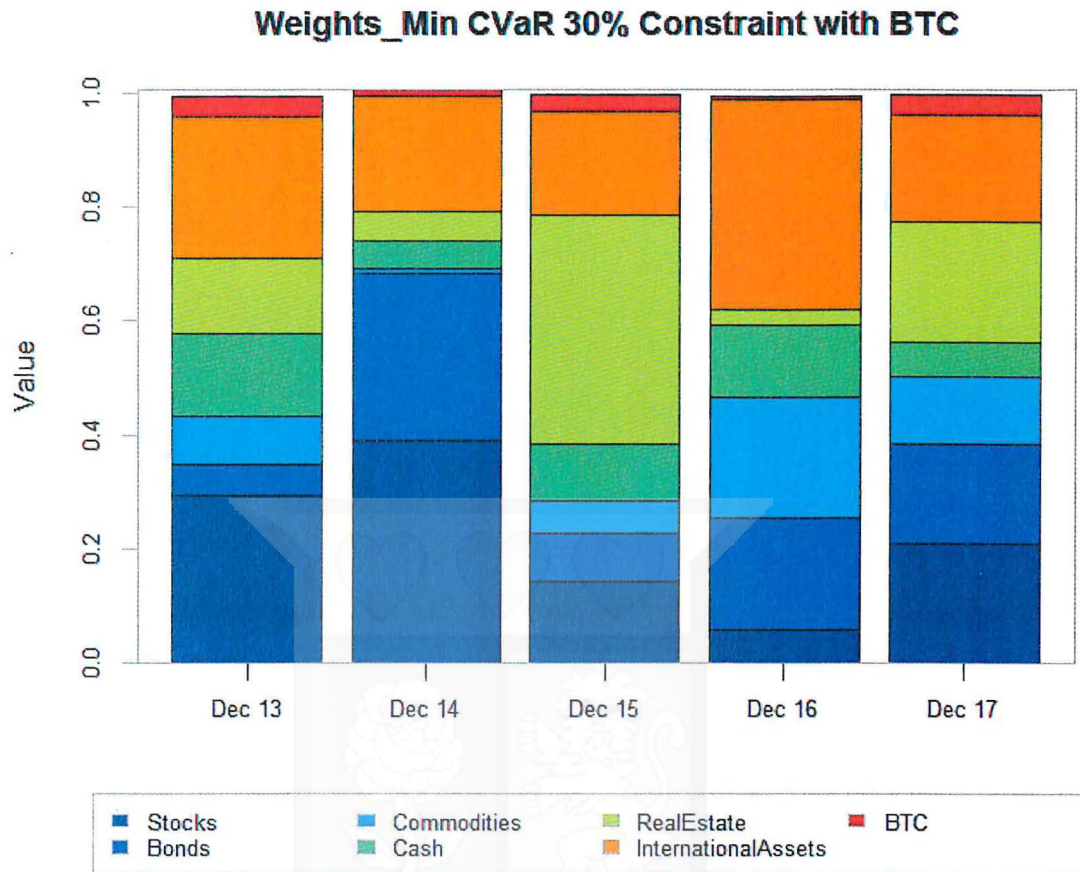


Figure 11: Weights of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

4.2.3 Risk Contribution

A comparison of the CVaR values and the risk contribution of the different asset classes is conducted so as to analyse how the inclusion of bitcoin impacts the different portfolios.

4.2.3.1 Minimum CVaR Concentration portfolio

Table 14 shows the CVaR values of the Minimum CVaR Concentration Portfolio over the different rebalancing dates. The CVaR values of the portfolio with bitcoin are higher than the values of the portfolio without bitcoin, with the exception of the last rebalancing date. This means that the portfolio with bitcoin is riskier.

| | <i>CVaR without BTC</i> | <i>CVaR with BTC</i> |
|------------|-------------------------|----------------------|
| 2013-12-01 | 0.0228 | 0.0314 |
| 2014-12-01 | 0.0265 | 0.0286 |
| 2015-12-01 | 0.0267 | 0.0326 |
| 2016-12-01 | 0.0228 | 0.0311 |
| 2017-12-01 | 0.0334 | 0.0294 |

Table 14: CVaR values of the Minimum Concentration Portfolio

Figure 12 shows the visual representation of the CVaR values.

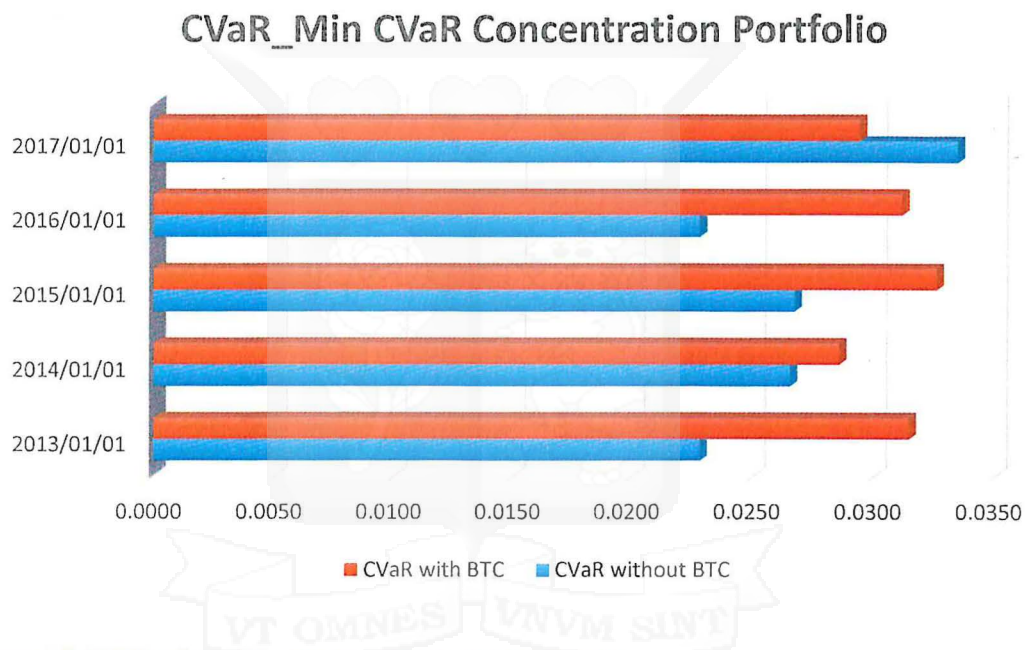


Figure 12: CVaR values of the Minimum Concentration Portfolio

Tables 15 and 16 represent the risk contribution of the different asset classes to the Minimum CVaR Concentration portfolios' CVaR values.

