STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES (SIMS)
MASTER OF SCIENCE IN DATA SCIENCE AND ANALYTICS
END OF YEAR EXAMINATION
DSA 8205: OPTIMIZATION FOR DATA SCIENCE

DATE: 10th February, 2026 **TIME**: 5:30 - 8:30 pm

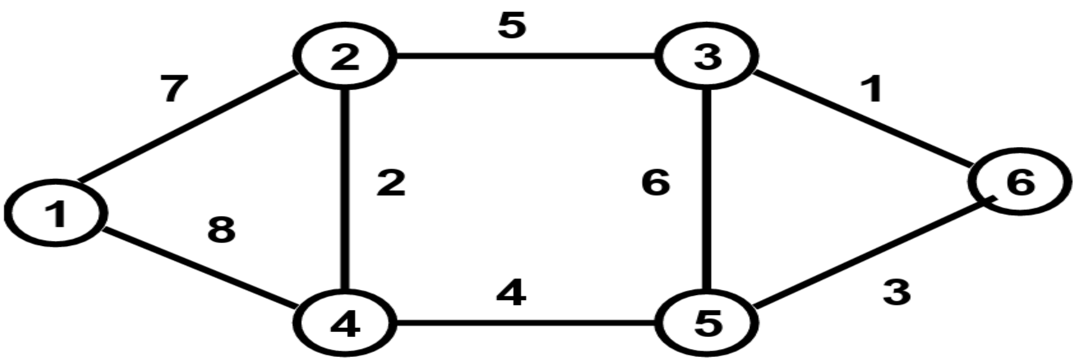
INSTRUCTIONS

1. This examination consists of **FOUR** questions.
2. Answer Question **ONE (COMPULSORY)** and any other **TWO** questions.
3. You may use a **SIMPLE CALCULATOR**. No **MOBILE PHONES** in the exams room.

Question One (30 Marks)

(i) What does it mean to perform sensitivity or "what if" analysis? (3 marks)

(ii) Consider the Figure below. What is the minimum cost spanning tree? List the arcs of the minimum cost spanning tree in the order in which they are added by Kruskal's algorithm. You do not need to show steps of the algorithm. (7 marks)



(iii) Solve the following NLP problem graphically. (6 marks)

$$
\begin{aligned}
\textbf{Maximize} \quad & Z = 2x_1 + 3x_2 \\
\textbf{Subject to:} \quad & x_1 x_2 \le 8 \\
& x_1^2 + x_2^2 \le 20 \\
& x_1 \ge 0; x_2 \ge 0
\end{aligned}
$$

(iv) Use the gradient search method to solve the following. (7 marks)

$$f(\mathbf{x}) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2$$

(v) What is linear programming? (2 marks)

(vi) Explain how one can use Hungarian method an assignment problem. (5 marks)

Question Two (15 Marks)

(i) Given the following quadratic programming problem,

$$\begin{array}{ll} \textbf{Max} & \mathrm{f(x_1, x_2)} = 15\mathrm{x}_1 + 30\mathrm{x}_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2 \\ \textbf{Subject to:} & \mathrm{x}_1 + 2\mathrm{x}_2 \leq 30 \\ & \mathrm{x}_1 \geq 0; \mathrm{x}_2 \geq 0 \end{array}$$

State the Karush-Kuhn-Tucker (KKT) conditions for the above problem. (5 marks)

(ii) Find the solution to the above problem using Karush-Kuhn-Tucker (KKT) method. (10 marks)

$$\begin{array}{ll} \text{Max} & f(x_1, x_2) = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2 \\ \text{subject to} & x_1 + 2x_2 \leq 30, \\ & x_1, x_2 \geq 0. \end{array}$$

Question Three (15 Marks)

(i) Explain five components of a linear programming problem. (5 marks)

(ii) Find the maximum value of the system below using the simplex method (7 marks)

$$\begin{array}{ll} \text{Maximize,} & Z = 20x_1 + 10x_2 \\ \text{subject to} & 4x_1 + x_2 + x_3 = 30, \\ & 2x_1 + 3x_2 + x_3 \leq 60, \\ & x_1 + 2x_2 + 3x_3 \leq 60, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

(iii) Carry out sensitivity analysis of question in (ii) above. (3 marks)

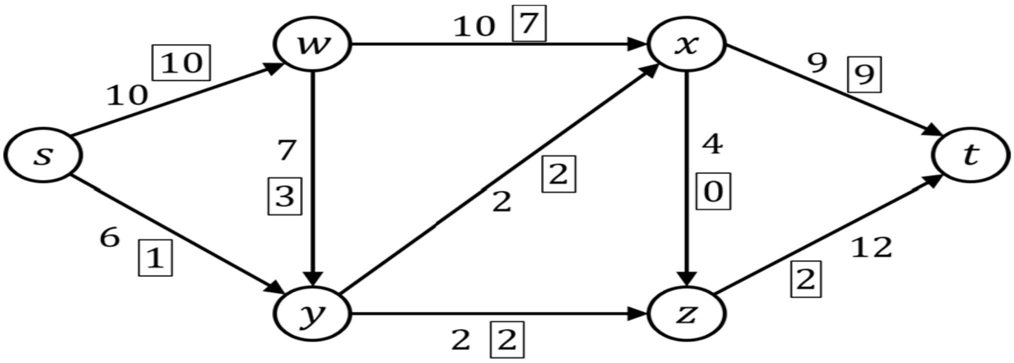
Question Four (15 Marks)

(i) Let $G = (V, E)$ be a flow network, with a source node s, target node t, and positive integer edge capacities $c(e)$ for each edge $e \in E$. For each of the following claims, state whether it is true or false. If it is true, explain why. If it is false, draw (and brie y explain) a ow network that shows a counterexample.

(a) Claim 1: If f is a max flow, then f saturates every edge into t (*i.e.*, $f(e) = c(e)$ for all edges e into t). (3 marks)

(b) Claim 2: Let (A, B) be a min cut. If we multiply the capacity of every edge in G by 2, then (A, B) is still a min cut. (3 marks)

(ii) Here is a flow network. Numbers in boxes represent the amount of ow currently being sent along an edge, and "unadorned" numbers represent edge capacities.



(a) Draw the residual graph for this flow. (3 marks)

(b) What is the definition of an augmenting path? (2 marks)

(c) Is the current flow in this graph/network a max flow? Use the residual graph to explain why or why not. (4 marks)

END