Strathmore
UNIVERSITY

**SCHOOL OF COMPUTING AND ENGINEERING SCIENCES**
**BACHELOR OF SCIENCE IN BUSINESS INFORMATION TECHNOLOGY/ BACHELOR OF SCIENCE IN COMPUTER NETWORKS AND CYBER SECURITY**
END OF SEMESTER EXAMINATION
**BBT 1105/ CNS 1104: Discrete Mathematics**

**Date: 1st November, 2024** **Time: 2 Hours**

**Instructions**

1. This examination consists of **FIVE** questions.
2. Answer **Question ONE (COMPULSORY)** and any other **TWO** questions.

## QUESTION ONE (30 MARKS)

(a) Evaluate the following expression $11000 \wedge (01011 \vee 11011)$ [2 Marks]

(b) Write the dual statement for each of the following

(i) $A = (A \cup B) \cap (A \cup \emptyset)$
(ii) $(\overline{p} \vee \overline{q}) \wedge (F \vee p) \wedge p \equiv p \vee \overline{q}$ [4 Marks]

(c) List the elements of the following sets [3 Marks]

(i) {x: x is a positive integer less than 12}
(ii) $\{x \in \mathbb{Z} : \; -3 \leq x < 7\}$
(iii) {x: x is an integer such that $x^2 = 2$}

(d) Given the function $h(x) = 4x^2 - 7x + 1$, find:

(i) $h(-3)$ [2 Marks]
(ii) $h(1 - 3t)$ [2 Marks]

(e) A website password consists of any 6 letters followed by any 3 digits. Suppose that no letters or digits can be repeated, how many passwords are possible? [3 Marks]

(f) Find the first five terms of the sequence defined by the following recurrence relation and initial conditions

$$a_n = a_{n-1} + 3a_{n-2}, \quad a_0 = 1, \quad a_1 = 2$$

[2 Marks]

(g) Determine whether f is a function from the set of all bit strings to the set of integers in each of the following cases. Provide a reason for your answer.

(i) f(S) is the position of 0 in bit S

(ii) f(S) is the number of 1 bits in S [4 marks]

(h) Draw the graph represented by the following adjacency matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$ [3 Marks]

(i) Sixteen persons have first names Amani, Pendo, Nathan, Tanya and last names Robertson, Locken and Carpenter. Show that at least two people have the same first and last names. [3 Marks]

(j) Is it possible for a function from $\{1, 2\}$ to $\{a, b, c, d\}$ to be onto? Explain [2 Marks]

## QUESTION TWO (20 MARKS)

(a) Translate each of the following statements into logical expressions using predicates, quantifiers and logical operators. Let the domain consist of students in the Discrete Mathematics class [3 Marks]

(i) Everyone in the Discrete Mathematics class has a smart phone.

(ii) Somebody in the class knows ten programming languages

(iii) There is a student in the class who plays soccer but cannot swim

(b) For each of the following propositions determine whether it is a tautology, contradiction or a contingency

(i) $p \implies \neg q$ [2 marks]

(ii) $(p \implies q) \vee (\neg p \implies q)$ [4 Marks]

(c) Verify the following statement of logical equivalence. [6 Marks]
$(\neg p \wedge \neg q) \vee (T \wedge p) \vee p \equiv p \wedge \neg q$

(d) Establish the validity of the argument with the following four premises

$$p \implies q; \quad q \implies (r \wedge s); \quad \neg r \vee (\neg t \vee u); \quad p \wedge t$$

and conclusion

$$\therefore u$$

[5marks]

## QUESTION THREE (20 MARKS)

(a) Determine the domain of the following functions

(i) $z(x) = x^2 + x - 3$ [1 Marks]

(ii) $h(x) = \sqrt{x-8}$ [2 Marks]

(iii) $k(x) = \frac{x-1}{(x-1)(3x+1)}$ [3 Marks]

(b) Given $f(x) = 4x - 1$ and $g(x) = \sqrt{3+2x}$, find

(i) $(f \circ g)(x)$ [2 Marks]

(ii) $(f \circ f)(x)$ [2 Marks]

(iii) $(f \circ g)(11)$ [2 Marks]

(c) Determine the inverse of the function $v(x)$, where [3 Marks]

$$v(x) = x^3 + 5$$

(d) How many bytes are required to encode 3500 bits of data? [2 Marks]

(e) The resale value $R$ (in dollars) of a certain type of industrial equipment has been found to behave according to the function $R(t) = 900,000e^{-0.01t}$, where $t$ is the number of years since original purchase. Determine the number of years it will take for the value of the piece of equipment to reach $ 600,000. [3 Marks]

## QUESTION FOUR (20 MARKS)

(a) What can you say about sets A and B if we know that

(i) $A \cup B = A$

(ii) $A \cap B = A$

(iii) $A - B = B - A$ [3 Marks]

(b) Show that $A \cap (B \cap \overline{A}) = \emptyset$ [3 Marks]

(c) Find $\bigcup_{i=1}^{\infty} A_i$ if for every positive integer i,

(i) $A_i = \{-i, i\}$

(ii) $A_i = [-i, i]$, that is the set of real numbers x with $-i \leq x \leq i$ [4 Marks]

(d) A mobile service provider surveyed 200 of its customers to determine the way they learned about a new tariff. The survey showed that 90 learned about the tariff from radio, 95 from television, 95 from newspapers, 40 from radio and television, 45 from radio and newspapers, 25 from television and newspapers, and 15 from all three types of media. Find,

(i) the number of customers who learned of the tariff from at least one of the three types of media

(ii) the number of customers who learned of the tariff from exactly two of the three types of media [4 Marks]

(e) Prove by the Principle of Mathematical Induction that for any positive integer n,

$$1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$$

[6 Marks]

## QUESTION FIVE (20 MARKS)

(a) Find a solution to the following recurrence relation $a_n = a_{n-1} + 2, \quad a_0 = 3$ [4 Marks]

(b) Let a, b and c be integers. Show that if $a|b$ and $b|c$ then $a|c$ [3 Marks]

(c) Determine whether the relations represented by the following adjacency matrices are equivalence relations. In each case provide reasons for your answer. (i) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ [6 Marks]

(d) Suppose that a connected planar graph has eight vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph? [3 Marks]

(e) Use the Euclidean algorithm to find $gcd(1529, 14038)$ [4 Marks]

**END OF PAPER**