4.2.3.1.1 Minimum CVaR Concentration portfolio without Bitcoin

In this portfolio, real estate has a negative risk contribution while the other asset classes have a positive risk contribution to the portfolio's CVaR. International assets have the highest risk contribution to the portfolio's CVaR ranging from 22% to 32% of the overall portfolio CVaR.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|
| 2013-12-01 | 0.2446 | 0.1135 | 0.2220 | 0.2214 | -0.0353 | 0.2338 |
| 2014-12-01 | 0.0434 | 0.2798 | 0.1630 | 0.2391 | -0.0082 | 0.2830 |
| 2015-12-01 | 0.1444 | 0.1723 | 0.2855 | 0.1239 | -0.0205 | 0.2945 |
| 2016-12-01 | 0.2080 | 0.3252 | 0.2162 | 0.0751 | -0.0479 | 0.2235 |
| 2017-12-01 | 0.1790 | 0.0024 | 0.2133 | 0.2842 | 0.0000 | 0.3211 |

Table 15: Asset Risk Contribution of the Minimum CVaR Concentration portfolio without Bitcoin

Figure 13 shows the visual representation of the asset risk contribution.

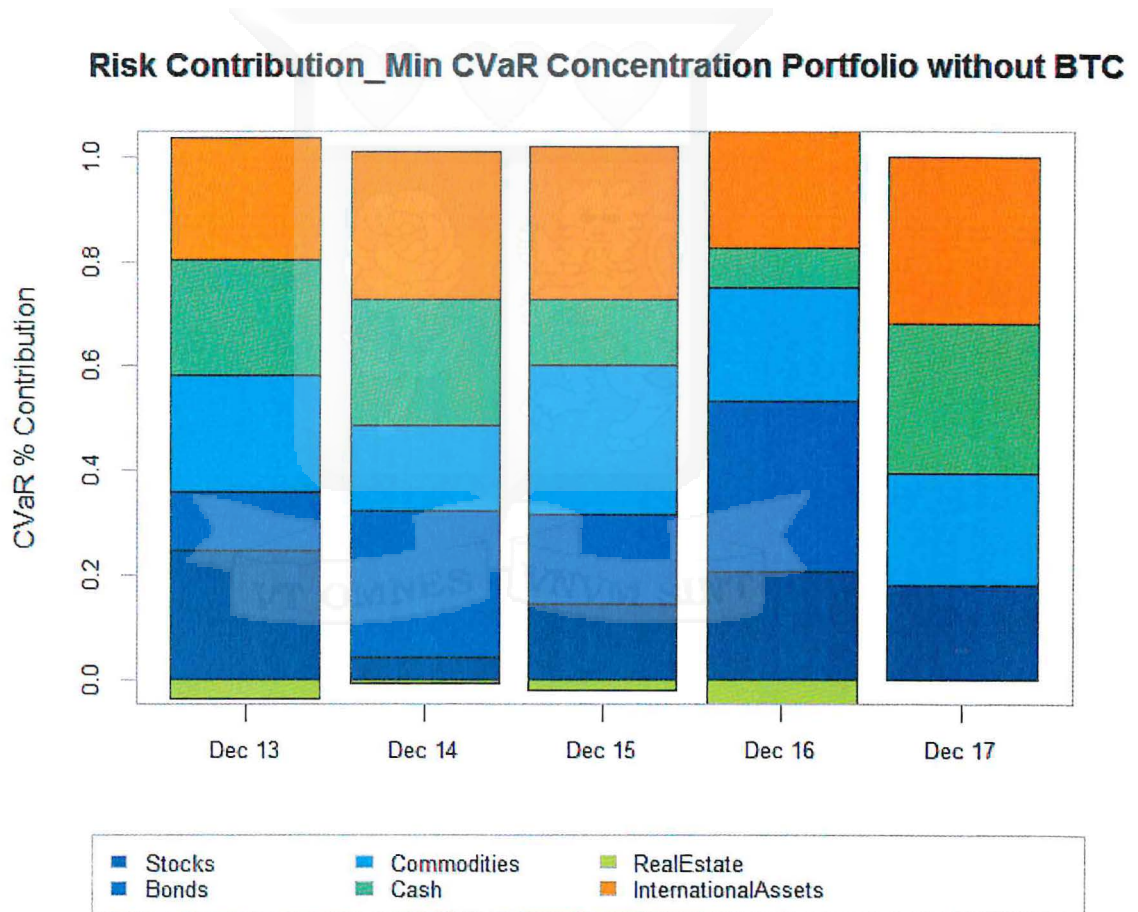


Figure 13: Asset Risk Contribution of the Minimum CVaR Concentration portfolio without Bitcoin

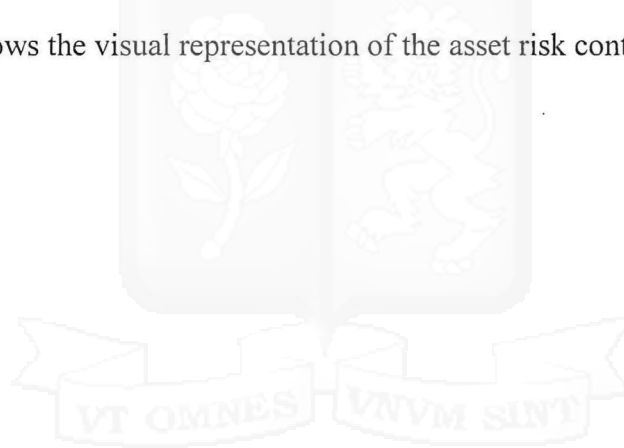
4.2.3.1.2 Minimum CVaR Concentration portfolio with Bitcoin

In this portfolio, real estate has negative risk contribution to the portfolio's CVaR while the other asset classes' risk contribution to the portfolio CVaR varies. Bitcoin's risk contribution is relatively high compared to its weight allocation. This shows that even though this portfolio allocates relatively low weights to bitcoin (3%-5.7%), the low weight of bitcoin carries with it relatively high risk (11%-23%).

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> | <i>BTC</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|------------|
| 2013-12-01 | 0.1274 | 0.0000 | 0.3473 | 0.2098 | -0.0304 | 0.2318 | 0.1142 |
| 2014-12-01 | 0.0073 | 0.1010 | 0.2006 | 0.2285 | -0.0024 | 0.3339 | 0.1311 |
| 2015-12-01 | 0.0648 | 0.1543 | 0.2668 | 0.2138 | -0.0083 | 0.1182 | 0.1905 |
| 2016-12-01 | 0.2019 | 0.2376 | 0.0858 | 0.1217 | -0.0054 | 0.1312 | 0.2271 |
| 2017-12-01 | 0.1772 | 0.1590 | 0.2734 | 0.1322 | -0.0068 | 0.1181 | 0.1469 |

Table 16: Asset Risk Contribution of the Minimum CVaR Concentration portfolio with Bitcoin

Figure 14 shows the visual representation of the asset risk contribution.



Risk Contribution_Min CVaR Concentration Port. with BTC

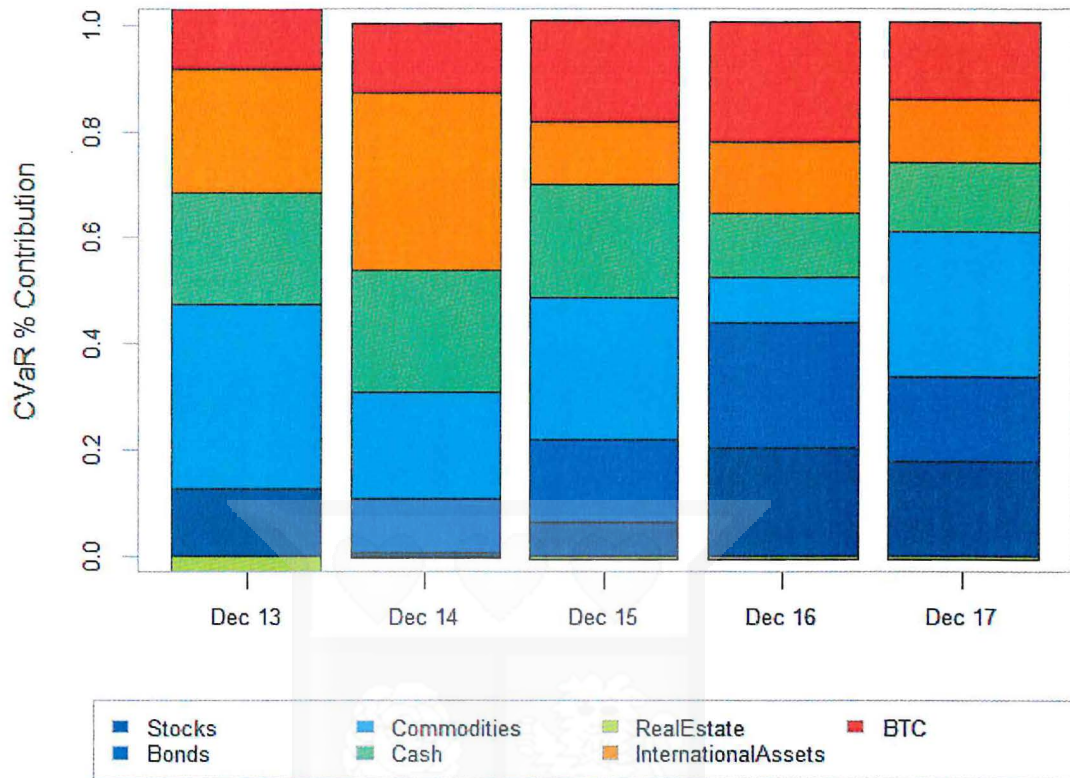


Figure 14: Asset Risk Contribution of the Minimum CVaR Concentration portfolio with Bitcoin

4.2.3.2 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint

Table 17 shows the CVaR Values of the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint over the different rebalancing dates. In this portfolio, only two of the CVaR values of the portfolio with bitcoin are higher than those of the portfolio without bitcoin.

The CVaR values of this portfolio are also considerably lower than those of the Minimum CVaR Concentration Portfolio with Bitcoin. This can be attributed to the fact that this portfolio has additional constraint that hinders asset risk contribution higher than 30%. This can also be related to how the risk of the Minimum CVaR portfolio under a 30% CVaR allocation constraint is relatively lower (0.0567) than the risk of the Minimum CVaR Concentration Portfolio (0.1091) as shown in Table 6 and 8. This

means that the Minimum CVaR portfolio under a 30% CVaR allocation constraint is less risky.

| | <i>CVaR without BTC</i> | <i>CVaR with BTC</i> |
|------------|-------------------------|----------------------|
| 2013-12-01 | 0.0228 | 0.0314 |
| 2014-12-01 | 0.0265 | 0.0286 |
| 2015-12-01 | 0.0267 | 0.0326 |
| 2016-12-01 | 0.0228 | 0.0311 |
| 2017-12-01 | 0.0334 | 0.0294 |

Table 17: CVaR Values of the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint

Figure 15 shows the visual representation of the CVaR values.

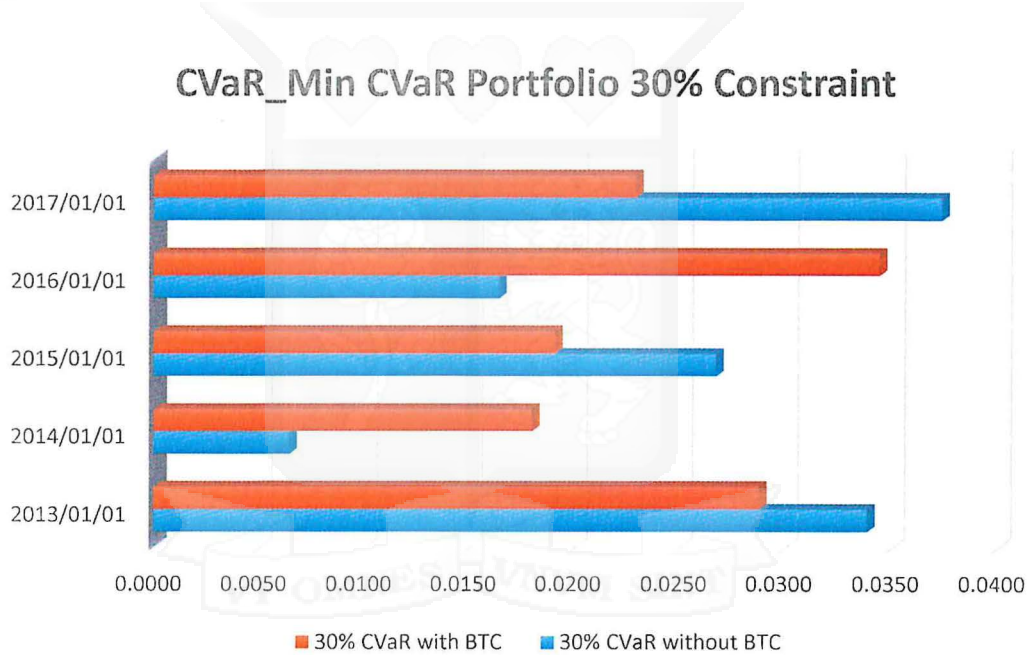


Figure 15: CVaR Values of the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint

Table 18 and 19 represent the assets' risk contribution to the portfolio CVaRs of the Minimum CVaR Portfolios under an upper 30% CVaR allocation constraint.

4.2.3.2.1 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

In this portfolio, the risk contribution of the asset classes to the portfolio CVaR does not follow a particular pattern. Real estate has the lowest risk contribution apart from one rebalancing year, 2014.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|
| 2013-12-01 | 0.2962 | 0.3223 | 0.2827 | 0.1281 | -0.0148 | -0.0145 |
| 2014-12-01 | 0.1543 | 0.0709 | 0.1778 | 0.2059 | 0.2211 | 0.1700 |
| 2015-12-01 | 0.1789 | 0.2709 | 0.0062 | 0.2964 | -0.0173 | 0.2650 |
| 2016-12-01 | 0.2290 | 0.0472 | 0.3072 | 0.2150 | -0.0987 | 0.3004 |
| 2017-12-01 | 0.2954 | -0.0176 | 0.3068 | 0.2443 | -0.0056 | 0.1766 |

Table 18: Asset Risk Contribution of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

Figure 16 shows the visual representation of the asset risk contribution.



Risk Contribution_Min.CVaR 30% Constraint without BTC

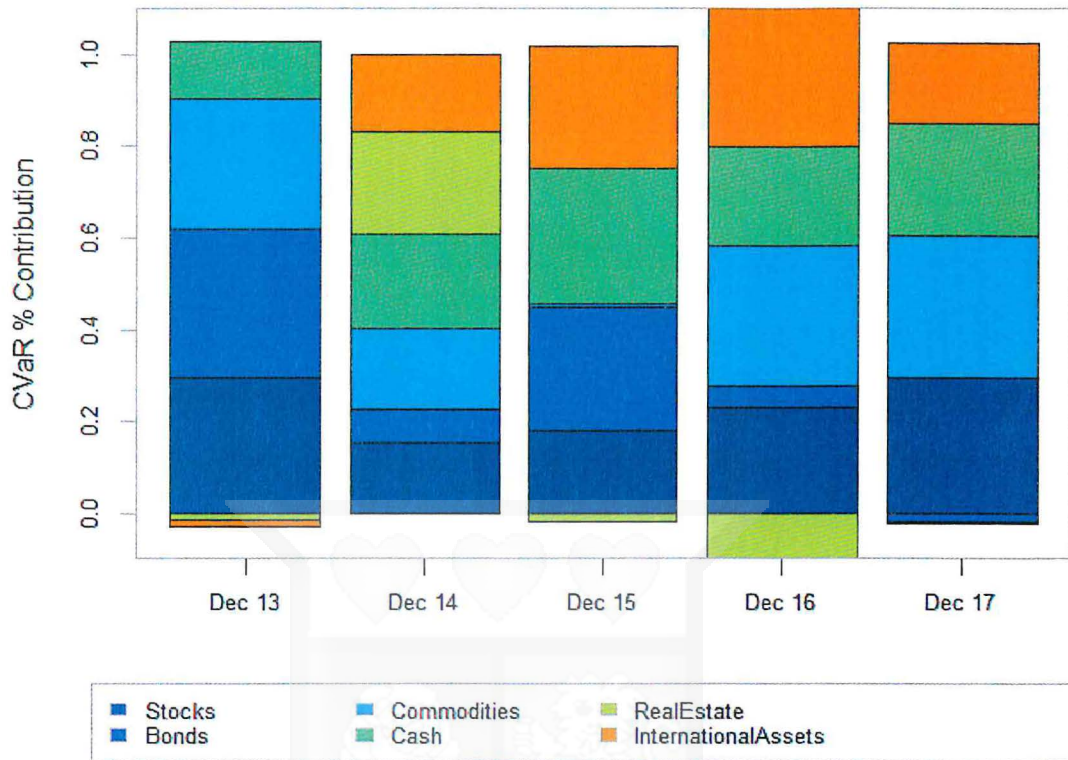


Figure 16: Asset Risk Contribution of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint without Bitcoin

4.2.3.2.2 Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

In this portfolio, real estate has negative risk contribution to the portfolio's CVaR across the board while the other asset classes' risk contribution varies. The risk contribution of bitcoin is relatively high as it ranges from (-0.8% to 28%) given that the weight allocation of bitcoin ranges from (0.6%-3.6%). A weight allocation of 3.6% yields a risk contribution of 28% which implies that bitcoin is a very risky asset class.

| | <i>Stocks</i> | <i>Bonds</i> | <i>Commo- dities</i> | <i>Cash</i> | <i>Real Estate</i> | <i>Int. Assets</i> | <i>BTC</i> |
|------------|---------------|--------------|--------------------------|-------------|------------------------|------------------------|------------|
| 2013-12-01 | 0.2319 | -0.0244 | 0.2936 | 0.1990 | -0.0234 | 0.1998 | 0.1235 |
| 2014-12-01 | 0.2750 | 0.2722 | 0.0413 | 0.1843 | -0.0137 | 0.2128 | 0.0281 |
| 2015-12-01 | 0.1107 | 0.0388 | 0.1237 | 0.2619 | -0.0768 | 0.2591 | 0.2827 |
| 2016-12-01 | 0.0227 | 0.1647 | 0.3020 | 0.2692 | -0.0041 | 0.2540 | -0.0084 |
| 2017-12-01 | 0.1827 | 0.1015 | 0.2217 | 0.0729 | -0.0491 | 0.2314 | 0.2389 |

Table 19: Asset Risk Contribution of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

Figure 17 shows the visual representation of the asset risk contribution.

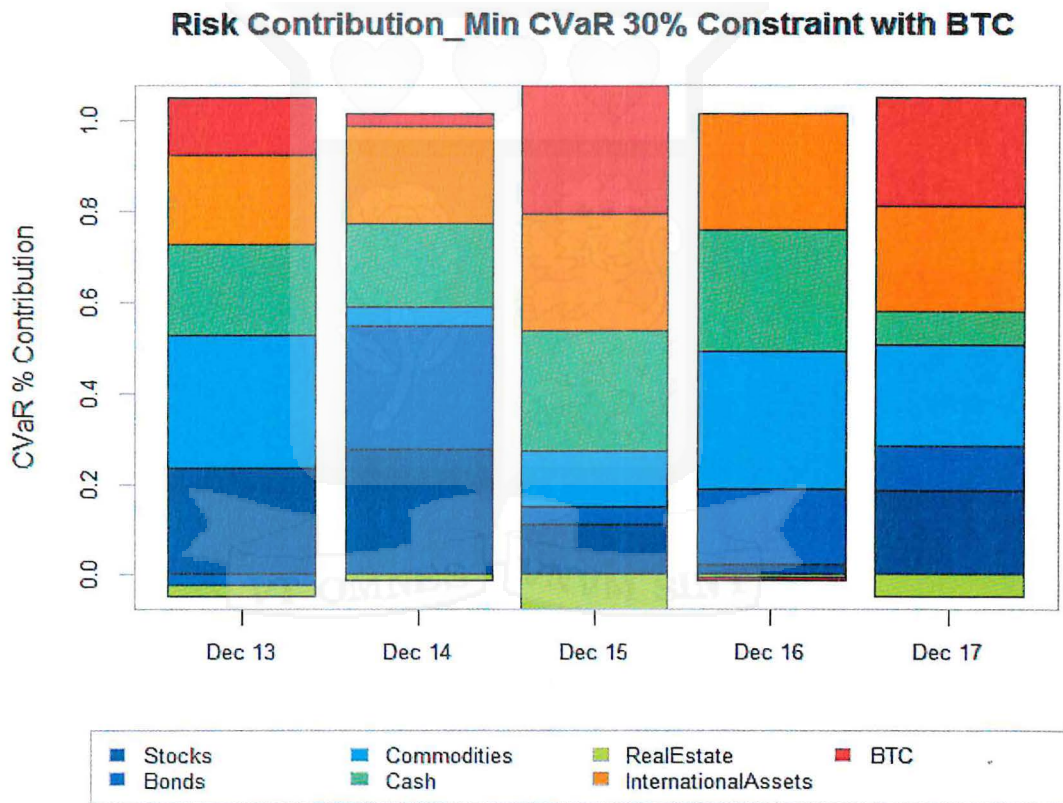


Figure 17: Asset Risk Contribution of the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint with Bitcoin

CHAPTER FIVE

5. DISCUSSION AND CONCLUSION

5.0 Introduction

The aim of this chapter is to link the results of this study to the research objectives, to link the results to previous literature, to deduce the appropriate conclusions, implications, limitations and recommendations of this research.

5.1 Conclusion

Unlike traditional portfolios, portfolios of this age should incorporate a balanced mix of traditional and alternative investments so as to achieve diversification. This study analyses the inclusion of bitcoin as an alternative asset in a South African portfolio due to its current popularity. The inclusion of different types of asset classes to a portfolio can have a beneficial impact on portfolio diversification and thus, this study aims to analyse the effects the inclusion of bitcoin has on the South African portfolio.

This study aims to answer how the return and risk measures of an already diversified portfolio are affected by the inclusion of bitcoin, how the asset allocation of a well-diversified portfolio is affected by the addition of bitcoin and how the risk contribution of assets in a well-diversified portfolio is affected by the inclusion of bitcoin. In order to answer these questions a portfolio consisting of stocks, bonds, commodities, cash, international assets and real estate is constructed. The inclusion of bitcoin on the portfolio is then analysed.

In this study, an alternative method to Mean-Variance Optimization is utilized due to its assumption that the data series is normally distributed. In order to account for bitcoin's highly non-normal return distribution, the Mean-CVaR Portfolio Optimization, which is more a robust approach, is utilized. So instead of using variance as an objective function in constructing optimal portfolios, Conditional Value-at-Risk (CVaR) is used. Two different frameworks are used namely: Minimum CVaR Concentration Portfolio and the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint.

The descriptive statistics show that the return series of bitcoin differ extremely from the return series of the other asset classes. Bitcoin's return series exhibit significantly

higher standard deviation (0.3141) and variance (0.0986), positive skewness (2.1514) and excess kurtosis (8.5639). The results also show that bitcoin has low correlation with the other asset classes. This implies that constructing a portfolio using these assets would lead to diversification as the volatility of the overall returns will be reduced which will in turn reduce the portfolio's standard deviation.

The study's results show that an inclusion of bitcoin increases both the return and the risk of the portfolio. In the Minimum CVaR Concentration Portfolio, the return and risk of the portfolio without bitcoin is 0.0501 and 0.0540 respectively, while the return and risk of the portfolio with bitcoin is 0.1527 and 0.1091 respectively. In the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint, the return and risk of the portfolio without bitcoin is 0.0633 and 0.0453 respectively while the return and risk of the portfolio with bitcoin is 0.0789 and 0.0567 respectively.

The descriptive results of bitcoin align with Briere et al. (2015) and Eisl et al. (2015) as they also state the presence of higher standard deviation, positive skewness and excess kurtosis. The results of the risk-return performance measures also align with the above mentioned literature as they state that including bitcoin increases not only the expected return but also the risk of the portfolios. Schut (2017), Briere et al. (2015) and Eisl et al. (2015) similarly state that bitcoin has low correlations with the other asset classes.

The portfolio optimization on the different frameworks shows that bitcoin is included in the optimal portfolios with relatively low weighting. In the Minimum CVaR Concentration Portfolio, the weight of bitcoin ranges from 3% to 5.7% while the highest weight in the portfolio is allocated to international assets (43%). In the Minimum CVaR Portfolio under an upper 30% CVaR allocation Constraint, the weight of bitcoin ranges from 0.6% to 3.6%, while the highest weight in the portfolio is allocated to stocks (39%). Schut (2017) and Klabbbers (2017) also stated that bitcoin has a relatively low weight allocation of on average 0% and 5%.

It is interesting to note that even with the low weightings, bitcoins' risk contribution to the portfolio CVaR is relatively high. In the Minimum CVaR Concentration Portfolio, bitcoin's risk contribution ranges from 11% to 23%. In the rebalancing date 2016-12-01, both bitcoin and bonds have a risk contribution of 23% but the variation in weights is

extreme in that, the weight of bonds is 24% and the weight of bitcoin is 5.7%. In the Minimum CVaR Portfolio under an upper 30% CVaR allocation constraint, the risk contribution of bitcoin ranges from -0.8% to 28%.

Eisl et al. (2015) and Carpenter (2016) concluded that bitcoin should be included in optimal portfolios because the additional risk in the portfolio that is as a result of bitcoin is overcompensated by high returns leading to better risk-return ratios. Gangwal (2016) also concluded that the inclusion of bitcoin yielded a higher return ratio meaning that bitcoin's returns offset its high volatility. Klabbers (2017) and Schut (2017) concluded that bitcoin is an effective diversifier.

In conclusion, this study finds that portfolios with bitcoin outperform the portfolios without bitcoin. The inclusion of bitcoin leads to the diversification effect that reduces the volatility in returns and in turn the risk of the portfolio. The results also show that including a relatively small amount of bitcoin in a well diversified portfolio has a positive impact on the return. The inclusion of bitcoin generally appears to be an attractive investment that can substantially increase the return of an efficient portfolio.

5.2 Limitations of the study

The data collected and utilized in this study spans from July 23, 2010 to December 29, 2017 because bitcoin's historical price data became available on July 23, 2010. Therefore, the findings on this study read as evidence and are specific for this time span only. Bitcoin cannot be considered as a mature asset therefore, there might be changes in the nature of bitcoin over time. The findings of this study are therefore restricted to this particular time span.

5.2 Implications of the findings

This study offers suggestive evidence for the inclusion of bitcoin in a well-diversified portfolio. Bitcoin has low correlation with other asset classes in a well-diversified portfolio and it increases the risk-return ratios of a portfolio. Investors can include bitcoin as an alternative asset to diversify their portfolios as it acts as a diversifier and leads to the increase of the portfolio return. It is important to note that the weight allocation to bitcoin is quite low.

5.4 Recommendations of the study

The following are possible areas for further research. First, sophisticated forecasting techniques such as Bayesian methods or Neural Network should be utilized for scenario generation instead of the use of historical data, which is used in this study. These forecasting techniques enable the estimation of how the sequence of observations will continue into future.

Another avenue for further study would be research on bitcoin as an alternative asset when bitcoin is already a mature asset. The current bitcoin performance is perceived to be heavily influenced by the peak at the end of 2013 and the rapid growth in 2016 and 2017 due to speculation in the market.

The following issue should be considered for future action. Policymakers and regulators in emerging markets should foster investments in bitcoin due to the advantages of the inclusion of bitcoin in well-diversified portfolios. They should provide investors with the opportunity to invest in bitcoin as it contributes positively to the investors' portfolio performance in general.



References

- Acerbi, C., & Tasche, D. (2002). Expected shortfall: a natural coherent alternative to value at risk. *Economic notes*, 31(2), 379-388.
- Alexander, G. J., & Baptista, A. M. (2004). Economic Implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis. *Journal of Economic Dynamics & Control* 26, 1159 - 1193.
- Artzner, P., Delbaen, F., Eber, J. M., & D., H. (1999). Coherent Measures of Risk. *Mathematical Finance*, 9,, 203-228.
- Baur, D., & Lucey, B. (2010). Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. *The Financial Review*, 45, ., 217-229.
- Baur, D., & Mc Dermontt, T. (2010). Is gold a safe haven? International evidence. *Journal of banking and finance*, 34, (8), 1886-1898.
- Baur, D., Lee, A., & Hong, K. (2015). Bitcoin: Currency or Investment? *Working paper*.
- Belousova, J., & Dortfleitner, G. (2012). On the diversification benefits of commodities from the perspective of euro investors. *Journal of Banking & Finance*, 36(9), 2455 - 2472.
- Benati, S., & Rizzi, R. (2007). A mixed integer linear programming formulation of the optimal mean/value-at-risk portfolio problem. *European Journal of Operational Research* 176, 423 - 434.
- Bitcoin.org. (2017). Retrieved from Bitcoin: <https://bitcoin.org/en/faq#what-is-bitcoin>
- Borio, C. V., & Bergh, P. V. (1993). The Nature and Management of Payment System risks: An international perspective. *BIS Economic Papers*.
- Boudt, P., Ardia, D., Mullen, K., Carl, P., & Peterson, B. (2010). Differential evolution (deoptim) for non-convex portfolio optimization. *Mimeo*.
- Bouoiyour, J., & Selmi, R. (2015). What Does BitCoin Look Like? *MPRA Paper No. 58091. Germany: University*.

- Bouoiyour, J., & Selmi, R. (2017). *Are Trump and Bitcoin Good Partners?* Working paper.
- Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. (2017). On the hedge and safe haven properties of Bitcoin: is it really more than a diversifier? *Finance Research Letters*, 20, 192-198.
- Briere, M., Burgues, A., & Signori, O. (2010). Volatility Exposure for Strategic Asset Allocation. *Journal of Portfolio Management*, 105 - 116.
- Briere, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual Currency, tangible return: Portfolio diversification with bitcoins. *CEB Working Paper*.
- Campbell, R., Huisman, R., & Koedijk, K. (2001). Optimal Portfolio Selection in a value-at-risk framework. *Journal of Banking & Finance* 25, 1789 - 1296.
- Carpenter, A. (2016). *Portfolio Diversification with Bitcoin*. James Madison University.
- Cavusgil, T. (1997). Measuring the Potential of Emerging Markets: An Indexing Approach. *Business Horizons*, Vol. 40, No.1 .
- Ciaian, P., Rajcaniova, M., & Kancs, D. (2016). The economics of BitCoin price formation,. *Applied Economics*, 48, (19),, 1799-1815.
- Danielsson, J. (2002). The emporor has no clothes: limits to risk modeling. *Journal of Banking and Finance* 26, 1273 - 1296.
- Dixon, S. (2017). *Enabling Payments to and from Kenya*. Retrieved from <http://blog.bnktothefuture.com/bnktothefuture-case-study-bitpesa/>
- Dyhrberg, A. (2015). *Bitcoin, Gold and the Dollar – a GARCH Volatility analysis*. University College Dublin, School of Economics.
- Eisl, A., Gasser, S. M., & Weinmayer, K. (2015). *Caveat Emptor: Does Bitcoin Improve Portfolio Diversification*. Vienna: WU Vienna University of Economics and Business.
- Elwell, C. K., Murphy, M. M., & Seitzinger, M. V. (2015). Bitcoin: Questions, Answers, and Analysis of Legal Issues. *Congressional Research Service*.

- Embrechts, P., & Hofert, M. (2014). Statistics and quantitative risk management for banking and insurance. *Annual Review of Statistics and Its Application*, 493-514.
- Fabozzi, F., Gupta, F., & Markowitz, H. (2002). The legacy of modern portfolio theory. *Journal of Investing*, 7-22.
- Ferdi, B., Brett, S., & Jen, S. (2015). Art investment as a portfolio diversification strategy in South Africa. *ERSA Working paper 537*.
- Gaivoronski, A.-A., & Pflug, G. (2004). Value-at-risk in portfolio optimization: properties and computational approach. *Journal of Risk* 7, 1 -31.
- Gangwal, S. (2016). Analyzing the Effects of Adding Bitcoin to Portfolio. *International Journal of Social, Behavioral, Educational, Economic, Business and Industrial Engineering Vol:10, No:10, 3509 - 3522*.
- Gibson, R. C. (1990). *Asset allocation: Balancing financial risk*. Homewood: Dow Jones-Irwin.
- Gobel, J., Keeler, H. P., Krzesinski, A. E., & Taylor, P. G. (2015). Bitcoin Blockchain Dynamics: the Selfish-Mine Strategy in the Presence of Propagation Delay.
- Grise, C., & Nitschka, T. (2013). *On financial risk and the safe haven characteristics of Swiss franc exchange rates*. Swiss National Bank Working Papers.
- Halaburda, H., & Gandal, N. (2014). *Competition in the cryptocurrency market*. NET Institute Working Paper .
- Hillier, D., Draper, P., & Faff, R. (2006). Do precious metals shine? An investment perspective, . *Financial analyst journal*, 62, (2),, 98-106.
- Jorion, P. (2001). *Value at risk*. McGraw Hill.
- Kaushal, R. (2016). Bitcoin: First Decentralised Payment System. *International Journal of Engineering and Computer Science* , 16514 -16517.
- Kemp, C., Reid, L., & Romero, M. (n.d.). *In Blockchain we Trust: A Comparison of the investment potential of Bitcoin and Ether over a five-year time horizon*. Middlebury Institute of International Studies at Monterey.

- Klabbers, S. (2017). *Bitcoin as an investment asset :The added value of bitcoin in a global market portfolio*. Radboud Universiteit Nijmegen.
- Kristoufek, L. (2013). Bitcoin Meets Google Trends and Wikipedia: Quantifying the Relationship between Phenomena of the Internet Era. *Scientific Reports* 3 (3415).
- Krokhmal, P., Palmquist, J., & Uryasev, S. (2001). *Portfolio Optimization with Conditional Value-at-Risk Objective and Constraints*.
- Krokhmal, P., Uryasev, S., & Zrazhevsky, G. (2002). Portfolio Optimization with conditional value-at-risk criterion. *Journal of Risk*, 4 (2).
- Larsen, N., Mausser, H., & Uryasev, S. (2002). Algorithms for optimization of value-at-risk. *Kluwer Academic Publishers*, 127 -157.
- Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *Journal of Finance*, 587 - 615.
- Mangram, M. E. (2013). A Simplified Perspective of the Markowitz. *Global Journal of Business Research*, 59 - 70.
- Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance* 7(1), 77-91.
- Markowitz, H. (1959). *Portfolio Selection; Efficient Diversification of Investments*. New Haven, Connecticut: Yale University Press.
- Markowitz, H. M. (2010). *Portfolio Theory: As I Still See It*.
- McNeil, A., Frey, R., & Embrechts, P. (2005). *Quantitative Risk Management*. Princeton University Press.
- Meggison, W. (1996). A historical overview of research in finance. *Journal of Finance*, 323 - 346.
- Miller, R. (1998). *Selling to Newly Emerging Markets*. New York: Quorum Books.
- Mishkin, F. (2009). *The economics of Money, Banking and Financial Markets: Global edition. 9th edition*. Pearson education.

- Miskolczi, P. (2016). *Differences between Mean-Variance and Mean-CVaR Portfolio Optimization Models*. University of Debrecen.
- Mody, A. (2004). What is an Emerging Market? *IMF Working Paper*.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, 34, 768-783.
- MSCI. (2017). *MSCI Emerging Markets Index*. MSCI.
- Nakamoto, S. (2008). *Bitcoin: A Peer-to-Peer Electronic Cash System*.
- Njuguna, E. M. (2014). Adoption of Bitcoin in Kenya , A case study of Bitpesa.
- Pflug, G. (2000). *Some Remarks on the Value-at-Risk and the Conditional Value-at-Risk*. In "*Probabilistic Constrained Optimization: Methodology and Applications*". Ed. S. Uryasev Kluwer Academic Publishers.
- Popper, N. (2015). *Digital Gold: The Untold Story of Bitcoin*". London: Penguin.
- Prapinmongkolkarn, W. (2008). Portfolio Optimization via Conditional Value-at-Risk.
- Premchand, A., & Choudhry, A. (2015). Future of Payments - ePayments. *International Journal of Emerging Technology and Advanced Engineering*.
- Ranaldo, A., & Söderlind, P. (2010). Safe haven Currencies (digest summary), . *Review of Finance*, 14, (3), 385 – 407.
- Raymaekers, A. (2015). Cryptocurrency Bitcoin: Disruption, challenges and opportunities. *Journal of Payments Strategy and Systems*, 30-40.
- Rockafellar, R. T., & Uryasev, S. .. (2000). (2000). "Optimization of Conditional Value-At-Risk.". *The Journal of Risk*, Vol. 2, No. 3, 21- 41.
- Rogojanu, A., & Badea, L. (2014). The issue of competing currencies. Case study – Bitcoin. *Theoretical and Applied Economics*, 21, (1), 103–114.
- Ross, S., Westerfield, R., & Jaffe, J. (2002). *Capital market theory: An overview. Corporate finance (6th ed.)*. New York: NY: McGraw-Hill.

- Ruppert, D. (2010). *Statistics and Data Analytics for Financial Engineering*. New York: Springer New York Dordrecht Heidelberg London.
- Schut, M. (2017). *Bitcoin analysis from an investor's perspective*. Erasmus University Rotterdam.
- Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 425 - 442.
- Sirila, D. (2014). The Pleasures and Perils of New Money in Old Pockets; M-PESA and Bitcoin in Kenya. *Harvard Law School*.
- Sjolin, S. (2014). *You aren't Investing in Africa and you're missing out*. MarketWatch.
- Smart Kenya. (2014, July 28). *Bitpesa Chief Operating Officer, Charlene Chen Talks Bitpesa and Bitcoin*. Retrieved from http://bitcoinagile.com/66DBE/bitpesa-chief-operating-officer-charlene-chen-talks-bitpesa-and-bitcoin_stream
- Swan, M. (2015). *Blockchain; Blueprint for a new economy*. O'Reilly Media, Inc.
- Szegö, G. (2002). Measures of risk. *Journal of Banking & Finance*, 26(7), 1253- 1272.
- Tobin, J. (1958). Liquidity preference as behavior toward risk. , . *Review of Economic Studies*, 65-85.
- van Wijk, D. (2013). What can be expected from the BitCoin? *Working Paper No. 345986*. Rotterdam: Erasmus Rotterdam Universiteit.
- Vyshnavi, R. (2016). Bitcoins as a Payment System: Problems and Challenges Developments with Particular Reference to India. *International Business Management*, 4863 - 4878.
- Wright, A., & Filippi, P. D. (2015). Decentralised Blockchain technology and the rise of flex cryptography. *Decentralised Blockchain Technology*.
- Xiong, J. X., & Idzorek, T. (2010). *Mean-Variance Versus Mean-Conditional Value-at-Risk Optimization: The Impact of Incorporating Fat Tails and Skewness into the Asset Allocation Decision*. Ibbotson Associates, Inc.